Adjusting discount rate for Uncertainty

- The Issue

- A simple approach: WACC
  — Weighted average Cost of Capital

- A better approach: CAPM
  — Capital Asset Pricing Model

Semantic Caution

Uses of the words “risk” and “uncertainty”

- Traditional Engineering assumes
  — variability in outcomes leads to bad events
  — equates uncertainty with downside, with “risk”

- But: variability may give upside opportunity
  — so, we should generally think of “uncertainty”
  — I will try to use this term whenever possible

- This presentation uses “risk” where the economic literature uses this term
Background: Aversion to “Risk”

- What is “risk aversion”?  
- People prefer projects with less variability in return on investment  
- Thus: people require some premium (extra payment) before they will accept projects with more uncertainty  
- The result: people will want to adjust discount rate for uncertainty  

- See example…

Consider this example...

- Consider two investments of $1000  
  - Savings account with annual yield of 5%  
  - Stock with a 50:50 chance of $1200 or $900 in a year

<table>
<thead>
<tr>
<th></th>
<th>Bank</th>
<th>Stock</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>$10.00</strong></td>
<td><strong>$10.00</strong></td>
<td><strong>$10.00</strong></td>
</tr>
<tr>
<td><strong>$10.50</strong></td>
<td><strong>0.5</strong></td>
<td><strong>0.5</strong></td>
</tr>
<tr>
<td><strong>$12.00</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>$9.00</strong></td>
<td></td>
<td></td>
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</tbody>
</table>
Investors Prefer Less Uncertainty

- Expected returns are identical:
  - Savings account = 5%
  - Stock = \( \frac{[0.5\times(1200 + 900) - 1000]}{1000} \times 100\% = 5\% \)

- Which would you prefer?

- In general, for same return, investors prefer project with more reliable, less uncertain returns

- What if stock had a 75% chance of selling for $1200? At some higher return, we prefer uncertain project

General Perspective on Risk vs Return

- Two key observations regarding preferences

  - Non-satisfaction
    - For a given level of risk, the preferred alternative is one with the highest expected return (A > C)

  - Risk Aversion
    - For a given level of return, the preferred alternative is one with the lowest level of risk (A > B)
Adjusting discount rate for Uncertainty -- simple approach

- Weighted Average Cost of Capital (WACC)

- Recall: WACC represents average return
  \[ = R \text{ for equity (Equity %)} + R \text{ on Bonds (Bond %)} \]

- Returns on Equity and Bonds depend on “risk” of company. Established company generally more certain than start-up

- Thus: WACC represents risk of company

When is WACC good risk adjustment?

- WACC represents an average for company

- ... So, it may be appropriate for average projects

- What discount rate applies to unique projects?

- More generally, how do we define appropriate discount rates for projects?

- Note: Since projects differ in risk, it is reasonable for a company to use several discount rates!
Adjusting Discount Rate for “risk”: A Better Approach

- Development of Capital Asset Pricing Model (CAPM)
  - Assumptions about investor’s view of risky investments
  - Risk characteristics and components
  - Principle of diversification
  - Beta: a formal metric of risk
  - The CAPM relationship between risk and expected return
  - The Security Market Line and expected return for individual investments

- Use of CAPM principles for project evaluation

Some Observations on how returns vary with uncertainty

- “Risk-free” rate defined as return if no variability
- Investments with greater variability are riskier
- Variability and expected return are correlated
- Suggestive data from a few years ago:

<table>
<thead>
<tr>
<th>Security</th>
<th>Expected Return %</th>
<th>Variability: Standard Deviation of Expected Returns (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk free</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>U.S Treasuries</td>
<td>7.7</td>
<td>3.3</td>
</tr>
<tr>
<td>Fixed Income</td>
<td>9.0</td>
<td>9.0</td>
</tr>
<tr>
<td>Domestic Equity</td>
<td>12.7</td>
<td>18.5</td>
</tr>
<tr>
<td>International Equity</td>
<td>12.9</td>
<td>19.4</td>
</tr>
<tr>
<td>Real Estate</td>
<td>12.9</td>
<td>16.9</td>
</tr>
<tr>
<td>Venture Capital</td>
<td>18.6</td>
<td>33.0</td>
</tr>
</tbody>
</table>
Greater Variability
=> Greater Expected Return

- An upward trend

A Note on “risk-free” rate

- In one sense the “risk-free” rate is theoretical
  - what investment is entirely free of risk?
  - Note: you may be sure of getting money back, but may have lost due to inflation...

- In options analysis, “risk-free” rate needs a number
  - this is taken to be rate of US Government bonds
  - on grounds that these are safest investments
    (do not ask me to defend this view)
  - this rate depends on life of the bond, that is, the time to maturity (such as 6 months, 10 years...)
Components of Uncertainty

- Useful to recognize 2 types of uncertainties
- Using standard terms:
  - **Market Risk** (systematic, non-diversifiable)
    - Investments tend to fluctuate with outside markets
    - Declines in the stock market generally affect all stocks
  - **Unique or Project Risk** (idiosyncratic, diversifiable)
    - Individual characteristics of investments affect return
    - An investment might be better or worse than overall market trends, because of its special characteristics

- What compensation should investors demand for each type?

Diversification

- A collection of projects (a portfolio) ‘diversifies’ the variability in return (has different ones)

- It reduces Unique Risks

- Why is this?

- Because ups in one project counterbalance downs in others thus lowering variability of portfolio
Role of Diversification

- Consider this example of two stocks:
  - A: Expected return = 20%,
    Standard Deviation of Expected Returns = 20%
  - B: Expected Return = 20%
    Standard Deviation of Expected Returns = 20%

- If portfolio has equal amounts of A and B
  - Expected return = 0.5*20% + 0.5*20% = 20%
  - What is Standard Deviation?

- In general, standard deviation of return on portfolio is NOT average of that of individual stocks!

Standard Deviation for a Portfolio

- Portfolio standard deviation is not a weighted average

- Portfolio standard deviation
  \[ \sigma_p = \sqrt{\sum_i \sum_j x_i x_j \sigma_i \sigma_j \rho_{ij}} \]
  for a portfolio of N investments, with i, j = 1 to N
  \( x_i, x_j = \) Value fraction of portfolio represented by investments i and j
  \( \sigma_i, \sigma_j = \) Standard deviation of investments i and j
  \( \rho_{ij} = \) Correlation between 
  \( \rho_{jj} = 1.0 \)
Standard Deviation of 2 Stock Portfolio

- Invest equal amounts in two stocks
  - For both A & B: Expected Return = 20%, Standard Deviation = 20%

\[ \sigma_p = \sqrt{(0.5)(0.5)(0.2)(0.2)(1)+ (0.5)(0.5)(0.2)(0.2)(1)+(2)(0.5)(0.5)(0.2)(0.2)\rho_{ab}} \]

- Portfolio standard deviation depends on correlation of A, B (\(\rho_{ij}\))

<table>
<thead>
<tr>
<th>Correlation Between A &amp; B</th>
<th>Portfolio Standard Deviation</th>
<th>Portfolio Expected Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.0%</td>
<td>20%</td>
</tr>
<tr>
<td>0.5</td>
<td>17.3%</td>
<td>20%</td>
</tr>
<tr>
<td>0</td>
<td>14.1%</td>
<td>20%</td>
</tr>
<tr>
<td>-1</td>
<td>0.0%</td>
<td>20%</td>
</tr>
</tbody>
</table>

Conclusions from Example

- Most investments not perfectly correlated (correlation, \(\rho_{ij} < 1\))
- Holding portfolio reduces standard deviation of value of portfolio, thus reduces “risk”
- With negative correlation, can eliminate all “risk”
Generalization for Many Stocks

- Formula for standard deviation $\sigma_p$ of portfolio
  \[ \sigma_p = \sqrt{\sum_i \sum_j x_i x_j \sigma_i \sigma_j \rho_{ij}} = \sqrt{\text{portfolio variance}} \]

- For a portfolio of N stocks in equal proportions ($x_i = x_j = 1/N$)
  - $N$ weighted variance terms, $i = j \rightarrow \sigma^2_i$
  - $(N^2-N)$ weighted cov. terms, $i, j \rightarrow \sigma_i \sigma_j \rho_{ij}$

- $\text{Var}(P) = N^2*(1/N)^2 \times \text{Average Variance} + (N^2-N)*$
  \[ (1/N)^2 \times \text{Average Covariance} \]

- $\text{Var}(P) = (1/N) \times \text{Av. Variance} + [1-(1/N)] \times \text{Av. Covariance}$

Implications of diverse portfolio

- For large $N$, $1/N \Rightarrow 0$
  - Average variance term associated with unique risks becomes irrelevant !!!
  - This is fundamentally important: investors do not need worry about uncertainties of individual projects. They can diversify out of them.
  - Covariance term associated with market risk remains. This is what investors must focus on!
Defining a Formal Measure of Risk

- Investors expect compensation for systematic, undiversifiable (market) risk
- Standard deviation of returns reflects market & unique risks
- Need method to extract market portion of risk
- Define reference point: the market portfolio (MPf), which is the full set of available securities
  \[ r_m = \text{Expected return for MPf} \]
  \[ \sigma_m = \text{Standard deviation of expected returns on MPf} \]
- Beta: index of investment risk compared to MPf:
  \[ \beta_i = \rho_{i,m} \sigma_i / \sigma_m \]

What Does Beta Imply?

- By definition, the market portfolio has beta = 1.0
- Beta describes the relative variability of returns
  — Concerned with correlated (systematic) portion of returns
  — If investment amplifies movements in MPf \( \beta > 1 \)
  — If attenuates, movements in MPf \( \beta < 1 \)
- Greater Beta reflects market risk of an investment
  \( \Rightarrow \) higher returns for investments with higher betas
- Beta calculated for either individual investments or portfolios
- Portfolio beta = weighted average of individual betas
Efficient Frontier for Investments

- Example demonstrated role of diversification
- Combinations of many securities result in optimum
  - Maximum return for given risk level
  - Minimum risk for given level of return
- Sub-optimal combinations lie below, to right of frontier

Combining Risk-Free and Risky Investments

- Investors can mix “risky” and “risk-free” investments to balance return and “risk”
- For any combination of risk-free and risky investing
  - Expected return is weighted average of risk-free (Rf) and portfolio return (Rp)
  - Standard deviation of Rf = 0
  - $\sigma_{mix} = x_p \sigma_p$
**CAPM Defines Premium due to Risk**

- The line representing best returns for risk is the CAPM line.
- This is crux of Capital Asset Pricing Model -- it gives price (risk premium) for assets.

**Determining Discount rate for Individual Investments**

- CAPM models maximized expected return.
- Beta indexes risk of individual investment to market portfolio.
- Market portfolio is tangent point in CAPM.
- Relation between beta and individual expected return results in:
Relation of Expected Return and Beta

- **Security Market Line (SML)**
  - \( R_p = R_f + B_p (R_m - R_f) \)
  - \( R_m \) is expected return of market portfolio
  - \( R_m - R_f \) is the market risk premium
  - \( B_p \) = beta of investment to be evaluated
- For the market portfolio, \( B_m = B_p = 1 \)
- For other investments, expected return scales with \( B_p \)

Implementing the CAPM: From Theory to Project Evaluation

- **Theory**: Project discount rate should be based on project beta
  - Investors can diversify away unique project risks
  - Adjustment apparent if project is carbon-copy of firm (McDonald’s #10,001) \( \Rightarrow \) WACC applies
- **Practice**: adjustment not trivial on most projects
  - Consider past experiences, returns in comparable industries
  - Detail unique aspects of specific project
  - Apply information to adjust discount rate
A General Rule for Managers

- CAPM translates to a simple rule:
  
  Use risk adjusted discount rate to calculate NPV for projects, Accept all positive NPV projects to maximize value

- Shareholders can avoid unique risks by diversifying, holding multiple assets

- If projects valued properly, wealth is maximized

Difficulties in Practice

- Estimating project beta may not trivial
- Budget constraints conflict with positive NPV rule
- Employees worry about unique project risks
  - Career can be adversely affected by bad outcomes
  - Generally cannot diversify (limited to few projects)
  - Issue might be addressed through proper incentives
- Reliance on past results to dictate future choices
- Individuals, companies are often “risk positive”
  - Entrepreneurs
  - Sometimes may “bet the company”
Summary

- CAPM adjusts discount rates for uncertainty
  - Models maximum expected return for level of “risk”
  - Based on observations of securities markets
- Unique “risks” can be diversified
- Investors expect compensation for “market risk”
- Standard deviation of returns reflects both market & unique risks
- Beta is index of market part of investment risk
- Security Market Line relates expected return to beta
  \[ R_p = R_f + B_p(R_m - R_f) \]
- Moving from theory to practice can be problematic