Utility Assessment

- Basic Axioms
- Example
- Interview Process
- Procedures
  - Conventional
  - New
- Discussion

Utility Function - U(X)

- Definition:
  - U(X) is a Special V(X),
  - Defined in an Uncertain Environment

- It has a Special Advantage
  - Units of U(X) DO measure relative preference
  - CAN be used in meaningful calculations
Basic Axioms of $U(X)$ (1)

- Probability
  - Probabilities exist - can be quantified
  - More is better

\[
A = \frac{p'}{1-p'} X_1 \quad B = \frac{p''}{1-p''} X_1
\]

If $X_1 > X_2$: $A > B$ if $p' > p''$

is preferred to

Basic Axioms of $U(X)$ (2)

- Preferences
  - Linear in Probability
    (substitution/independence) - Equals can be substituted if a subject is indifferent between $A$ and $B$

\[
p \quad \frac{A}{C} = \quad \frac{B}{C}
\]

Not a good assumption for small $p$ (high consequences)!
Cardinal Scales (1)

- Units of interval are equal, therefore averages and arithmetic operations are meaningful

- Two types exist
  - Ratio
    Zero value implies an absence of phenomenon
    e.g., Distance, Time
    note: \( F'(x) = a F(x) \)
    defines an equivalent measure (e.g., meters and feet)

Cardinal Scales (2)

- Ordered Metric
  Zero is relative, arbitrary for example: Temperature

- define two points:
  0 degrees C - freezing point of pure water
  100 degrees C - boiling point of pure water at standard temperature and pressure
  0 degrees F - freezing point of salt water
  100 degrees F - What?
  Note: \( f'(x) = a f(x) + b \) (e.g. \( F = (9/5) C + 32 \))
  equivalent measures under a positive linear transformation
Consequences of Utility Axioms

- Utility exists on an ordered metric scale
- To measure, sufficient to
  - Scale 2 points arbitrarily
  - Obtain relative position of others by probability weighting -- Similar to triangulation in surveying
  - For Example: Equivalent = (X*, p; X*)

How do we Measure Utility?

- Since it is empirical -- Measure
- Since it is personal -- Measure Individuals
- Solution: Some form of Interview
  - oral
  - computer based
Interview Issues (1)

- Put person at ease
  - this individual is expert on his values
  - his opinions are valued
  - there are no wrong answers
  - THIS IS NOT A TEST!!

- Scenario relevant to
  - person
  - issues to be evaluated

Interview Issues (2)

- Technique for obtaining equivalents: BRACKETING

- Basic element for measurement: LOTTERIES
**Nomenclature**

- **Lottery**
  A risky situation with outcomes $0_j$ at probability $p_j$
  Written as $(0_1, p_1; 0_2, p_2; ...)$

- **Binary Lottery**
  A lottery with only two branches, entirely defined by $X_U, p_U, X_L$
  $p(X_L) = 1 - p_U$
  Written as $(X_U, p_U; X_L)$

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**Nomenclature (cont’d)**

- **Elementary Lottery**
  Lottery where one outcome equals zero, that is, the status quo
  written as $(X, p)$
Utility Measurement
Conventional Method

- Certainty Equivalent - Balance $X_i$ and a lottery
  - Define $X^*$ - best possible alternative on the range
  - Define $X_*$ - worst possible alternative on the range
  - Assign convenient values - $U(X^*) = 1.0; U(X_*) = 0.0$
  - Conduct data collection/interview to find $X_i$ and $p$
    - Note: $U(X_i) = p$
    - Generally $p = 0.5$
    - 50:50 lotteries
  - Repeat, substituting new $X_i$ into lottery, as often as desired e.g. $X_2 = (X_1, 0.5; X)$

Utility Measurement
New Method (1)

- Avoid Certainty Equivalents to Avoid “Certainty Effect”
- Consider a “Lottery Equivalent”
  - Rather than Comparing a Lottery with a Certainty
  - Reference to a Lottery is Not a Certainty
- Thus
  - $p_e$ $X^*$
  - $1-p_e$ $X_*$
  - $0.5$ $X_i$
  - $0.5$ $X^*$
- Vary “$p_e$” until Indifferent between Two Lotteries. This is the “Lottery Equivalent”
Utility Measurement
New Method (2)

- Analysis
  \((X^*, P_e; X_i) \sim (X_i; P; X_i)\)

\[ P_e U(X^*) + (1-P_e)U(X_i) = P U(X_i) + (1-P) U(X_i) \]
\[ P_e [U(X^*) - U(X_i)] = P [U(X_i) - U(X_i)] \]
\[ P_e = P U(X_i) \]

\[ U(X_i) = P_e/P; \text{ or } U(X_i) = 2P_e \text{ when } P = 0.5 \]

- Graph

Big Advantage - Avoids Large Errors (+/- 25% of “Certainty Equivalent” Method)

Example of Measurement

- Scenario
  Your rich, eccentric relative offers you \(X\) for sure or a 50:50 chance to get ____

- Bracketing
  if \(X = \) ____
  would you take it?
  would someone else?
  Your indifference point is
  Other person’s is

- Interpretation:
  \[ U(x) \]
  \[ 1 \]
  \[ 0 \]
Lotteries -- Central to Utility Measurement

- Uncertainty
  - Basis for Assessment of Utility
  - Motivates Decision Analysis
- Lottery - Formal Presentation of Uncertain Situation
- Utility Assessment - Compares Preference of Alternative of Known Value with Alternative of Known Value
- How Does One Extract Utility Information from Interview Data?
- How Does One Construct Lottery Basis for Interview?

“Buying and Selling Lotteries”

- Observable Feature of Daily Existence
- Obvious One Include:
  - Buying Lottery tickets
  - Gambling; Other Games of Chance
  - Purchase of Insurance
- Subtler Ones Are:
  - Crossing a Street against the Lights
  - Exceeding the Speed Limit
  - Illegal Street Parking
  - Smoking; Overeating; Drug-Taking
- Question: How to Analyze This Behavior?
Two Basic Lottery Transactions (1)

- Buying of Lotteries
  - In Absence of Transaction, Subject “Holds” an Object of Value
  - In Exchange for the Lottery, Subject Gives Up Valued Object
  - Buying “Price” Defines Net Value of Purchased Lottery

Two Basic Lottery Transactions (2)

- Selling of Lotteries
  - In Absence of Transaction, Subject “Holds” a Lottery
  - In Exchange for the Lottery, Subject Receives a Valued Object
  - Selling “Price” Defines Value of Sold Lottery

- Analytically Distinct Transactions; Must be Treated Differently
Selling Lotteries (1)

- Generally Easier to Understand
- Initially, Subject Holds a Lottery
  Example, You Own a 50:50 Chance to Win $100

\[ \Pr = 0.5 \]

\[ \begin{array}{c}
\$100 \\
\$0 \\
\end{array} \]

Selling Lotteries (2)

- Subject Agrees to Exchange (Sell) this Lottery for No Less Than $SP = Selling Price
  Example: $30

\[ \Pr = 0.5 \]

\[ \begin{array}{c}
$100 \\
$30 \sim \\
$0 \\
\end{array} \]

This is Called an “Indifference Statement”
Selling Lotteries - Alternative View (1)

- Another way to look at lottery transactions is to express them as decision analysis situations. Selling a lottery can be represented as follows:

\[ \text{Keep} \quad \begin{cases} \text{Sell} \quad \$30 & \text{p} = 0.5 \quad \$0 \end{cases} \]

- When the two alternative strategies are equally valued, then we can construct an indifference statement using the two sets of outcomes.

Selling Lotteries Alternative View (2)

- Based on this Indifference Statement, Utility Values can be determined

\[ p = 0.5 \quad \begin{cases} \text{Sell} \quad \$30 & \text{Keep} \quad \$100 \end{cases} \]

\[ \begin{align*}
\text{Set } U(\$0) &= 0.0 \quad \text{and } U(\$100) = 1.0. \\
\text{Translate the Indifference Statement into a Utility Statement: } & U(\$30) = 0.50 \quad U(\$0) \neq 0.50 \quad U(\$100) \\
\text{Solve for } U(\$30) \quad & U(\$30) = 0.50 \ (0) + 0.50 \quad U(\$100) = 0.50
\end{align*} \]
Buying Lotteries (1)

- The “Other” Side of the Transaction
- Subtle, but Critical Analytical Difference
- Source of Difference:
  Buying Price Changes Net Effect of Lottery
- Example: Look at the Buyer in the Last Example
  This Lottery was Purchased for $30
  \[ p = 0.5 \]
  \[ \begin{array}{c} \$100 \\ \hline \$0 \end{array} \]

What is the Appropriate Indifference Statement?
Buying Lotteries (2)

- Indifference Statement
  \[
  p = 0.5 \\
  \$0 \sim \$70 \\
  \$0 \sim \$-30
  \]

  Must Explicitly Consider “Do Nothing” vs Net Outcomes

- Note:
  Net Outcomes, Not Original Outcomes,
  Determine Indifference Statement
  
  Set \( U(-\$30) = 0 \); \( U(\$70) = 1 \)
  
  \[
  U(\$0) = 0.5 \ U(-\$30) + 0.5 \ U(\$70)
  \]
  
  \( U(\$0) = 0.5 \)

Buying Lotteries (3)

- Again, recast the buying situation as a decision tree
  \[
  p = 0.5 \\
  \$100 - \$30 \\
  \$0 - \$30
  \]
  
  \[
  D \\
  \text{Buy} \\
  \text{Do Nothing}
  \]

- If the buyer is just indifferent between the two decision outcomes, then the following indifference statement must hold
  \[
  p = 0.5 \\
  \$0 \sim \$70 \\
  \$0 \sim \$-30
  \]
Buying Lotteries (4)

- Resulting Utility Function is Different

- Seller

- Buyer

- This Should Not be Surprising. If the Utility Functions were Not Different, the Transaction would Not Have Taken Place!

Exercises:

Buying and Selling Lotteries

- Given a Transaction, Generate the Indifference Statement

  - Buy this Lottery for $35
  - Sell this Lottery for $50
  - Pay Someone $30 to Take This Lottery

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Utility</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy this Lottery for $35</td>
<td>-15</td>
</tr>
<tr>
<td>Sell this Lottery for $50</td>
<td>165</td>
</tr>
<tr>
<td>Pay Someone $30 to Take This Lottery</td>
<td>0</td>
</tr>
</tbody>
</table>

  - $200
  - 0.5
  - 0.75
  - 0.5

Engineering Systems Analysis for Design
Richard de Neufville, Joel Clark, and Frank R. Field
Massachusetts Institute of Technology
Utility Assessment
Slide 30 of 32
### Indifference Statements

Let

\[ U(\$165) = 1 \]
\[ U(-\$50) = 0 \]

Then

\[ U(\$0) = 0.50 \]
\[ U(\$50) = 0.75 \]
\[ U(-\$30) = 0.25 \]