The CAPM
(Capital Asset Pricing Model)

NPV Dependent on Discount Rate Schedule

- Discussed NPV and time value of money
- Choice of discount rate influences decisions
- WACC may be appropriate for average projects
- What discount rate applies to unique projects?
CAPM: A Basis for Adjusting Discount Rates for Risk

- Development of the Capital Asset Pricing Model
  - Assumptions about investor's view of risky investments
  - Risk characteristics and components
  - Principle of diversification
  - Beta: a formal metric of risk
  - The Capital Asset Pricing Model relationship between risk and expected return
  - The Security Market Line and expected return for individual investments

- Use of CAPM principles for project evaluation

- Comparison of utility theory and CAPM

Motivation for CAPM: Investors Prefer Less Risk

- Consider two investments
  - Deposit $10 in a savings account with annual yield of 5%
  - Buy stock for $10 with a 50-50 chance of selling for $12 or $9 in one year

- Which is more attractive to risk-averse investors?
  - Expected return for savings account = 5%
  - Expected return for stock = (0.5*(12+9) - 10)/10*100% = 5%

- For same return, investors prefer less risky savings account

- What if stock had a 75% chance of selling for $12?
Motivation for CAPM (2)

Bank
$10.00 $10.50

Stock
$10.00

0.5 $12.00

0.5 $9.00

How do Investors Regard Risk and Return?

- Two key observations regarding preferences

- Non-satisfaction
  - For a given level of risk, the preferred alternative is one with the highest expected return ($A > C$)

- Risk Aversion
  - For a given level of return, the preferred alternative is one with the lowest level of risk ($A > B$)
Risk Metrics: An Empirical Observation

- Investors expect compensation for variability (risk)
- Risk-free rate defined as return if no variability
- More risky securities priced to return premium
- Correlation between variability and expected return

<table>
<thead>
<tr>
<th>Security</th>
<th>Expected Return %</th>
<th>Variability: Standard Deviation of Expected Returns (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk free</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>U.S Treasuries</td>
<td>7.7</td>
<td>3.3</td>
</tr>
<tr>
<td>Fixed Income</td>
<td>9.0</td>
<td>9.0</td>
</tr>
<tr>
<td>Domestic Equity</td>
<td>12.7</td>
<td>18.5</td>
</tr>
<tr>
<td>International Equity</td>
<td>12.9</td>
<td>19.4</td>
</tr>
<tr>
<td>Real Estate</td>
<td>12.9</td>
<td>16.9</td>
</tr>
<tr>
<td>Venture Capital</td>
<td>18.6</td>
<td>33.0</td>
</tr>
</tbody>
</table>

Relationship Between Variability and Expected Return

- An upward trend
- Argument based on aggregate performance of groups
- CAPM models expectations for individual investments
Components of Risk

- Finance defines 2 risks (standard deviation)
- Market Risk (systematic, non-diversifiable)
  - Investments tend to fluctuate with outside markets
  - Declines in the stock market and the price of Microsoft might be correlated
- Unique or Project Risk (idiosyncratic, diversifiable)
  - Individual characteristics of investments affect return
  - Microsoft might increase in price, despite a decline in the overall stock market
- Diversify by holding a portfolio of many investments
- What compensation should investors demand for each type?

Role of Diversification

- Unique risks are reduced by holding an investment portfolio
- Example: Two Stocks
  - A: Expected return = 20%, Standard Deviation of Expected Returns = 20%
  - B: Expected Return = 20%
    Standard Deviation of Expected Returns = 20%
- Consider portfolio with equal amounts of A and B
  - Expected return = 0.5*20% + 0.5*20% = 20%
  - Standard Deviation?
Standard Deviation for a Portfolio

- Portfolio standard deviation is not a weighted average.

- Portfolio standard deviation

\[ \sigma_p = \sqrt{\sum_i \sum_j x_i x_j \sigma_i \sigma_j \rho_{ij}} \]

for a portfolio of N investments, with i, j = 1 to N

\(x_i, x_j\) = Value fraction of portfolio represented by investments i and j

\(\sigma_i, \sigma_j\) = Standard deviation of investments i and j

\(\rho_{ij}\) = Correlation between investments i and j

\(\rho_{jj} = 1.0\)

Example: Standard Deviation for a 2 Stock Portfolio

- Invest equal amounts in two stocks
  —For both A & B:  Expected Return = 20%, Standard Deviation = 20%

\[ \sigma_p = \sqrt{(0.5)(0.5)(0.2)(0.2)(1) + (0.5)(0.5)(0.2)(0.2)(1) + (2)(0.5)(0.5)(0.2)(0.2)\rho_{ab}} \]

- Portfolio standard deviation depends on correlation of A, B

<table>
<thead>
<tr>
<th>Correlation Between A &amp; B</th>
<th>Portfolio Standard Deviation</th>
<th>Portfolio Expected Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.0%</td>
<td>20%</td>
</tr>
<tr>
<td>0.5</td>
<td>17.3%</td>
<td>20%</td>
</tr>
<tr>
<td>0</td>
<td>14.1%</td>
<td>20%</td>
</tr>
<tr>
<td>-1</td>
<td>0.0%</td>
<td>20%</td>
</tr>
</tbody>
</table>
Example: Standard Deviation for a 2 Stock Portfolio (2)

- Most investments not perfectly correlated (correlation < 1)
- Holding portfolio leads to risk reduction
- With negative correlation, can eliminate all risk

Generalization for Portfolio with Many Stocks

- General formula for standard deviation of portfolio returns
  \[ \sigma_P = \sqrt{\sum_i \sum_j x_i x_j \sigma_i \sigma_j \rho_{ij}} = \sqrt{\text{portfolio variance}} \]

- For a portfolio of N stocks in equal proportions \((x_i = x_j = 1/N)\)
  - N weighted variance terms, \(i = j \rightarrow \sigma_i^2\)
  - \((N^2-N)\) weighted cov. terms, \(i \neq j \rightarrow \sigma_i \sigma_j \rho_{ij}\)

- \(\text{Var}(P) = N^*(1/N)^2* \text{Average Variance} + (N^2-N)^*(1/N)^2* \text{Average Covariance}\)
- \(\text{Var}(P) = (1/N)*\text{Av. Var.} + [1-(1/N)^*] \text{ Av. Cov.}\)
Generalization for Portfolio with Many Stocks (2)

\[ \sigma_p = \sqrt{\frac{1}{N}} \text{ Average Variance} + (1-\frac{1}{N}) \text{ Average Covariance} \]

- For large \( N \), \( \frac{1}{N} \Rightarrow 0 \)
  - Average variance term associated with unique risks becomes irrelevant
  - Average covariance term associated with market risk remains

Defining a Formal Measure of Risk

- Investors expect compensation for systematic, undiversifiable (market) risk
- Standard deviation of returns reflects market & unique risks
- Need method to extract market portion of risk
- Define a reference point: the market portfolio
  - The full set of available securities
  - \( r_m = \text{Expected return for market portfolio} \)
  - \( \sigma_m = \text{Standard deviation of expected returns on market portfolio} \)
- Beta: index of investment risk compared to market portfolio
  \[ \beta_i = \rho_{im} \sigma_i / \sigma_m \]
What Does Beta Imply?

- By definition, the market portfolio has beta = 1.0
- Beta describes the relative variability of returns
  - Concerned with correlated (systematic) portion of returns
  - If investment amplifies movements in market portfolio beta > 1
  - If attenuates, movements in market portfolio beta < 1
- Beta reflects market risk of an investment
  - Investors expect higher returns for increased market risk
  - Thus, higher returns for investments with higher betas
- Can be calculated for individual investments or portfolios
- Portfolio beta = weighted average of individual betas

Investment Portfolios and the Efficient Frontier

- Example demonstrated role of diversification
- Combinations of many securities result in optimum
  - Maximum return for given risk level
  - Minimum risk for given level of return
  - Assumes no borrowing or lending
- Sub-optimal combinations lie below, to right of frontier
Combining Risk-Free and Risky Investments

- For any combination of risk-free and risky investing
  - Investor can mix investments in portfolio and risk-free to achieve desired return
  - Expected return is weighted average of risk-free (Rf) and portfolio return (Rp)
  - Standard deviation of Rf = 0
  - $\sigma_{mix} = x_p \sigma_p$

CAPM: Selecting a Portfolio to Maximize Returns for Risk

- Infinite number of portfolios, even on efficient frontier
- Tangent point yields optimum
- CAPM shows expected return for investment combinations
Determining Expected Return for Individual Investments

- CAPM models maximized expected return
- Beta indexes risk of individual investment to market portfolio
- Market portfolio is tangent point in CAPM
- Relation between beta and individual expected return results

How Does Expected Return Relate to Beta?

- Security Market Line (SML)
  - \( R_p = R_f + B_p (R_m - R_f) \)
  - \( R_m - R_f \) is the market risk premium
  - \( B_p \) is the beta of the portfolio or investment to be evaluated
- For the market portfolio, \( B_m = B_p = 1 \)
  - Total expected return is \( R_m \)
- For other investments, expected return scales with \( B_p \)
Differences Between Borrowing and Lending Rates

- Not typical to have same rate for borrowing and lending
- Risk-free rate generally unattainable for small investors
- Adjustments to model possible, minor, illustrated below
- Point of tangency shifts

<table>
<thead>
<tr>
<th>Return (%)</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Borrowing</td>
<td></td>
</tr>
<tr>
<td>Lending</td>
<td></td>
</tr>
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Implementing the CAPM: From Theory to Project Evaluation

- Relation between market risk and expected return
  - Investments have market risk and unique risk components
  - Market risk commands premium over risk-free rate
  - Unique risk is managed (averaged out) by diversification
- Project discount rate should be based on project beta
  - Investors can diversify away unique project risks
  - Adjustment apparent if project is carbon-copy of firm (McDonald’s #10,001) ==> WACC applies
- “Proper” adjustment not trivial on most projects
  - Consider past experiences, returns in comparable industries
  - Detail unique aspects of specific project
  - Apply information to adjust discount rate
A General Rule for Managers

- Portfolio theory translates to a simple rule for managers:
- Use risk adjusted discount rate to calculate NPV for projects,
- Accept all positive NPV projects to maximize value
  — Shareholders capable of diversifying unique risks by holding multiple assets
  — Positive NPV implies market risk in projects is expected to be compensated
  — If projects are properly valued, shareholder wealth is maximized

Limitations and Conflicts in Practice

- Estimating project beta may not trivial
- Budget constraints conflict with positive NPV rule
- Employees worry about unique project risks
  — Career can be adversely affected by bad outcomes
  — Not always in a position to diversify (limited to few projects)
  — Issue might be addressed through proper incentives
- Reliance on past performance to dictate future choices
- Individuals and companies are often “risk positive”
  — Entrepreneurs
  — Sometimes only choice is bet the company
How does Utility Theory Compare with CAPM?

- **Utility**
  - Applies a single discount rate for time value
  - Adjusts for risk preference of decision-maker
  - Utility is bottom-up and focused on individual preferences

- **CAPM**
  - Adjusts discount rate for overall aversion to market risk
  - No adjustment for risk preferences of decision-maker
  - Based on top-down, aggregate perspectives

- **Utility and CAPM**
  - Both value risky opportunities, accounting for risk aversion
  - Under the right circumstances, should give same results
  - “No double counting”

Summary

- **CAPM adjusts discount rates for risk**
  - Models maximum expected return for level of risk
  - Based on observations of securities markets

- Unique risks can be diversified

- Investors expect compensation for market risk

- Standard deviation of returns reflects both market & unique

- Beta is index of market part of investment risk

- Security Market Line relates expected return to beta
  - \( R_p = R_f + B_p(R_m - R_f) \)

- Moving from theory to practice can be problematic