Adjusting discount rate for Uncertainty

- The Issue

- A simple approach: WACC
  — Weighted average Cost of Capital

- A better approach: CAPM
  — Capital Asset Pricing Model

Semantic Caution
Uses of the words “risk” and “uncertainty”

- Traditional Engineering assumes
  — variability in outcomes leads to bad events
  — equates uncertainty with downside, with “risk”

- But: variability may give upside opportunity
  — so, we should generally think of “uncertainty”
  — I will try to use this term whenever possible

- This presentation uses “risk” where the economic literature uses this term
Background: Aversion to “Risk”

- What is “risk aversion”?  
- People prefer projects with less variability in return on investment  
- Thus: people require some premium (extra payment) before they will accept projects with more uncertainty  
- The result: people will want to adjust discount rate for uncertainty

- See examples…

Example

- A Simple Game:
  - I Am Ready to Give Away $1 On Coin Toss
  - If Heads, I Give Away; If Tails, I Keep Money
  - Probability of Heads = 50%
    Expected Value = $ 0.50

- How Much Would You, Individually, Pay Me For The Opportunity To Play This Game?
Slightly Different Example

- A Repeat of Simple Game:
  - I Am Ready to Give Away $10 On Coin Toss
  - If Heads, I Give Away; If Tails, I Keep Money
  - Probability of Heads = 50%
    Expected Value = $5

- How Much Would You, Individually, Pay Me For The Opportunity To Play This Game?

Interpretation of Example

- Averages Not The Basis For Most People’s Choice
- People Decide on the Basis of “Real Value” ≡ Utility
- They are “Risk Averse”, their Utility Typically Is Non-Linear
Consider this example...

- Consider two investments of $1000
  - Savings account with annual yield of 5%
  - Stock with a 50:50 chance of $1200 or $900 in a year

<table>
<thead>
<tr>
<th>Bank</th>
<th>$10.00</th>
<th>$10.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock</td>
<td>$10.00</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.5</td>
</tr>
</tbody>
</table>

Investors Prefer Less Uncertainty

- Expected returns are identical:
  - Savings account = 5%
  - Stock = \( \frac{0.5 \times (1200 + 900) - 1000}{1000} \times 100\% = 5\% \)

- Which would you prefer?

- In general, for same return, investors prefer project with more reliable, less uncertain returns

- What if stock had a 75% chance of selling for $1200? At some higher return, we prefer uncertain project
General Perspective on Risk vs Return

- Two key observations regarding preferences
  - Non-satisfaction
    - For a given level of risk, the preferred alternative is one with the highest expected return (A > C)
  - Risk Aversion
    - For a given level of return, the preferred alternative is one with the lowest level of risk (A > B)

Adjusting discount rate for Uncertainty -- simple approach

- Weighted Average Cost of Capital (WACC)
  - Recall: WACC represents average return
    = R for equity (Equity %) + R on Bonds (Bond %)
  - Returns on Equity and Bonds depend on "risk" of company. Established company generally more certain than start-up
  - Thus: WACC represents risk of company
When is WACC good adjustment for uncertainty?

- WACC represents average for company
- … So, it may be right for average projects

- What adjustment right for unique projects?
- More generally, how do we define discount rates for projects in uncertain world?

- Note: Logic is that since projects uncertainties differ, so should their discount rates. A company thus might use several!

Adjusting discount rate for Uncertainty a better approach

- The Capital Asset Pricing Model (CAPM)
  — Assumptions about investor attitudes
  — Components of Uncertainty
  — Principle of diversification
  — Beta – a formal measure of “risk”
  — CAPM relation between return and “risk”
  — Expected return from unique projects
- Use of CAPM for project evaluation
Some Observations on how returns vary with uncertainty

- “Risk-free” rate defined as return if no variability
- Investments with greater variability are riskier
- Variability and expected return are correlated
- Suggestive data from a few years ago:

<table>
<thead>
<tr>
<th>Security</th>
<th>Expected Return (%)</th>
<th>Variability: Standard Deviation of Expected Returns (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk free</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>U.S Treasuries</td>
<td>7.7</td>
<td>3.3</td>
</tr>
<tr>
<td>Fixed Income</td>
<td>9.0</td>
<td>9.0</td>
</tr>
<tr>
<td>Domestic Equity</td>
<td>12.7</td>
<td>18.5</td>
</tr>
<tr>
<td>International Equity</td>
<td>12.9</td>
<td>19.4</td>
</tr>
<tr>
<td>Real Estate</td>
<td>12.9</td>
<td>16.9</td>
</tr>
<tr>
<td>Venture Capital</td>
<td>18.6</td>
<td>33.0</td>
</tr>
</tbody>
</table>

Greater Variability => Greater Expected Return

- An upward trend
A Note on “risk-free” rate

- In one sense the “risk-free” rate is theoretical
  - what investment is entirely free of risk?
  - Note: you may be sure of getting money back, but may have lost due to inflation...

- In options analysis, “risk-free” rate needs a number
  - this is taken to be rate of US Government bonds
  - on grounds that these are safest investments
    (do not ask me to defend this view)
  - this rate depends on life of the bond, that is, the time to maturity (such as 6 months, 10 years...)

Components of Uncertainty

- Useful to recognize 2 types of uncertainties
- Using standard terms:
  - Market Risk (systematic, non-diversifiable)
    - Investments tend to fluctuate with outside markets
    - Declines in the stock market generally affect all stocks
  - Unique or Project Risk (idiosyncratic, diversifiable)
    - Individual characteristics of investments affect return
    - An investment might be better or worse than overall market trends, because of its special characteristics

- What compensation should investors demand for each type?
Diversification

- A collection of projects (a portfolio) ‘diversifies’ the variability in return (has different ones)

- It reduces Unique Risks

- Why is this?

- Because ups in one project counterbalance downs in others thus lowering variability of portfolio

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Role of Diversification

- Consider this example of two stocks:
  - A: Expected return = 20%
    Standard Deviation of Expected Returns = 20%
  - B: Expected Return = 20%
    Standard Deviation of Expected Returns = 20%

- If portfolio has equal amounts of A and B
  - Expected return = 0.5*20% + 0.5*20% = 20%
  - What is Standard Deviation?

- In general, standard deviation of return on portfolio is NOT average of that of individual stocks!
Standard Deviation for a Portfolio

- Portfolio standard deviation is not a weighted average

- Portfolio standard deviation
  \[ \sigma_p = \sqrt{\sum_i \sum_j x_i x_j \sigma_i \sigma_j \rho_{ij}} \]
  for a portfolio of N investments, with i, j = 1 to N
  \[ x_i, x_j = \text{Value fraction of portfolio represented by investments } i \text{ and } j \]
  \[ \sigma_i, \sigma_j = \text{Standard deviation of investments } i \text{ and } j \]
  \[ \rho_{ij} = \text{Correlation between investments } i \text{ and } j \]
  \[ \rho_{jj} = 1.0 \]

Standard Deviation of 2 Stock Portfolio

- Invest equal amounts in two stocks
  - For both A & B: Expected Return = 20%, Standard Deviation = 20%
  \[ \sigma_p = \sqrt{(0.5)(0.5)(0.2)(0.2)(1)+ (0.5)(0.5)(0.2)(0.2)(1)+ (2)(0.5)(0.5)(0.2)(0.2)\rho_{ab}} \]
- Portfolio standard deviation depends on correlation of A, B (\(\rho_{ij}\))

<table>
<thead>
<tr>
<th>Correlation Between A &amp; B</th>
<th>Portfolio Standard Deviation</th>
<th>Portfolio Expected Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.0%</td>
<td>20%</td>
</tr>
<tr>
<td>0.5</td>
<td>17.3%</td>
<td>20%</td>
</tr>
<tr>
<td>0</td>
<td>14.1</td>
<td>20%</td>
</tr>
<tr>
<td>-1</td>
<td>0.0%</td>
<td>20%</td>
</tr>
</tbody>
</table>
Conclusions from Example

- Most investments not perfectly correlated (correlation, $\rho_{ij} < 1$)

- Holding portfolio reduces standard deviation of value of portfolio, thus reduces “risk”

- With negative correlation, can eliminate all “risk”

Generalization for Many Stocks

- Formula for standard deviation $\sigma_p$ of portfolio

\[
\sigma_p = \sqrt{\sigma_i^2 \Sigma_i \Sigma_j x_i x_j \sigma_i \sigma_j \rho_{ij}} = \sqrt{\text{portfolio variance}}
\]

- For a portfolio of $N$ stocks in equal proportions ($x_i = x_j = 1/N$)
  - $N$ weighted variance terms, $i \neq j \rightarrow \sigma_i^2$
  - $(N^2-N)$ weighted cov. terms, $i, j \rightarrow \sigma_i \sigma_j \rho_{ij}$

- $\text{Var}(P) = N^*(1/N)^2* \text{Average Variance} + (N^2-N)^* (1/N)^2* \text{Average Covariance}$

- $\text{Var}(P) = (1/N)^* \text{Av. Variance} + [1-(1/N)^*] \text{Av. Covariance}$
Implications of diverse portfolio

\[ \sigma_p = \sqrt{\frac{1}{N} \text{ Average Variance} + (1-\frac{1}{N}) \text{ Average Covariance}} \]

- For large \( N \), \( \frac{1}{N} \rightarrow 0 \)
  - Average variance term associated with unique risks becomes irrelevant !!!
  - This is fundamentally important: investors do not need worry about uncertainties of individual projects. They can diversify out of them.

- Covariance term associated with market risk remains. This is what investors must focus on!

Defining a Formal Measure of Risk

- Investors expect compensation for systematic, undiversifiable (market) risk
- Standard deviation of returns reflects market & unique risks
- Need method to extract market portion of risk
- Define reference point: the market portfolio (MPf), which is the full set of available securities
  
  \[ r_m = \text{Expected return for MPf} \]
  \[ \sigma_m = \text{Standard deviation of expected returns on MPf} \]

- Beta: index of investment risk compared to MPf:
  
  \[ \beta_i = \rho_{im} \sigma_i / \sigma_m \]
What Does Beta Imply?

- By definition, the market portfolio has beta = 1.0
- Beta describes the relative variability of returns
  - Concerned with correlated (systematic) portion of returns
  - If investment amplifies movements in MPf  beta > 1
  - If attenuates, movements in MPf  beta < 1
- Greater Beta reflects market risk of an investment
  => higher returns for investments with higher betas

- Beta calculated for either individual investments or portfolios
- Portfolio beta = weighted average of individual betas

Efficient Frontier for Investments

- Example demonstrated role of diversification
- Combinations of many securities result in optimum
  - Maximum return for given risk level
  - Minimum risk for given level of return

- Sub-optimal combinations lie below, to right of frontier
Combining Risk-Free and Risky Investments

- Investors can mix “risky” and “risk-free” investments to balance return and “risk”
- For any combination of risk-free and risky investing
  - Expected return is weighted average of risk-free (Rf) and portfolio return (Rp)
  - Standard deviation of Rf = 0
  - $\sigma_{mix} = x_p \sigma_p$

CAPM Defines Premium due to Risk

- The line representing best returns for risk is the CAPM line
- This is crux of Capital Asset Pricing Model -- it gives price (risk premium) for assets
Determining Discount rate for Individual Investments

- CAPM models maximized expected return
- Beta indexes risk of individual investment to market portfolio
- Market portfolio is tangent point in CAPM
- Relation between beta and individual expected return results in:

\[
R_p = R_f + B_p(R_m - R_f)
\]
- \(R_m\) is expected return of market portfolio
- \(R_m - R_f\) is the market risk premium
- \(B_p\) = beta of investment to be evaluated
- For the market portfolio, \(B_m = B_p = 1\)
- For other investments, expected return scales with \(B_p\)
Implementing the CAPM: From Theory to Project Evaluation

- **Theory**: Project discount rate should be based on project beta
  - Investors can diversify away unique project risks
  - Adjustment apparent if project is carbon-copy of firm (McDonald’s #10,001) ==> WACC applies

- **Practice**: adjustment not trivial on most projects
  - Consider past experiences, returns in comparable industries
  - Detail unique aspects of specific project
  - Apply information to adjust discount rate

A General Rule for Managers

- **CAPM** translates to a simple rule:
  - Use risk adjusted discount rate to calculate NPV for projects,
  - Accept all positive NPV projects to maximize value

- Shareholders can avoid unique risks by diversifying, holding multiple assets

- If projects valued properly, wealth is maximized
Difficulties in Practice

- Estimating project beta may not trivial
- Budget constraints conflict with positive NPV rule
- Employees worry about unique project risks
  — Career can be adversely affected by bad outcomes
  — Generally cannot diversify (limited to few projects)
  — Issue might be addressed through proper incentives
- Reliance on past results to dictate future choices
- Individuals, companies are often “risk positive”
  — Entrepreneurs
  — Sometimes may “bet the company”

Summary

- CAPM adjusts discount rates for uncertainty
  — Models maximum expected return for level of “risk”
  — Based on observations of securities markets
- Unique “risks” can be diversified
- Investors expect compensation for “market risk”
- Standard deviation of returns reflects both market & unique risks
- Beta is index of market part of investment risk
- Security Market Line relates expected return to beta
  — $\text{R}_p = \text{R}_f + \beta_p(\text{R}_m - \text{R}_f)$
- Moving from theory to practice can be problematic