Information Collection - Key Strategy

- **Motivation**
  - To reduce uncertainty which makes us choose “second best” solutions as insurance

- **Concept**
  - Insert an information-gathering stage (e.g., a test) before decision problems, as a possibility

![Diagram]

- **Operation of Test**
  - **New Information**
    - Revision of Prior Probabilities in Decision Problem
    - New Expected Values in Decision Problem
  - **EV (after test) > EV (without test)**
    - Because we can avoid bad choices and take advantage of good ones, in light of test results

- **Question:**
  - Since test generally has a cost, is the test worthwhile?
    - What is the value of information?
    - Does it exceed the cost of the test?
Essential Concept - it's complex!

- Value of information is an expected value
- Expected Value of information
  \[ = \text{EV (after test)} - \text{EV (without test)} \]
  \[ = \sum_k p_k (D_k^*) - \text{EV (D*)} \]
  Where \( D^* \) is optimal decision without test and \( D_k^* \) are optimal decisions after test, based on \( k \) test results, \( TR_k \), that revise probabilities from \( p_j \) to \( p_{jk} \)
- \( p_k \) = probability, after test, of \( k \)th observation

Example

<table>
<thead>
<tr>
<th>Test</th>
<th>Good</th>
<th>Medium</th>
<th>Poor</th>
</tr>
</thead>
</table>

Revise probabilities after each test result

Expected Value of Perfect Information  EVPI

- Perfect information is hypothetical – but simplifies!
- Use: Establishes upper bound on value of any test
- Concept: Imagine a “perfect” test which indicated exactly which Event, \( E_j \), will occur
  - By definition, this is the “best” possible information
  - Therefore, the “best” possible decisions can be made
  - Therefore, the EV gain over the “no test” EV must be the maximum possible
  an upper limit on the value of any test!
EVPI Example (1)

- Question: Should I wear a raincoat?
  - RC - Raincoat; RC - No Raincoat
- Two possible Uncertain Outcomes
  - (p = 0.4) or No Rain (p = 0.6)

```
    D
     /\          0.4
    C - R - 5
     \ 0.6  __________ 0.6
         /\          /\ 0.4
        R - NR -2  R - NR -10

    RC
```

- Remember that better choice is to take raincoat, EV = 0.8

EVPI Example (2)

- Perfect test
  - Says Rain p = 0.4 Take R/C 5
  - Says No Rain p = 0.6 No R/C 4

Why these probabilities? Because these are best estimates of results. Every time it rains, perfect test will say “rain”

- EVPI

\[
\text{EV (after test)} = 0.4(5) + 0.6(4) = 4.4 \\
\text{EVPI} = 4.4 - 0.8 = 3.6
\]
Application of EVPI

- A major advantage: EVPI is simple to calculate
- Notice:
  - Prior probability of the occurrence of the uncertain event must be equal to the probability of observing the associated perfect test result
  - As a “perfect test”, the posterior probabilities of the uncertain events are either 1 or 0
  - Optimal choice generally obvious, once we “know” what will happen
- Therefore, EVPI can generally be written directly
- No need to use Bayes’ Theorem

Expected Value of Sample Information  EVSI

- Sample information are results taken from an actual test  \( 0 \leq EVSI \leq EVPI \)
- Calculations required
  - Obtain probabilities of test results, \( p_k \)
  - Revise prior probabilities \( p_j \) for each test result \( TR_k \)
    \( \Rightarrow p_{jk} \)
  - Calculate best decision \( D_k^* \) for each test result \( TR_k \)
    (a k-fold repetition of the original decision problem)
  - Calculate EV (after test) = \( \sum_k p_k (D_k^*) \)
  - Calculate EVSI as the difference between EV (after test) - EV (without test)
- A BIG JOB
**EVSI Example (1)**

- Test consists of listening to forecasts
- Two possible test results
  - Rain predicted = RP
  - Rain not predicted = NRP
- Assume probability of a correct forecast = 0.7
  
  \[ p(RP/R) = p(NRP/NR) = 0.7 \]
  
  \[ p(NRP/R) = p(RP/NR) = 0.3 \]
- First calculation: probabilities of test results
  
  \[ P(RP) = p(RP/R) p(R) + p(RP/NR) p(NR) \]
  
  \[ = (0.7) (0.4) + (0.3) (0.6) = 0.46 \]
  
  \[ P(NRP) = 1.00 - 0.46 = 0.54 \]

**EVSI Example (2)**

- Next: Posterior Probabilities
  
  \[ P(R/RP) = p(R) (p(RP/R)/p(RP)) = 0.4(0.7/0.46) = 0.61 \]
  
  \[ P(NR/NRP) = 0.6(0.7/0.54) = 0.78 \]

Therefore,

- \[ p(NR/RP) = 0.39 \] (false positive – says it will happen and it does not)
- \[ p(R/RNP) = 0.22 \] (false negative – says it will not happen, yet it does)
EVSI Example (3)

- Best decisions conditional upon test results

\[
\begin{align*}
EV (RC) &= (0.61) (5) + (0.39) (-2) = 2.27 \\
EV (RC) &= (0.61) (-10) + (0.39) (4) = -4.54
\end{align*}
\]

EVSI Example (4)

- Best decisions conditional upon test results

\[
\begin{align*}
EV (RC) &= (0.22) (5) + (0.78) (-2) = -0.48 \\
EV (RC) &= (0.22) (-10) + (0.78) (4) = 0.92
\end{align*}
\]
**EVSI Example (5)**

- EV (after test) = \( p(\text{rain predicted}) \cdot \text{EV(strategy/RP)} + P(\text{no rain predicted}) \cdot \text{EV(strategy/NRP}) \)
  
  \[
  = 0.46 \cdot 2.27 + 0.54 \cdot 0.92 = 1.54
  \]

- EVSI = 1.54 - 0.8 = 0.74 < EVPI = 3.6

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**Practical Example:**

Is a Test Worthwhile? (1)

- If value is Linear (i.e., probabilistic expectations correctly represent value of uncertain outcomes)
  
  - Calculate EVPI
  
  - If EVPI < cost of test, Reject test
  
  - Pragmatic rule of thumb
    
    - If cost > 50% EVPI, Reject test (Real test are not close to perfect)

  - Calculate EVSI
  
  - EVSI < cost of test, Reject test

  - Otherwise, accept test
Is Test Worthwhile? (2)

- If Value Non-Linear (i.e., EV of outcomes does NOT reflect attitudes about uncertainty)
- Theoretically, cost of test should be deducted from EACH outcome that follows a test
  - If cost of test is known
    A) Deduct costs
    B) Calculate EVPI and EVSI (cost deducted)
    C) Proceed as for linear EXCEPT
      Question is if EVPI(cd) or EVSI(cd) > 0?
  - If cost of test is not known
    A) Use iterative, approximate pragmatic approach
    B) Focus first on EVPI
    C) Use this to estimate maximum cost of a test