Using Bayes Theorem to Get Probability Inputs for Decision Tree Analysis

Problem 15.1 Money bags

Bayes’ Theorem

Let E be the event of choosing the $640 bag, Ê the complementary (choose the $280 bag), $10 the event of drawing a $10 bill from the bag, $1 the event of drawing a $1 bill from the bag.

\[
P(E) = P(10) = 0.5 \cdot \frac{60}{100} + 0.5 \cdot \frac{20}{100} = 0.4
\]

\[
P(E \mid 10) = \frac{P(E \cap 10)}{P(10)} = \frac{0.5 \cdot 0.6}{0.4} = 0.75
\]

So you should choose the bag from which you draw the $10 bill.

Likelihood ratios

We know the following probabilities:

\[
P(10 \mid E) = 0.6 \quad P(1 \mid E) = 0.4
\]

\[
P(10 \mid Ê) = 0.2 \quad P(1 \mid Ê) = 0.8
\]

Note: these probabilities are complementary.

So we can compute the conditional likelihood ratios and the likelihood ratios for a single observation:
Then deduce the probability of picking the good bag:

\[
P(E/\{\$10, \$1, \$1\}) = \frac{LR_N}{1 + LR_N} = 0.43
\]

This time, choose the other bag, which has a probability of being the good bag of 1-0.43 = 0.57.

**Another $10 bill?**

\[
LR_N = LR_0 \cdot (CLR_{s10})^2 \cdot (CLR_{s1})^2 = 1 \times 3 \times 0.5^2 = 2.25
\]

\[
P(E/\{\$10, \$10, \$1, \$1\}) = \frac{LR_N}{1 + LR_N} = 0.69
\]

Since there is more $10 in the $640 bag, drawing another $10 bill from the same bag increases the probability that the bag from which you have drawn the two $10 bills is the good bag. So pulling out a second $10 bill would change your choice from part (b) above: choose the bag from which you have drawn the two $10 bills.

**Problem 16.2 Money bags, Take 2**

Probabilities needed to populate the tree:
\[ P(20/320) = 0.25 \quad P(100/320) = 0.75 \]
\[ P(20/1000) = 0 \quad P(100/1000) = 1.0 \]

\[ P(20) = P(20/320)P(320) + P(20/1000)P(1000) = 0.125 \]
\[ P(100) = 1 - P(20) = 0.875 \]
\[ P(1000/100) = \frac{P(100/1000)}{P(100)} P(1000) = 0.57 \]
\[ P(320/100) = \frac{P(100/320)}{P(100)} P(320) = 0.43 \]
\[ P(1000/20) = 0 \]
\[ P(320/20) = 1.0 \]

Please refer to the following decision tree established using Data TreeAge. The tree provides expected values of your \textbf{gain at the end of the game, taking into account the cost of the test}. Based on expected value, tree says: \textbf{take the wallet without doing the test}. Your expected gain is $660, which is $60 in addition to the $600 from the money bag.