LP and Sensitivity Analysis

**Problem 5.5. Meat Market**

Let $X_1$ denote the amount of regular hamburger made and sold for $p_1$ cents
$X_2$ denote the amount of premium hamburger made and sold for $p_2$ cents
$B_1$ denote the amount of beef purchased at $c_1$ cents
$B_2$ denote the amount of beet purchased at $c_2$ cents
$O$ denote the amount of oatmeal purchased at $c_3$ cents
$L$ denote the limit of beef available at $c_1$ cents
$D$ denote the budget in dollar

Max: \[ \text{Profit} = (p_1 X_1 + p_2 X_2) - (c_1 B_1 + c_2 B_2 + c_3 O) \]

Subject to:
\[
\begin{align*}
0.8X_1 + 0.7X_2 &\leq O \\
0.2X_1 + 0.3X_2 &\leq B_1 + B_2 \\
B_1 &\leq L \\
C_1 B_1 + C_2 B_2 + C_3 O &\leq 100 \cdot D \\
X_1, X_2, B_1, B_2, O &\geq 0
\end{align*}
\]

**Problem 6.6. Iron Alloys**

a) current price of master alloy #3 = $238
   opportunity cost of master alloy #3 = $17
   price to consider using = $238 - $17 = $221

b) cost of using steel scrap #1
   opportunity cost at $39/ton = $6.7/ton
   opportunity cost at $38/ton = $6.7/ton - $1/ton = $5.7/ton
   cost of using 0.1 ton steel scrap #1 = 0.1 \times 5.7 = $0.57

   value of relaxing composition constraint
   shadow price of relaxing carbon content = 2.4 ($/ton/\%) \text{ (Range: 3.3 – 3.8)}
   value of relaxing by 0.2\% = 0.2 \times 2.4 = $0.48

   Not accept. We will lose $0.57 - $0.48 = $0.09 if accepting.
c) Option 1:
0.2% more Chromium. Value = 0.2 \times 6.1 = $1.22/ton

Option 2:
0.1% more Chromium and 0.1% more Manganese.
Value = 0.1 \times 6.1 + 0.1 \times 5.0 = $1.11/ton
(0.2% more Manganese have a value of 0.2 \times 5.0 = $1.0/ton?)
But that exceed the range of (1.5% - 1.7%) for the shadow price of $5.0/ton/\%.
The shadow price will drop beyond the range when we relax the constraint, so the total value is less than $1.0/ton)