Adjusting Discount Rate for Risk

- The Issue

- A simple approach: WACC
  (weighted average cost of capital)

- A better approach: CAPM
  (Capital Asset Pricing Model)

Background: Aversion to Risk

- What is the implication?

- People prefer projects that are less risky

- Conversely, this implies that they require some premium before they will accept risk

- The result is that they will want to adjust discount rate for risk

- See following example
Consider this example...

- Consider two investments of $1000
  - Savings account with annual yield of 5%
  - Stock with a 50:50 chance of $1200 or $900 in a year

<table>
<thead>
<tr>
<th></th>
<th>Bank</th>
<th>Stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10.00</td>
<td>$10.50</td>
<td>0.5</td>
</tr>
<tr>
<td>$12.00</td>
<td>$9.00</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Investors Prefer Less Risk

- Expected returns are identical:
  - Savings account = 5%
  - Stock = \(\left\{0.5\left(1200 + 900\right) - 1000\right\}/1000\) = 5%

- Which would you prefer?

- In general, for same return, investors prefer less risky project

- What if stock had a 75% chance of selling for $1200? At some higher return, the risky project is preferred
General Perspective on Risk vs Return

- Two key observations regarding preferences

  - Non-satisfaction
    - For a given level of risk, the preferred alternative is one with the highest expected return (A > C)

  - Risk Aversion
    - For a given level of return, the preferred alternative is one with the lowest level of risk (A > B)

Adjusting Discount Rate for Risk: Simple Approach

- Weighted Average Cost of Capital (WACC)

- Recall Earlier Discussion: it represents Average Expected Return. First-order formula:
  \[ WACC = R_{equity} \text{ (Equity %)} + R_{bonds} \text{ (Bond %)} \]

- Returns on Equity and bonds depend on how risky the company is. An established proven organization will be able to raise money at lower rates than a start-up, for example.

- Therefore, WACC reflects risk of company...
When is WACC good risk adjustment?

- WACC represents an average for company
- ... So, it may be appropriate for average projects
- What discount rate applies to unique projects?
- More generally, how do we define appropriate discount rates for projects?
- Note: Since projects differ in risk, it is reasonable for a company to use several discount rates!

Adjusting Discount Rate for Risk: A Better Approach

- Development of Capital Asset Pricing Model (CAPM)
  - Assumptions about investor’s view of risky investments
  - Risk characteristics and components
  - Principle of diversification
  - Beta: a formal metric of risk
  - The CAPM relationship between risk and expected return
  - The Security Market Line and expected return for individual investments

- Use of CAPM principles for project evaluation

- Comparison of utility theory and CAPM
Risk Metrics: Empirical Observation

- Risk-free rate defined as return if no variability
- Investments with greater variability (standard deviation) are riskier
- Correlation between variability and expected return
- Suggestive (not current) data:

<table>
<thead>
<tr>
<th>Security</th>
<th>Expected Return %</th>
<th>Variability: Standard Deviation of Expected Returns (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk free</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>U.S Treasuries</td>
<td>7.7</td>
<td>3.3</td>
</tr>
<tr>
<td>Fixed Income</td>
<td>9.0</td>
<td>9.0</td>
</tr>
<tr>
<td>Domestic Equity</td>
<td>12.7</td>
<td>18.5</td>
</tr>
<tr>
<td>International Equity</td>
<td>12.9</td>
<td>19.4</td>
</tr>
<tr>
<td>Real Estate</td>
<td>12.9</td>
<td>16.9</td>
</tr>
<tr>
<td>Venture Capital</td>
<td>18.6</td>
<td>33.0</td>
</tr>
</tbody>
</table>

Greater Variability

=> Greater Expected Return

- An upward trend
Components of Risk

- Useful to recognize 2 types of risks
  - **Market Risk** (systematic, non-diversifiable)
    - Investments tend to fluctuate with outside markets
    - Declines in the stock market generally affect all stocks
  - **Unique or Project Risk** (idiosyncratic, diversifiable)
    - Individual characteristics of investments affect return
    - An investment might be better or worse than overall market trends, because of its special characteristics

- What compensation should investors demand for each type?

Diversification

- A collection of risks (a portfolio) ‘diversifies’ risks (has different ones)

- It reduces Unique Risks

- Why is this?

- Because ups in one project counterbalance downs in others thus lowering variability of portfolio
Role of Diversification

- Consider this example of two stocks:
  - A: Expected return = 20%,
    Standard Deviation of Expected Returns = 20%
  - B: Expected Return = 20%
    Standard Deviation of Expected Returns = 20%

- If portfolio has equal amounts of A and B
  - Expected return = 0.5*20% + 0.5*20% = 20%
  - What is Standard Deviation?
  - In general, it is NOT average of that of individual stocks!

Standard Deviation for a Portfolio

- Portfolio standard deviation is not a weighted average

- Portfolio standard deviation
  \[ \sigma_p = \sqrt{\sum_{i} \sum_{j} x_i x_j \sigma_i \sigma_j \rho_{ij}} \]
  for a portfolio of N investments, with i, j = 1 to N
  \( x_i , x_j \) = Value fraction of portfolio represented by investments i and j
  \( \sigma_i , \sigma_j \) = Standard deviation of investments i and j
  \( \rho_{ij} \) = Correlation between investments i and j
  \( \rho_{jj} = 1.0 \)
Standard Deviation of 2 Stock Portfolio

- Invest equal amounts in two stocks
  - For both A & B: Expected Return = 20%, Standard Deviation = 20%

\[ \sigma_p = \sqrt{0.5(0.5)(0.2)(0.2)(1) + 0.5(0.5)(0.2)(0.2)(1) + 2(0.5)(0.5)(0.2)(0.2)\rho_{ab}} \]

- Portfolio standard deviation depends on correlation of A, B (\(\rho_{ij}\))

<table>
<thead>
<tr>
<th>Correlation Between A &amp; B</th>
<th>Portfolio Standard Deviation</th>
<th>Portfolio Expected Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.0%</td>
<td>20%</td>
</tr>
<tr>
<td>0.5</td>
<td>17.3%</td>
<td>20%</td>
</tr>
<tr>
<td>0</td>
<td>14.1</td>
<td>20%</td>
</tr>
<tr>
<td>-1</td>
<td>0.0%</td>
<td>20%</td>
</tr>
</tbody>
</table>

Conclusions from Example

- Most investments not perfectly correlated (correlation, \(\rho_{ij} < 1\))
- Holding portfolio reduces standard deviation of value of portfolio, thus reduces risk
- With negative correlation, can eliminate all risk
Generalization for Many Stocks

- Formula for standard deviation $\sigma_p$ of portfolio

$$\sigma_p = \sqrt{\sum_i \sum_j x_i x_j \sigma_i \sigma_j \rho_{ij}} = \sqrt{\text{portfolio variance}}$$

- For a portfolio of $N$ stocks in equal proportions ($x_i = x_j = 1/N$)
  - $N$ weighted variance terms, $i = j \rightarrow \sigma_i^2$
  - $(N^2-N)$ weighted cov. terms, $i, j \rightarrow \sigma_i \sigma_j \rho_{ij}$

- $\text{Var}(P) = N^*(1/N)^2 \text{ Average Variance} + (N^2-N)^* (1/N)^2 \text{Average Covariance}$
- $\text{Var}(P) = (1/N) \text{Av. Variance} + [1-(1/N)^*] \text{Av. Covariance}$

Implications of diverse portfolio

$$\sigma_p = \sqrt{(1/N)^* \text{ Average Variance} + (1-(1/N)) \text{Average Covariance}}$$

- For large $N$, $1/N \Rightarrow 0$
  - Average variance term associated with unique risks becomes irrelevant !!!
  - This is fundamentally important: investors in companies do not need to worry about individual risks They can diversify out of them.

  - Average covariance term associated with market risk remains. This is what investors must focus on!
Defining a Formal Measure of Risk

- Investors expect compensation for systematic, undiversifiable (market) risk
- Standard deviation of returns reflects market & unique risks
- Need method to extract market portion of risk
- Define a reference point: the market portfolio (MP), which is the full set of available securities
  \[ r_m = \text{Expected return for MP} \]
  \[ \sigma_m = \text{Standard deviation of expected returns on MP} \]
- Beta: index of investment risk compared to MP:
  \[ \beta_i = \rho_{im} \sigma_i / \sigma_m \]

What Does Beta Imply?

- By definition, the market portfolio has beta = 1.0
- Beta describes the relative variability of returns
  — Concerned with correlated (systematic) portion of returns
  — If investment amplifies movements in MP  \( \beta > 1 \)
  — If attenuates, movements in MP  \( \beta < 1 \)
- Greater Beta reflects market risk of an investment
  \( \Rightarrow \) higher returns for investments with higher betas
- Beta calculated for either individual investments or portfolios
- Portfolio beta = weighted average of individual betas
Efficient Frontier for Investments

- Example demonstrated role of diversification
- Combinations of many securities result in optimum
  - Maximum return for given risk level
  - Minimum risk for given level of return
- Sub-optimal combinations lie below, to right of frontier

Combining Risk-Free and Risky Investments

- Investors can mix risky and risk-free investments to balance return and risk
- For any combination of risk-free and risky investing
  - Expected return is weighted average of risk-free (Rf) and portfolio return (Rp)
  - Standard deviation of Rf = 0
  - $\sigma_{\text{mix}} = x_p \sigma_p$

CAPM Defines Risk Premium

- The line representing best returns for risk is the CAPM line.
- This is crux of Capital Asset Pricing Model -- it gives price (risk premium) for assets.

Determining Discount rate for Individual Investments

- CAPM models maximized expected return
- Beta indexes risk of individual investment to market portfolio
- Market portfolio is tangent point in CAPM
- Relation between beta and individual expected return results in:
Relation of Expected Return and Beta

- **Security Market Line (SML)**
  - \( R_p = R_f + B_p (R_m - R_f) \)
  - \( R_m \) is expected return of market portfolio
  - \( R_m - R_f \) is the market risk premium
  - \( B_p \) = beta of investment to be evaluated

- For the market portfolio, \( B_m = B_p = 1 \)

- For other investments, expected return scales with \( B_p \)

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Implementing the CAPM: From Theory to Project Evaluation

- Project discount rate should be based on project beta
  - Investors can diversify away unique project risks
  - Adjustment apparent if project is carbon-copy of firm (McDonald’s #10,001) \( \implies \) WACC applies

- “Proper” adjustment not trivial on most projects
  - Consider past experiences, returns in comparable industries
  - Detail unique aspects of specific project
  - Apply information to adjust discount rate
A General Rule for Managers

- CAPM translates to a simple rule:
  
  **Use risk adjusted discount rate to calculate NPV for projects,**
  **Accept all positive NPV projects to maximize value**

- Shareholders can avoid unique risks by diversifying, holding multiple assets

- If projects valued properly, wealth is maximized

Difficulties in Practice

- Estimating project beta may not trivial
- Budget constraints conflict with positive NPV rule
- Employees worry about unique project risks
  - Career can be adversely affected by bad outcomes
  - Generally cannot diversify (limited to few projects)
  - Issue might be addressed through proper incentives
- Reliance on past results to dictate future choices
- Individuals and companies are often “risk positive”
  - Entrepreneurs
  - Sometimes may “bet the company”
Comparing Utility Theory with CAPM?

- **Utility**
  - Applies a single discount rate for time value
  - Adjusts for risk preference of decision-maker
  - Utility is bottom-up and focused on individual preferences

- **CAPM**
  - Adjusts discount rate for overall aversion to market risk
  - No adjustment for risk preferences of decision-maker
  - Based on top-down, aggregate perspectives

- **Utility and CAPM**
  - Both value risky opportunities, accounting for risk aversion
  - Do not “double count”, applying utility to CAPM!

Summary

- **CAPM adjusts discount rates for risk**
  - Models maximum expected return for level of risk
  - Based on observations of securities markets
- **Unique risks can be diversified**
- **Investors expect compensation for market risk**
- **Standard deviation of returns reflects both market & unique risks**
- **Beta is index of market part of investment risk**
- **Security Market Line relates expected return to beta**
  - \( R_p = R_f + B_p (R_m - R_f) \)
- **Moving from theory to practice can be problematic**