Today

- We will review some aspects of
  - NPV
  - Options Pricing
  - Decision Analysis
  - Simulation
- Important: ALL class notes and exercises are relevant for the exam, not only the following few pages!
- We will then further discuss the Antamina case

NPV

- Wish to make an economic case for a project
  - Go ahead with the project if there is no better alternative to spend your money on
- NPV - discounted future cash flows (present value) minus initial investment
- Condition for acceptance of the project is NPV > 0
- Simple and widely used in practice
NPV and discount rates

- But what discount rate should we use?
- Discount rate = risk-free rate + risk premium
- Discount rate increases with risk (measured e.g. by the standard deviation of cash flows)
  - What is the effect of an increased discount rate (due to increased risk) on NPV?
  - Can you imagine an example of a project whose value should increase with an increase in risk?
- Should discount rates be constant over time?
  - Should they change, depending on how the underlying uncertainties unfold? If so, how?

NPV and uncertainty

- The forecast is always wrong...
- The flaw of averages lurks everywhere...
- Should the value of a project be a number or a distribution?
  - Value at risk
- Distribution of NPV can be obtained by Monte Carlo simulation
  - Avoids the flaw of averages

NPV and flexibility

- NPV does not account for managerial activity
- “Value” of managerial flexibility is unclear if value is a distribution rather than a number
- Project design and management from this point of view:
  - Use design parameters and project management variables to shape the distribution of the project value so that it becomes acceptable
  - E.g. may wish to buy insurance to avoid the possibility of big losses, even though the expected value of the project becomes smaller with the insurance
Options

- An option is a right to a future action
  - Right but NOT an obligation
- Payments involved
  - Need to pay for the right NOW
  - Need to pay for carrying out the action (if I decide to do so) in the future

Types of options

- Put-type
  - Take action to guard against losses due to bad circumstances
- Call-type
  - Take action to enhance profits due to good circumstances
- Compound option
  - Rights to various actions to react flexibly to unfolding of the future

The new paradigm: Projects are options

- Real options paradigm: A project is a compound option on underlying uncertainties
  - Market uncertainties (demand, prices)
  - Industry uncertainties (M&A, innovations)
  - Technical uncertainties (R&D, technical risk)
  - Organisational uncertainties (key personnel)
  - Political uncertainties (regulations, wars)
- Multiple actions possible
  - Project management
Consequence of the new paradigm

- Flexibility is key to the value of a project
- Important consequences for project evaluation
  - Conservative NPV view can be way off by not taking flexibility into account!
- Important consequences for project design
  - Can improve the value of the project considerably by building in flexibility

Project valuation: Borrowing from Black-Scholes

- Financial options characteristics
  - Name of underlying asset (S)
  - Number of shares of the underlying asset optioned
  - Type (put or call)
  - Rule of exercise (European or American)
  - Expiration date (T)
  - Exercise price (K)
- Buyer of the option has to pay a price to writer of the option
- What is a fair price of an option?

Gambles

Gamble 1
- £6
- £2
- £3
- £6
- 1/2
- £2
- £2

Gamble 2 (risk free)
- £0
- £2

Option
You have learned...

- why the third gamble can be interpreted as a call option on Gamble 1 as underlying (what’s the strike price?)
- why aligning the expected value of the option with Gamble 1 allows for arbitrage
  - if the option is priced at £0.75 then both the option and Gamble 1 have an expected return of 33%
- how to construct a replicating portfolio for the payoffs of the option using Gamble 1 and the risk-free Gamble 2
- how to price the option by using the prices of the replicating portfolio

You understand

- that the value of the option is independent of the probabilities on the arcs
  - major assumption: payoffs of all three gambles are determined by the same flip of the coin, not three different flips!
- that the value of the option depends on the returns of Gamble 1 for the two scenarios (100% and −33% in the example) and on the risk-free interest rate (10% in the example)

Risk-neutral valuation

- You understand that the value of the replicating portfolio can be written in the form
  
  \[ x+y=q C_u+(1-q) C_d/(1+r) \]
  
  where
  
  \[ q=(1+r-d)/(u-d) \]
  
  with this q (called the risk-neutral probability), the value of the option is the discounted expected value
  - NPV calculation with appropriate probabilities

Page 5
Valuing options in a lattice

You know how to work backwards through a lattice to price an American-type option
- Find risk-neutral probabilities
- Find payoffs of end-nodes
- Repeat single-period risk-neutral valuation for every node, working backwards through the lattice
You know that the lattice valuation also gives you an optimal decision rule for exercising the option
You can do this for a stylized project such as the Simplico gold mine

How to get the u’s and d’s

The lattice model of stock movement gives (approximately) a log-normal distribution for returns
- If $\mu = \log(1+r)$, $\sigma^2 = \text{std dev}^2$, (expected log-return and standard deviation of log-return over 7 periods, say over $T=12$ months)
then this distribution can be (approximately) replicated in a lattice by choosing the parameters
$$u = e^{\mu \sigma \sqrt{T}} \quad d = e^{-\mu \sigma \sqrt{T}}$$
where $m$ is the number of periods that are represented by a single move in the lattice
- E.g., if $T=12$ months and the step length of the lattice is 1 week then $m=1/(52/12)=6/13$
- Only $u$ and $d=1/u$ are of interest in option pricing
- $u$ and $d$ depend only on the standard deviation of the log-returns

Examples

- Luenberger, Investment Science
  - Worked examples 12.7-12.10 in section 12.8
  - P.348, Exercise 9
### Comparison of determinants of financial option and real option

<table>
<thead>
<tr>
<th>Financial option</th>
<th>Real option</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price of underlying asset (stock price)</td>
<td>“Value” of underlying uncertainties (e.g., price of gold, value of cash flows of non-flexible project)</td>
</tr>
<tr>
<td>Exercise price</td>
<td>Cost to carry out the real option</td>
</tr>
<tr>
<td>Time to expiration</td>
<td>Time by which the real option has to be carried out</td>
</tr>
<tr>
<td>Dividend payments</td>
<td>Production revenues that are not re-invested in the project</td>
</tr>
</tbody>
</table>

### Decision analysis

- Financial options framework is
  - Difficult to communicate (where is the arbitrage opportunity?)
  - Narrow (what if there is no natural “value” of the underlying uncertainties?)
- Decision analysis
  - Easy to communicate
  - Well known
  - Very versatile

### You know…

- ...how to set up a decision tree with uncertainty and decision nodes
  - Can apply discount rate to unfolding uncertainties (takes time)
- ...how to evaluate a decision tree, given decision rules for each node (e.g. maximise expected value or maximise the minimal payoff)
- ...what the difference is to financial option pricing
  - Decision tree: typically you decide first and then uncertainty is revealed
  - Option pricing: first uncertainty is revealed and then you decide to exercise or not
- Exercise: Set up lattice valuation of American option as decision analysis problem
  - Set up decision tree
  - What are the probabilities on uncertainty branches?
  - What is the decision criterion
Simulation

- Why are we interested in simulation?
- What are the benefits of simulations?
- What are the limitations?
- How to sample?
  - Make assumptions, e.g., normal distributions?
  - Sample from historic data?
  - Use subjective probabilities?
- How to deal with statistical dependence?
  - Build a model of uncertainties (e.g., lattice model of gold-price movement as in the case of Simplicio)
- What is a sensitivity analysis and why is it necessary?

Exercise

- How do the techniques
  - NPV
  - Financial Options Pricing
  - Decision Analysis
  - Simulation
  complement each other in analysing flexibility and uncertainty in projects?
- Think in terms of examples...

Preparing for an exam...

- This is how I prepared for my exams when I was a student:
  - Build group of 3-4 classmates.
  - Reserve a whole day for a meeting (e.g. over the weekend)
  - Everyone in the group sets an exam before the day of the day of the meeting
  - All of you do the 3 exams under exam conditions on the day of the meeting (use ½ day with breaks)
  - After this, discuss the results over the next ½ day.
  - After the day – go out and have a good drink or two...
- Most important: Setting the exam, not doing it is the most educating activity!
Use the bulletin board...

- Please make use of the bulletin board on the web if you have questions AND please answer your classmates questions if you think you can do that!!!
- I will mainly monitor the process from now on, i.e., will only interfere if I realise that things are going wrong…
- I will continue to answer questions about the projects on the bulletin board