NPV Assignment

Context: You are a homeowner in Cambridge MA looking for a home furnace. You have to choose between a natural gas furnace and an oil burner.

Part 1 – Determine your opportunity cost of capital (OCC)

You now have $15,000 in cash.

You wish to determine your discount rate for the heating device. You evaluate this rate based on other opportunities you would have to give up in order to buy the furnace. Looking at your financial records, the following investment opportunities are available:

- Since you are an avid Patriots fan, you could buy two season tickets for $2,500 and get a 10% discount;
- You could insulate your house for $10,000, which would save about $1,500/yr in fuel cost;
- You could pay off your $50,000 30-year mortgage at a 8% yearly rate; and
- You could lend $10,000 to two MIT alums starting a new business off of their graduate research. They guarantee the amount to double in 10 years.

Given the cash available, and assuming 2% annual inflation, what is your minimum OCC for the heating device?

Solution: The Patriots tickets provide an annual return of 10%, net of inflation. This return becomes 12% with inflation. To see this, if tickets are bought a year later at the same price, they are worth less than $2,500 in present value term due to inflation. This is like saving an additional 2% on the ticket cost (you pocket the inflation return). It can also be argued that the return is 10% if tickets are bought now instead of a year later, which does not affect ranking of opportunities. House insulation provides a 15% annual return net of inflation, which gets to 17% with inflation (with same reasoning as above). The mortgage payment provides 8% annual return, including inflation (8% is a nominal rate and banks typically manage to include inflationary pressure). The business investment opportunity provides approximately 7% annual return nominal, thus including inflation:

\[
10,000(1 + r)^{10} = 20,000 \Rightarrow r = \frac{\frac{1}{2}^{\frac{1}{10}}}{1} = 7\%
\]

Rank ordering the projects gives:

<table>
<thead>
<tr>
<th>Project</th>
<th>Size, $</th>
<th>Annual return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insulation</td>
<td>10,000</td>
<td>17%</td>
</tr>
<tr>
<td>Patriots tickets</td>
<td>2,500</td>
<td>12%</td>
</tr>
<tr>
<td>Mortgage</td>
<td>50,000</td>
<td>8%</td>
</tr>
<tr>
<td>Business</td>
<td>10,000</td>
<td>7%</td>
</tr>
</tbody>
</table>
For a $15,000 investment, your OCC is \((0.17 \times 10,000 + 0.12 \times 2,500 + 0.08 \times 2,500)/15,000 \approx 15\%\).

**Part 2 – Choice of technology**

Assume that the natural gas burner costs $6,000, and the oil burner only $5,000. Based on U.S. average heating consumption census data\(^1\), you estimate your annual house consumption to about 60 million BTUs. You also estimate the natural gas and heating oil prices using current spot prices on the New York Mercantile EXchange (NYMEX)\(^2\) – as of the day you are doing the exercise. Since you really enjoy Cambridge, you plan on living here for the next forty years.

1. Since other investment opportunities would provide a return at least equal to your OCC, you use this rate to discount future expenses and find the cost NPV for each technology choice.

Note here that if students make the case that OCC for the first $5,000-$6,000 among the list of available investments is 17\% - which corresponds to $10,000 investment opportunity in insulation - it is accepted. The point here is to use the weighted OCC because you are looking at any investment opportunity that have to give you at least the return provided by already available investment opportunities. This may point may however not have been clarified in writing the assignment, which is why it is accepted.

2. As done in the Asphalt vs. Concrete lecture, quantify the effect of the choice of discount rate over time. Using Data Tables in Excel®, measure the NPV difference between the two technologies (NPV\(_{\text{gas}}\) – NPV\(_{\text{oil}}\)) for discount rates between 0\% and 25\%.

See Excel spreadsheet for questions 1 and 2.

3. For low discount rates, which technology is best? As the discount rate increases, is it as obvious which one to choose? Discuss the effect of the discount rate on investment choice. How does that relate to technology and policy making?

For low discount rates, the natural gas technology is more appropriate since the fuel costs less, even though the burner is more expensive. As the discount rate increases, the impact of future fuel expenses is lessened. Thus, the benefit from lower natural gas prices diminishes with higher discount rates to slowly converge towards NPVs equal to the heating oil device, which requires a higher fuel price. Thus the curve showing the difference in NPVs converges toward zero.

4. Cut the valuation horizon to twenty years. Using your OCC, how is the NPV affected? Why? For discount rates between 0\% and 25\%, what do you observe?

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1. [http://www.eia.doe.gov/emeu/recs/recs97_ce/t2_4c.html](http://www.eia.doe.gov/emeu/recs/recs97_ce/t2_4c.html)
2. Suggested website: [http://www.oilnergy.com/1gnymex.htm](http://www.oilnergy.com/1gnymex.htm). Note that 1 gallon of heating oil is equivalent to 139,000 BTUs.
As noted in the Asphalt vs. Concrete lecture slides, cutting the time horizon does not change much the NPV for a fixed and relatively high discount rate of 15% (both curves converge to the same NPV difference). This is because expenses incurred after year 20 do not weigh much in the NPV valuation with a high discount rate. For low discount rates however (0%-15%), future expenses after year 20 weigh more in the valuation. Therefore, for low discount rates, the NPV difference between the two technologies is larger for a 40 year horizon than for a 20 year horizon.

Purposes of the assignment:
  a. Determine your OCC and use it in technology investment decision.
  b. Get acquainted with typical NPV calculations.
  c. Get practical feel for the long-term effect of discount rate on choice of technology.
  d. Realize that policy-makers and/or designers may affect this choice through selection of discount rate.