Problem 1 (40 points)

The following information is given about air traffic at a particular runway:

(a) Aircraft are classified into 3 types: heavy (H), large (L), and small (S).

(b) Relevant aircraft characteristics are as follows:

<table>
<thead>
<tr>
<th>Aircraft type</th>
<th>Approach speed (knots)</th>
<th>Mix (%)</th>
<th>Runway occupancy time on landing (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>150</td>
<td>30</td>
<td>60</td>
</tr>
<tr>
<td>L</td>
<td>120</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>S</td>
<td>90</td>
<td>20</td>
<td>40</td>
</tr>
</tbody>
</table>

(c) The length of the final approach to the runway is 6 n. miles.

(d) The minimum separation requirements (in nautical miles) between consecutive landing aircraft on final approach are given by the matrix below (rows indicate the leading aircraft and columns the trailing aircraft):

<table>
<thead>
<tr>
<th></th>
<th>H</th>
<th>L</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>4</td>
<td>5</td>
<td>6*</td>
</tr>
<tr>
<td>L</td>
<td>2.5</td>
<td>2.5</td>
<td>4</td>
</tr>
<tr>
<td>S</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

[*= This particular separation applies only when the leading aircraft is at the runway threshold; in other words, when the leading heavy airplane reaches the runway, the trailing small airplane must be at least 6 n. miles behind it; note that all the other separation requirements in the matrix apply throughout the final approach.]

(e) Two landing aircraft cannot occupy the runway simultaneously.

(f) The deviation from assigned separations between successive airborne aircraft on final approach is described by the probability density function (pdf) of Figure 1. Time 0 on this Figure corresponds to the assigned arrival time of the second of the two aircraft at some given point. For example, if a landing aircraft B is supposed to be 100 seconds behind another aircraft A at the time when A reaches the runway threshold, then, according to Figure 1, B will actually be between 80 and 120 seconds behind A and the distance will be distributed statistically as a random variable with a triangular pdf. [We
shall assume, as an approximation, that the deviations for successive pairs of aircraft are statistically independent; in other words, if for the pair A-B (“A followed by B”) the aircraft B is, e.g., 12 seconds "early" (i.e. at -12 seconds in Figure 1) this does not have implications for where aircraft C is, relative to B, in the next pair of landing aircraft B-C.

(g) It is desired by Air Traffic Control (ATC) to leave sufficient additional separation ("buffer") between successive landing aircraft so as to have a probability of violating the minimum airborne separation requirements (see 4 above) equal to 0.10. Note that the same size buffer will be used for all aircraft pairs.

![Fig. 1](image)

Part 1 (10 points): Find the runway’s capacity (arrivals per hour) when it is used only for arrivals. [Note that this is a simple application of the model we presented in the class, with the complication that you now have to compute the length of the buffer \( b \) on the basis of the information provided in (f) and (g). By contrast, in the lecture notes you were simply told that \( b=10 \) seconds.]

[Note: In all the remaining parts of this problem assume that there is no additional separation ("buffer") between arriving aircraft; in other words, now \( b=0 \). Stated differently, we assume that air traffic controllers can always achieve the minimum required separation between landing aircraft, as dictated by the constraints in (c) and (d).]

Assume now that during peak departure periods, ATC sends some Type S aircraft to this runway for take-off. Specifically, they try to insert as many type S departures as possible, between consecutive arrivals while observing the following rules:

(h) The departures will not in any way affect the arrival rate; in other words, separations between successive arrivals, as shown under (d), will not be increased in any way in order to accommodate departures.

(i) A departure inserted between two arrivals can begin its takeoff roll only after the leading arrival has exited the runway and must lift off the runway before the trailing arrival touches down on the runway. Note that the time needed for an arriving aircraft to exit the runway is given by the runway occupancy time on arrival shown under (c). Assume also that the time during which an arriving aircraft occupies the runway,
provides sufficient time for a departing aircraft of Type S to enter the runway and get ready to begin its take-off roll. (In other words, the takeoff roll can begin immediately after the preceding arriving aircraft exits the runway.)

(j) The runway occupancy time on takeoff of type S aircraft is 50 seconds. This is the time from beginning of takeoff roll to lifting off the runway.

(k) If two or more departures are to be inserted between a pair of arriving aircraft, the minimum separation time between consecutive departures of type S aircraft is 50 seconds.

Part 2 (10 points): Compute the number of departures of type S aircraft that can be performed per hour on the runway under the conditions and rules just described.

This airport is sometimes forced to use only a single runway during IFR weather periods. Thus, the runway must accommodate both landings and takeoffs during these periods. Assume that the traffic mix (H, L, S) for departing aircraft is identical with the traffic mix for arriving aircraft. The following rules/assumptions apply:

(l) The local air traffic controllers use an operations-sequencing strategy of alternating landings and takeoffs on the runway. This means that during periods of continuous demand a landing is always followed by a takeoff, which is then followed by a landing, etc. Thus, when the minimum required time gap between two landing aircraft, \(i\) and \(j\), is not sufficient to insert a takeoff, the time gap will be increased by ATC appropriately, so that a takeoff can be inserted.

(m) The minimum separation requirement (in seconds) between two departing aircraft is 90 seconds; in other words, a takeoff cannot be initiated within less than 90 seconds of the initiation of the preceding takeoff. [Note: Under our operations sequencing strategy, exactly one arrival always takes place between two departures.]

(n) Takeoffs wait next to the threshold of the runway. As soon as a landing aircraft crosses the runway threshold, the next departing aircraft enters the runway and prepares for the takeoff run. It takes 30 seconds for a departing aircraft to enter the runway and set up for take-off. (Note that, in the meanwhile, the arriving aircraft that just landed is moving down the runway toward a runway exit.)

(o) A takeoff run cannot begin before the preceding landing aircraft has exited the runway.

(p) Once a takeoff run begins, the runway occupancy time for all departing aircraft (time from the beginning of the takeoff run to lifting off the runway) is equal to 60 seconds. [Note: This assumption supersedes assumption (j) above.]
(q) A landing aircraft is not allowed to cross the runway threshold unless the runway is clear of all landing or departing aircraft. (Note that this is the only "departure-followed-by-arrival" separation requirement.)

(r) Even if there is enough time to insert two or more takeoffs between two landings, the controllers will only insert one takeoff.

**Part 3** (10 points): Find the capacity of this runway (total number of landings and takeoffs per hour) when it is used for both arrivals and departures in the manner described.

**Part 4** (10 points): Suppose that assumption (p) is modified, so that it is now assumed that departing aircraft of type H occupy the runway for 70 seconds, of type L for 60 seconds and of type S for 50. Repeat Part 3 to compute the capacity of the runway (total number of landings and takeoffs per hour).

[Some guidance for Part 3 of this problem:

As the problem states, the strategy used is to alternate arrivals and departures (one departure between every pair of arrivals and vice versa). This means that the separation between successive arrivals may have to be increased sometimes to make possible the insertion of a departure.

To illustrate, let's look at the case of the arrivals pair "H followed by S" (H-S): In this case the required separation between the arriving aircraft due to the airborne separation requirement is given by \((6/90)\times3600\) seconds = 240 seconds. (Note that in this case, the separation requirement applies when the leading airplane (H) has reached the runway threshold, i.e., is about to touch down on the runway -- see problem statement.) At the same time, the trailing arriving airplane (S) cannot touch down on the runway before the leading airplane (H) has cleared the runway (it takes 60 seconds for this) AND the departing airplane has completed its take-off run and has lifted off the runway (this takes an additional 60 seconds). In other words, the required separation in the H-S case is given by max (240, 120) = 240 seconds. This will permit the insertion of a departure between the two arrivals. Note that this does not require increasing the separation between the two arriving aircraft, compared to the arrivals-only problem. However, an increase in the separation between two arriving aircraft may be necessary in other cases.]

**Problem 2:** [Intersecting Runways: A simple version.] (15 points)

An airport consists of two intersecting runways whose approximate geometric layout is shown on Figure 2. Runway 01 is used only for arrivals and runway 10 only for departures. We want to compute the runway capacity of the airport under the conditions described below. (Some of the information provided may not be particularly useful in doing the problem.)
Just one (!) type of aircraft uses the airport. Aircraft of this type are characterized by: a final approach speed of 150 knots; runway occupancy times of 50 seconds on arrival (time between the touchdown of the arriving aircraft and the instant when it exits the runway); and runway occupancy times of 60 seconds on departure (time between the beginning of the takeoff roll of the aircraft and its lift-off from the runway).

The separation required between successive landing aircraft of the type in question while flying on final approach is 4 nautical miles (nmi). This indicates the minimum acceptable distance between successive landing aircraft at any point during their final approach. The length, $n$, of the final approach is 5 nmi.

Successive departing aircraft of the type in question must be separated by 90 seconds between the beginning of take-off rolls.

During peak periods this runway system is operated by alternating arrivals and departures: an arrival on runway 01 is always followed by a departure on runway 10, which is always followed by an arrival on runway 01 and so on. An arrival on runway 01 must be at least 1 nmi away from the beginning of runway 01 at the instant when the preceding departure crosses the intersection of the two runways. Similarly, a departure roll on Runway 10 cannot begin before the preceding arrival crosses the intersection of the two runways.
Assume that all arriving aircraft need 20 seconds from the instant they cross the beginning of Runway 01 to the instant when they cross the intersection of the two runways. Similarly assume that all departing aircraft require 30 seconds from the instant they begin their takeoff roll on Runway 10 to the instant when they cross the intersection of the two runways.

(a) What is the capacity of this system in terms of total number of movements (landings and takeoffs) per hour? Note that air traffic controllers will adjust the separations between successive operations to make sure that no separation requirement mentioned above will be violated. For example, an arriving aircraft must (a) be separated by at least 4 nmi from the preceding arrival during final approach, (b) cannot touch down on the arrival runway before the preceding arrival has exited the runway and (c) must be at least 1 nmi from the beginning of Runway 01 at the instant when the preceding departure crosses the intersection of the runways.

(b) Repeat part (a) with the following data for the characteristics of the aircraft:

- Approach speed = 120 knots
- Runway occupancy time on arrival = 45 seconds
- Runway occupancy time on departure = 50 seconds
- Minimum separation between successive arriving aircraft on final approach = 2.5 nmi
- Minimum separation between beginning of take-off roll of successive departing aircraft = 60 seconds

All other information (mode of operation, times to cross intersections, etc.) same as in part (a).

**Problem 3:** (15 points) [This problem is an illustration of one type of action that is being considered to increase airport capacity.]

Consider once again the data given in the first part of Problem 1 above (arrivals only), but replace assumptions (f) and (g) with the simple assumption that the buffer \( b = 0 \) seconds.

(a) For the sake of simplifying computation, assume that there are only two types of aircraft, H and S, and that type H aircraft constitute 40% of the traffic and type S 60%. Assume that all the other data given in the problem (6 n. mile approach path, separation requirements, approach speeds, runway occupancy times, etc.) are still valid.

(b) Instead of one runway, we have two independent parallel runways for arrivals. (Departures are accommodated on other runways, dedicated to takeoffs.)

**Part 1:** (5 points) Compute the single-runway arrival capacity at this airport assuming a 40%-60% traffic mix at each runway and first-come, first-served sequencing. Then the total arrival capacity of the two independent runways (call this \( A \)) is twice the single-runway capacity you have just computed.
Part 2: (20 points) Can a total arrival capacity greater than $A$ be achieved by judiciously allocating the population of aircraft to the two runways? In other words, suppose air traffic controllers decide not to have a 40%-60% traffic mix on both runways, but instead have different traffic mixes at each of the runways. (Service would still be first-come, first-served at each of the runways.) Can you give a numerical example, in which, by choosing carefully the traffic mix at each of the runways, a total capacity, $B$, which is greater than $A$ can be achieved? (Note, however, that the total capacity, $B$, that you compute must be consistent with a 40%-60% overall traffic mix for the entire airport.) Any example satisfying these conditions ($B > A$ for an overall 40%-60% traffic mix) will receive full credit. You don’t need to produce an elegant solution to this problem, but you should explain your reasoning. In practice, multi-runway airports increase their capacity by judiciously allocating different traffic mixes to their runways.

Problem 4: (30 points)

This is not a “problem” in the traditional sense, but a multiple-choice test developed by the FAA for checking your understanding of the FAA’s Airport Design Standards. Please answer all the questions on the attached FAA test with the exception of questions 10, 24, 27, 28 and 29. The intent here is to give you an incentive to at least familiarize yourself with the Airport Design Standards as described in Chapter 9 of the FAA’s training manual on Airport Planning Criteria -- a large portion of which will be distributed as a class handout. While reviewing that handout, you may also wish to use Sections 9-2, 9-3, 9-5, 9-6, 9-7, 9-8 and 9-9 of the de Neufville and Odoni textbook for background information.