Application Portfolio

Flexibility in Automotive Assembly Systems

Hadi Zaklouta

Fall 2009

Abstract

This application portfolio examines multiple vehicle assembly system designs for producing SUV and small cars. Two dimensions of flexibility were examined, capacity decision making flexibility as well as production flexibility for a total of four system design possibilities. Using a set of demand and demand growth projections derived from past trends in the automotive market, a decision tree analysis and a binomial lattice analysis were conducted to evaluate the benefits of introducing flexibility within a six year time frame. Parameters useful for comparison in the analysis were Expected Net Present Values, min and max NPV and return on investment given by ENPV/CapEx. The analyses indicate that with the given the projected vehicle demands, production flexibility does not increase profitability. However, capacity decision making flexibility does add value to the system by capturing the upside of demand projections. This was more the case in the small car assembly system than the SUV assembly system. The analyses have demonstrated that the binomial lattice model is a more practical tool than the decision tree model despite the necessary approximations and simplifications that had to be undertaken.

Table of Contents

System Description

The engineering system in my analysis is an automotive assembly system for manufacturing SUV and small cars using either separate facilities or shared facilities. A recent trend in the automotive industry has been to switch over from dedicated assembly lines capable of producing single product styles to flexible assembly systems that can produce any of several styles on the same line using shared tools and fixtures. The main motivation for the increased interest in flexible systems lies in the fact that it is better at dealing with demand uncertainties existing in the current automotive markets. One can imagine the extreme case where product A has higher demand than expected and product B less than predicted. In this case, a flexible assembly system could prevent any excess or scarcity of production capacity that would occur in a single style approach. Although a flexible line may be more expensive, this approach may have a positive effect on costs per unit of assembly and profits. In this application portfolio, I wish to explore these anticipated effects in detail.

In this analysis, I wish to compare the profitability performance of a two product assembly system to the performance of a single style assembly system. I will be conducting an expected profit analysis of both systems under uncertain demand/growth conditions. The goal of the analysis is to see if and when production flexibility is desirable.

The two principal design levers in the system are flexibility and capacity choice. The system can be chosen to be either flexible or single style and capacity decisions are made based on expected demand projections. The simplified assembly systems are modeled as parallel lines of equipment, where each line consists of all processes involved in assembling a car. Each assembly line has a maximum annual production capacity and capacity decisions are made by choosing an optimum number of lines that would maximize expected profits. Cost functions will allow us to see how the choice of flexibility and capacities will lead to different unit costs and therefore different profits depending on actualized demand.

Uncertainties that will influence the performance of the system in terms of profitability are the uncertainties in initial demand and projected demand growth rates. Demand that is actualized after the capacity investment decision is made will affect unit cost of assembly, either increasing it by lagging behind production capacity or decreasing it by being higher than anticipated.

This portfolio will evaluate inflexible assembly systems as well as assembly systems designed with flexibilities along the two dimensions described above. It will go through a decision tree analysis as well as a binomial lattice analysis of all system designs to explore the added value of introducing flexibility.

Sources of Uncertainty

The uncertainties the system faces are uncertainties in future demand projections. This is divided into two separate uncertainties, uncertainty in initial/first year demand and uncertainty in the following growth rates. The auto producer is entering the market with its new products and naturally there is uncertainty over initial market share. Moreover, following the first production period the product demands may follow different growth trends.

Demand uncertainties will have direct impacts on per vehicle costs of production and overall profitability given a fixed investment capacity. In the case where demand lags behind a chosen production capacity and there is excess capacity, unit costs are higher than they could be if capacity was less. Similarly, in the case where demand is far ahead of chosen production capacity, there is a missed opportunity of increased profitability.

The problem with predicting automotive demand is that it depends on a lot of uncertain factors including gasoline price, state of the economy, competitive forces from abroad and consumer perception of both the company and the specific product.

This application portfolio considers a hypothetical average US automotive manufacturer. For the first demand period, this manufacturer's expected SUV and small car demand value is set to the total SUV and small car sales in the US in the year 2009 divided by 5; representing the biggest five car manufacturers in the US (GM, Ford, Chrysler, Toyota and Honda). In other words, the company is assumed to operate in an oligopoly with four other competitors, each with an equal share of the market. According to the Wall Street Journal, total market size for small cars and SUVs in the US in the period September 2008-September 2009 was approximately 1.5million small cars and 670,000 SUVs[1]. Therefore the company's first period expected demands are set to be approximately 300,000 for small cars and 130,000 for SUVs. Standard deviations of around 10% of the expected value for the normally distributed annual small car and SUV demands are reasonable given the high volatility of automobile demand in the US illustrated in Figure 1[2]. Large fluctuations in the value of the Dow Jones Index and oil prices as seen in Figures 2-3 and the presumed correlated fluctuating impact on auto sales also justify this standard deviation.

Figure 1: Total Auto Sales in the US market in the period 2008-November 2009.

---

[1] http://online.wsj.com/mdc/public/page/2_3022-autosales.html#autosalesA. Accessed, October 4th, 2009.

[2] Ibid. accessed, November 29th, 2009.

Figure 2: Dow Jones Index in period 2005-2009

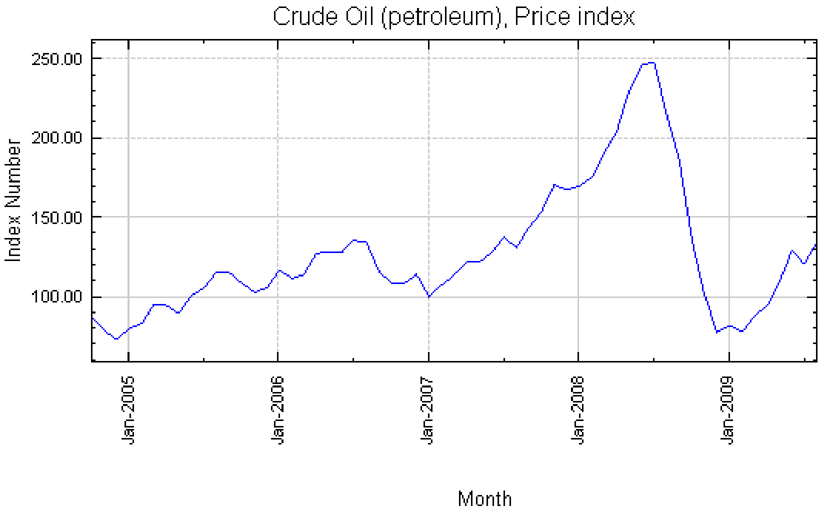


Figure 3: Crude Oil price in period 2005-2009

The second source of uncertainty under consideration is uncertainty in demand growth rates following the initial demand described above. Given the recent downward trend in sales illustrated in the referenced Wall Street Journal article[3], a general stagnation or decline in upcoming expected demand for SUVs and small cars seems reasonable. Meanwhile, the expected demand growth rates chosen for this analysis will depend on first year actualized demand but range from -5% to +4% for small car demand and between -4% and -15% for SUV demand. The exact probabilities of growth rates are illustrated in Figure 5. The referenced Wall Street Journal indicates an annual decline in sales of around 40% for SUVs and around 20% for small cars. This analysis uses values smaller in magnitude because the Wall Street Journal value only indicates a one year trend, one which demonstrated the highest economic downfall and may therefore be unreasonably high. A lack of hard data does not allow for a regression analysis to get growth rate projections but the assumption is that the decline of sales for either product will dampen in the future due to reviving gasoline prices and economic conditions. Moreover, based on the Wall Street Journal record the assumption is that there is a sharper projected decline in SUV demand compared to small car demand perhaps due to concerns about fuel inefficiency and gasoline prices as mentioned earlier.

---

[3] Ibid.

Figure 4 below summarizes the value of the parameters chosen for this analysis. Meanwhile Figure 5 illustrates the probability distribution of first year demand and Figure 6 shows the probabilities of different growth rates given an actualized first year demand scenario. This approach using discrete probabilities rather than a continuous distribution is implemented to make a decision tree analysis and a binomial lattice analysis possible. These probabilities were chosen in a manner that shifts the expected value of growth in a ranked manner from very low to very high rates depending on actualized first year market demand e.g. very high first year market demand will result in a very high expected value growth rate. It is also important to note that all probabilities where chosen in a manner that results in an approximately normal distribution of outcome possibilities.

	Expected Demand in first period	**S.Dev of Exp.Demand**	**Subsequent growth rate**
small cars	300000	10%	-5% to+4%
SUVs	130000	10%	-4 to -15%

Figure 4: Demand parameter estimates for this model

	SUV		**Small Car**	
Market Demand	**Demand**	**P(D)**	**Demand**	**P(D)**
Very High	200000	0.10	425,000	0.10
High	160000	0.25	375,000	0.25
Average	130000	0.30	300,000	0.30
Low	100000	0.25	220,000	0.25
Very Low	60000	0.10	180,000	0.10
EV	130000		300,000	
S.Dev.		12,665		28,755
Standard Deviation (%)		10%		10%

Figure 5: First Year Demand for different market conditions and associated probabilities

YR 1 Market Demand/Growth rate	**SUV growth rates and probabilities**				
	-0.30	**-0.20**	**-0.10**	**0.00**	**0.10**
Very High	0.05	0.1	0.275	0.325	0.25
High	0.05	0.15	0.35	0.3	0.15
Average	0.1	0.2	0.3	0.225	0.175
Low	0.15	0.225	0.35	0.2	0.075
Very low	0.2	0.3	0.3	0.15	0.05

YR1 Market Demand/Growth rate	**Small Car growth rates and probabilities**				
	-0.20	**-0.10**	**0.00**	**0.08**	**0.15**
Very High	0.05	0.1	0.275	0.325	0.25
High	0.05	0.15	0.35	0.3	0.15
Average	0.1	0.2	0.3	0.225	0.175
Low	0.15	0.225	0.35	0.2	0.075
Very low	0.2	0.3	0.3	0.15	0.05

Figure 6: Probabilities of subsequent year growth rates given actualized year 1 demands

System Design Description

The vehicle assembly system designs in this analysis are organized along the two dimensions of production flexibility and capacity decision making flexibility. This analysis explores how these two dimensions impact the profitability of the system.

Assembly line production flexibility refers to whether the assembly lines can produce single style vehicle or whether they can accommodate more vehicle designs. As mentioned in the first AP section, the advantage of this flexible approach is that under uncertain demand scenarios one may be able to minimize excess capacity when the vehicle demands are negatively correlated. In the case where, for example, the demand of one product declines, the other product’s demand will grow proportionally, a single style assembly line approach would result in excess capacity for the first product’s assembly line and insufficient capacity for the second vehicle. This translates into higher per unit cost for the first product and a missed profit opportunity with regards to the second product. A multi-style flexible assembly line that can produce both products eliminates both these problems by having one product’s fall in demand and production compensated by another product’s increase in demand and production. In this analysis the optimum production capacities for the single style and multi style assembly systems are chosen for a situation where decision making is either inflexible or flexible over the systems’ production lifetime.

Capacity decision making flexibility refers to whether production capacity decisions can be adjusted based on observed demand. For the sake of simplicity, the assembly plant is modeled as a parallelized line system in which capacity is defined in integer multiples of a single line maximum assembly output. In the first scenario, the investment decision is fixed in a sense that no future lines can be added to the assembly plant. This could be due to many reasons including land or building constraints. The idea is that the producer will invest in the lowest cost land and building based on expected demand. The model will find the optimum fixed capacity investment that would serve the expected demand throughout the production life of the assembly plant whilst maximizing profits. This capacity optimization approach is applied to both the single style assembly line model for SUVs and small cars separately and to the flexible assembly line model in which vehicles A and B are assembled on the same lines. In the second scenario allowing for flexible capacity decision making, expansion costs are deferred to the future if they are needed. Figure 7 below shows the different flexibility scenarios under consideration:

		Capacity decision making flexibility	
		Yes	**No**
Assembly line flexibility	**Yes**	I	II
	No	III	IV

Figure 7: Flexibility scenarios under analysis

In all scenarios across the two dimensions of capacity decision making flexibility and assembly line production flexibility, the planner/investor faces one decision variable that would allow him to minimize cost per unit assembled. They have to decide on the number of production lines to invest in which determines the expected production capacity in multiples of a fixed value depending on assembly line type, i.e. whether it’s single style SUV or small car or multi style.

System Design Parameters

In this section, the cost functions and parameters used in the analysis are explained in detail.

Although the number of working hours a year is assumed to be the same across all assemblies, each assembly type will have its own maximum achievable capacity per line. This is because the rate (output/unit time) at which an assembly line operates is limited by the maximum cycle time within the assembly processes. In other words, the slowest process in an assembly line limits its rate of output. There is the simplification of fixed equipment allocation for every assembly process, which is to say that one cannot add equipment to a process in order to speed it up. Meanwhile, the principal maximum capacities are referred to as Cap_{small} and Cap_{SUV} for single style assembly lines and Cap_{Multi} for the multi style assembly line and have fixed values.

The cost per unit assembly in any scenario will depend on the particular assembly system variable costs as well as on the annual depreciation of the fixed costs distributed over the minimum of production capacity and demand.

The unit cost, $C_{u,i}$, for *i* where *i* is either single style small car or single style SUV assembly is described in equation 1. B_n in the equation refers to the annual depreciation of the building accommodating *n* number of lines. This depreciation, in millions of dollars, is described in equation 2, has a non-linear relationship with *n* and is the source of economies of scale. Meanwhile, $C_{f,i}$ in equation 1 refers to the depreciation of the fixed cost of equipment and tools per line associated with assembly system *i*. Note that due to the difference in time scale examined in the following analyses, in the decision tree analysis the fixed costs are depreciated over two years while in the lattice model they are depreciated over six years. The depreciation of equipment scales linearly with number of lines *n*. The building and equipment depreciation values are the fixed costs in the system and are distributed over x_i, the produced quantity of *i* which is the minimum of the chosen maximum production capacity and demand. $C_{v,i}$, on the other hand, represents the variable cost of producing one unit of product *i*.

$$C_{u,i} = \frac{B_n + n \cdot C_{f,i}}{x_i} + C_{v,i} \tag{1}$$

where $x_i = \min\left(n \cdot Cap_i, d_i\right)$

for *i* ∀ small car, SUV

$$B_n = 10 + 3n^{0.8} \tag{2}$$

The cost function for small car and SUVs on a multi style assembly system is represented in equation 3 where the variables are essentially the same as in equation 1. Note that in the flexible assembly line, the fixed costs of tools and equipment are higher than either of the two single style approaches because it is a new and more sophisticated technology. In addition, the fixed costs are distributed over the total produced quantity x_T which is the minimum of the chosen maximum production capacity and the total demand of both products. The variable cost, $C_{v,i}$, however depends on the product whose unit cost we are considering.

$$C_{u,i} = \frac{B_n + n \cdot C_{f,multi}}{x_T} + C_{v,i} \tag{3}$$

where $x_T = \min\left(d_{small} + d_{SUV}, n \cdot Cap_{multi}\right)$

$i \, \forall$ small car, SUV

The ultimate goal of the analysis is to maximize for profit. This is captured in the set up below. $P_{u,I}$ refers to the unit price of product i and x_i is the produced amount of product i. Meanwhile $C_{u,i}$ refers to the unit cost of production described in the equations for cost 1 or 3.

$$\max \sum_i x_i \cdot (P_{u,i} - C_{u,i})$$

$i \, \forall$ small car, SUV or both

Expected profits in both the flexible assembly system and the single style assembly system will be maximized by choosing appropriate capacity investments depending on demand and growth projections. NPVs of the assembly systems will be compared in the case where production capacity decision is made in the first term only and in the case where production capacity can be increased each term if necessary. There is a slight complication with the profit maximization approach for flexible assembly lines in that when maximizing for profit, production capacity favors the product with the highest profit margin which in this case is the small car (see Figure 8). This prioritization is incorporated into the analysis.

The chosen prices, line capacities, variables costs and equipment costs are provided in Figure 8 below and are typical values in the auto industry.

	SUV	**Small Car**	**Multiproduct assembly**
Unit Price ($k)	30	20	-
Capacity of line (1000s)	30	50	40
Variable Cost ($k)	20	12	-
Cost of Equipment/line ($m)	60	50	75

Figure 8: Prices, variable costs of product types and annual capacities and equipment costs per line of single vehicle style assemblies and multistyle assembly.

Decision Tree Analysis

This section goes through the decision tree analyses for the scenarios depicted in Figure 7 along the two dimensions of flexibility in design.

In the decision tree analyses there are five defined first year demand scenarios for either product ranging from very high demand to very low demand. Although the demand values differ in the SUV and small car models, the probabilities associated with being in each state are the same. This simplification becomes useful when considering multistyle assembly lines. The first year demand scenarios and associated probabilities are depicted in Figure 5 from earlier. Meanwhile, there are five defined possible growth rate categories for period two ranging from very high growth rates to very low growth rates although the set of five possible growth rates are different in the case of SUVs and small cars. The probabilities of achieving a certain growth rates in the second year also depend on the actualized demand from the first year. These probabilities are depicted in Figure 6 and are independent of vehicle type.

In this model, capacity decisions are restricted to five variants for each of the single style assembly systems as well as the multistyle assembly system. These variants are listed in Figure 9 below and could be arbitrary, but in this case, are derived by optimizing capacity for maximum expected value of profits over the two year periods based on first year demand states.

Plant Design	SUV_Single Style	SMALL CAR_Single Style	MultiStyle
Largest	7	9	16
Large	6	8	14
Average	5	6	11
Small	4	5	8
Smallest	2	4	6

Figure 9: 5 design variants for each assembly type in terms of number of assembly lines in plant derived by optimizing capacity for maximum expected value of profits

Given the plant design variants, the projected first year demands and their probabilities as well as the anticipated growth rate with their probabilities, one can now construct decision tree analyses that would allow for decision making that maximizes expected value of profits.

Inflexible in Capacity Decision Making

Here, scenarios II and IV from Figure 7 are under investigation. Decision trees that do not incorporate flexibility in capacity decisions have one decision node at the beginning of period one followed by two stages of chance nodes describing the possible demand outcomes at the end of year 1 and 2. Figure 10 below is a partially expanded decision tree that describes single style or multi style (product flexible) assembly systems. The probabilities of first year demands ranging from very low demand to very high demand are labeled on the branches coming off the first chance node, but the probabilities of achieving a certain rate of growth for year 2 are just labeled P1-P5 for the sake of clarity, as these will depend on the achieved demand in year 1. Although not depicted in Figure 10, there are a total of $5^3 = 125$ payoffs possible for each assembly system. Each decision tree was rolled back two chance nodes by sequentially multiplying payoffs with branch probabilities and getting a value for expected profitability at a previous chance node. Appendix A outlines an example decision tree calculation to explain how ENPV and VARG data is derived. Finally, the branches representing all decisions were ranked by expected profitability and an optimum capacity decision was identified for each assembly system. These decisions are depicted in Figure 11.

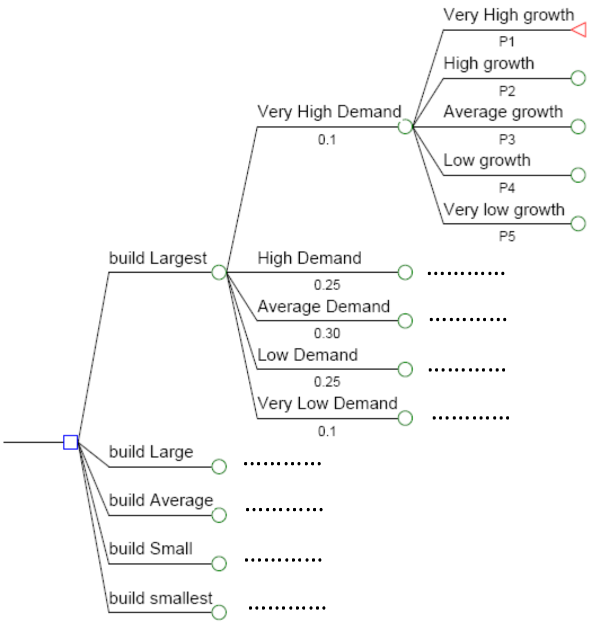


Figure 10: Partially filled decision tree depicting an assembly system with inflexible capacity decision making, where capacity decisions are fixed in the first period. There are a total of 125 payoffs.

Assembly System	Best Strategy	# Lines	NPV of Expected Profit
SUV Single Style	Build Large	6	$1,756,057,475
Small Car Single Style	Build Large	8	$3,424,910,740
Multi Style	Build Large	14	$4,391,184,914

Figure 11: Expected profit-maximizing decisions for single style and multistyle assembly systems in case where capacity decisions are fixed in first year.

It is important to note that the multistyle assembly system did not raise expected profits as anticipated. Its NPV of expected profit value is less than the sum of single style NPV of expected profits. This may have been caused by the oversimplifications regarding demand and growth probabilities being equal. In the multi style assembly analysis the assumption made was that the demands were to a certain extent correlated although the range of possible growth rates were defined differently for each product. This meant that actualized high first year demand or high second year growth rate of SUVs corresponded to a similar trend in small car demand although of different magnitude. The unexpected result may have also been caused by the fact that both products defined growth rate range is not different enough to justify flexible assembly technology. If SUVs growth rates were strictly negative and small car growth rates strictly positive with no overlap as opposed to both products demonstrating expected stagnation or decline, results may indicate a more favorable NPV of expected profits for the multi style assembly technology. All these factors may defeat the purpose and benefit of having multi style assembly lines as discussed in AP1. The high flexibility upcharge is therefore not offset by cost reduction gains under demand uncertainty if the products were to be perfectly negatively correlated with very different growth rate ranges and little overlap.

Flexible Capacity Decision Making

Here, scenarios I and III from Figure 7 are under investigation. Decision trees that incorporate flexibility in making capacity decisions have two decision nodes and two chance nodes as seen in Figure 12. The first decision node addresses the primary decision to invest in capacity to meet first year demand. Following the first decision node is a chance node with five possible first year demand scenarios. Based on actualized first year demand, a decision, represented by the second decision node, is then made to either upgrade the plant capacity to any of the possible upgrades or to remain at the capacity chosen in period 1. This decision node is then followed by a chance node with five possible growth scenarios. The demand probabilities and growth probabilities based on actualized demand for each product are the same as in scenarios II and IV discussed earlier. Figure 12 is a partially expanded decision tree for flexible decision making over two production periods. In this model, second period decisions have a lower capacity limit defined by the decision made in the first year. That is to say that plant investments cannot be reversed, but only expanded. For example, having built the largest capacity in period one, the decision in period two is limited to staying at largest. Meanwhile, if decision 1 was to build large, decision 2 could be to either upgrade to largest and so on. This results in a total of 375 payoffs. Each decision tree was initially rolled back once to obtain optimal second period capacity decisions given any given first year decision based on expected value of profits. The tree was then rolled back a second time to determine optimal first year decisions using expected profits as a criteria. Appendix B provides an example decision tree evaluation for flexible capacity decision making. The optimal decisions for each assembly system under analysis are described in Figure 13.

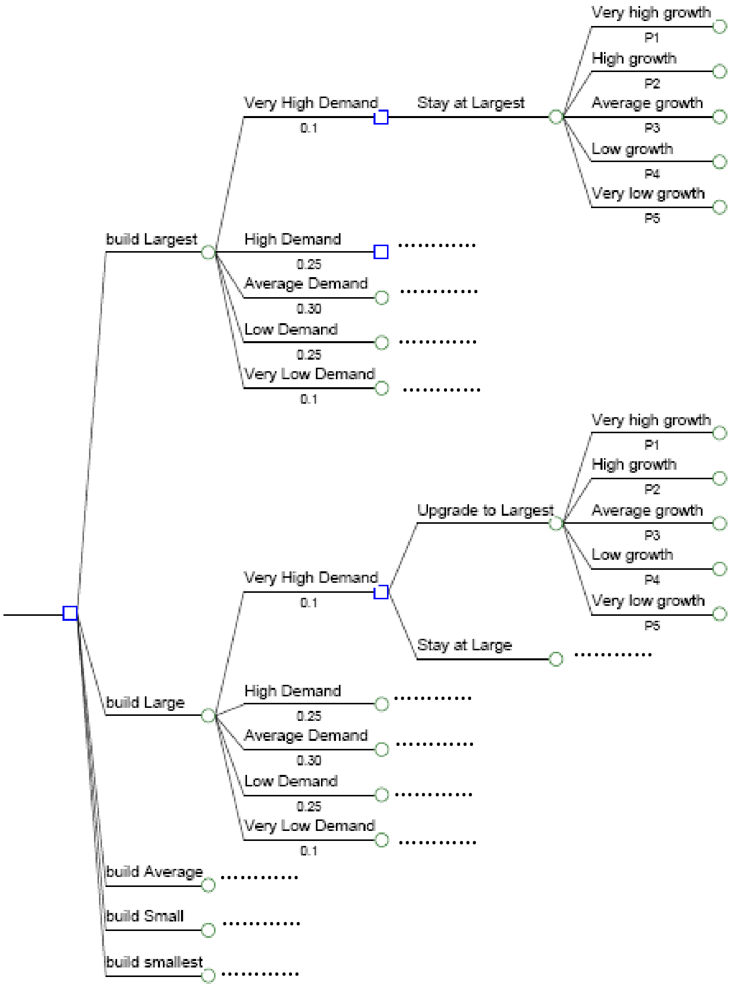


Figure 12: Partially filled decision tree depicting an assembly system with flexible capacity decision making, where capacity decisions are made in the first period and second period upgrades are possible. There are a total of 375 payoffs

SUV Single Style ULTIMATE FLEXIBLE STRATEGY

D1	followed by	D2	if Yr 1 Market Demand is:	NPV of Expected Profits
6 lines		6-Large	Very Low	
"Build Large"		6-Large	Low	
		6-Large	Average	
		6-Large	High	
		7- Largest	Very High	$1,759,733,287

Small Car Single Style ULTIMATE FLEXIBLE STRATEGY

D1	followed by	D2	if Yr 1 Market Demand is:	NPV of Expected Profits
8 lines		8-Large	Very Low	
"Build Large"		8-Large	Low	
		8-Large	Average	
		8-Large	High	
		9- Largest	Very High	$3,440,404,665

Multi style ULTIMATE FLEXIBLE STRATEGY

D1	followed by	D2	if Yr 1 Market Demand is:	NPV of Expected Profits
14 lines		14- Large	Very Low	
"Build Large"		14- Large	Low	
		14- Large	Average	
		14- Large	High	
		16- Largest	Very High	$4,419,756,155

Figure 13: Expected profit-maximizing dynamic decision set for single style and multistyle assembly systems in case where capacity decisions made in first year can be upgraded in the second year.

Again, this decision tree analysis has shown that the multistyle assembly line's NPV of expected profits is less than the sum of the NPV of expected profits in the single style assembly lines for the same set of reasons discussed earlier. It is also interesting to note that the optimal decision sets in the flexible systems are equivalent for either single style assembly lines; build large first followed by upgrading to largest only if actualized first year market demand is very high. This could be attributed to the fact that the probabilities of the demands and growth rates are simplified to be the same in both products although the values of demands and growth rates are different. Consequently because the product demand developments are assumed to be positively correlated, the multi style assembly plant decisions follow a similar trend.

Figure 14 depicts the NPV of expected profits for the analyzed scenarios identified in Figure 7, exploring both dimensions of capacity decision making flexibility and production flexibility. Flexibility in capacity decision making increased NPV of profits in all assembly systems. Production flexibility, defined by the ability to produce multiple product types on the same line, did not increase NPV of expected profits if you compare it to the sum of single style assembly lines of either product. On the contrary, it decreased NPV of expected profits due to simplifications done in the analysis regarding product demand correlations as well as the defined set of possible growth rates. This analysis demonstrates the inappropriateness of flexible production lines for the modeled product demand in this analysis as the demands are not negatively correlated enough.

		SUV NPV of profits	Small Car NPV of profits
Single Style	INFLEXIBLE OVER TIME	$1,756,057,475	$3,424,910,740
	FLEXIBLE OVER TIME	$1,759,733,287	$3,440,404,665
	INCREASE IN profits (%)	0.209321794	0.452389179

Multi Style	Inflexible	$4,391,184,914
	Flexible	$4,419,756,155
	INCREASE IN profits (%)	0.650649919

Figure 14: NPV of Expected profits for all scenarios identified in Figure 7 along the dimensions of flexibility in production and flexibility in capacity decision making.

Figure 15, 16 and 17 show the VARG curves constructed from the decision tree analysis for SUV and small car single style assembly lines as well as multistyle assembly lines respectively, considering both inflexible and flexible capacity decision making. All inflexible designs are shown as well as the optimal flexible decision set identified in Figure 13. All other flexible decision sets are discarded. The expected net present values are shown as vertical lines and as one can tell, the flexible capacity decision making option raises the expected net present value in all cases.

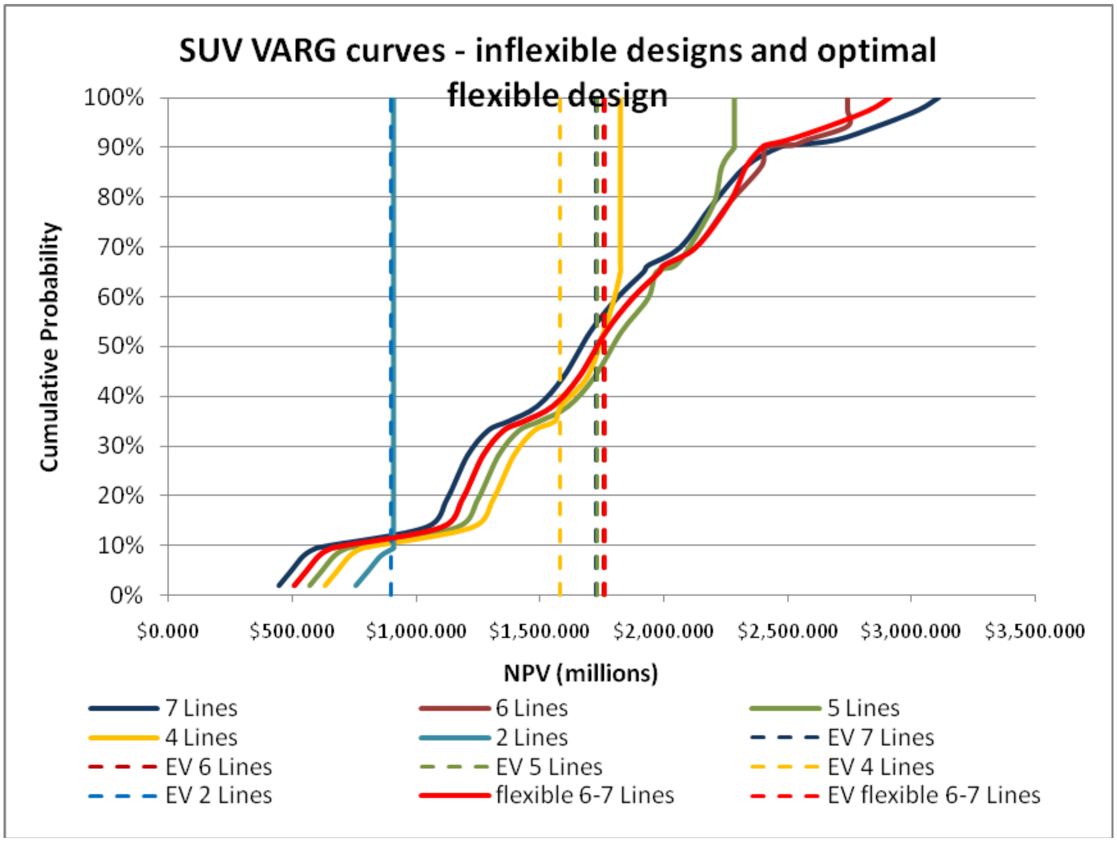


Figure 15: VARG curves for all SUV inflexible designs (2,4,5,6 or 7 lines) and for SUV flexible decision making design (6 lines expandable to 7 if needed). Expected net present values are shown as dashed vertical lines.

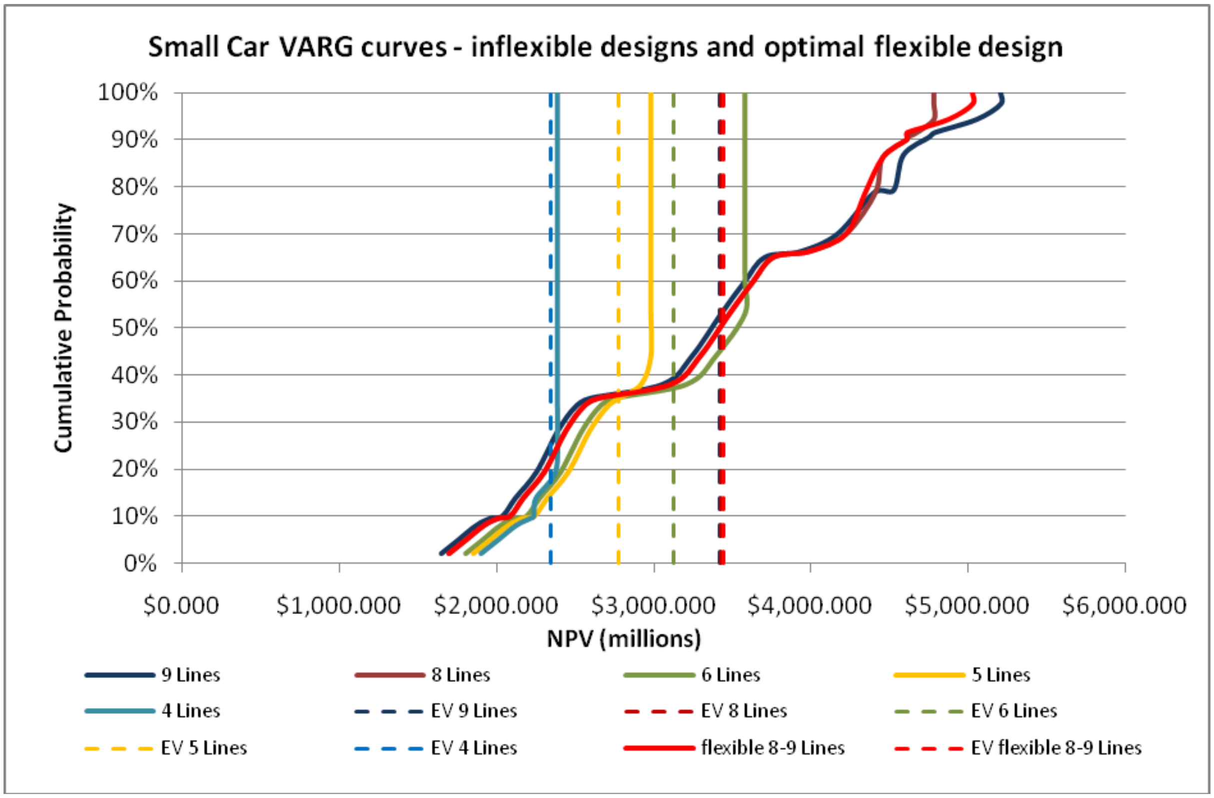


Figure 16: VARG curves for all small car inflexible designs (4,5,6,8 or 9 lines) and for small car flexible decision making design (8 lines expandable to 9 if needed). Expected net present values are shown as dashed vertical lines.

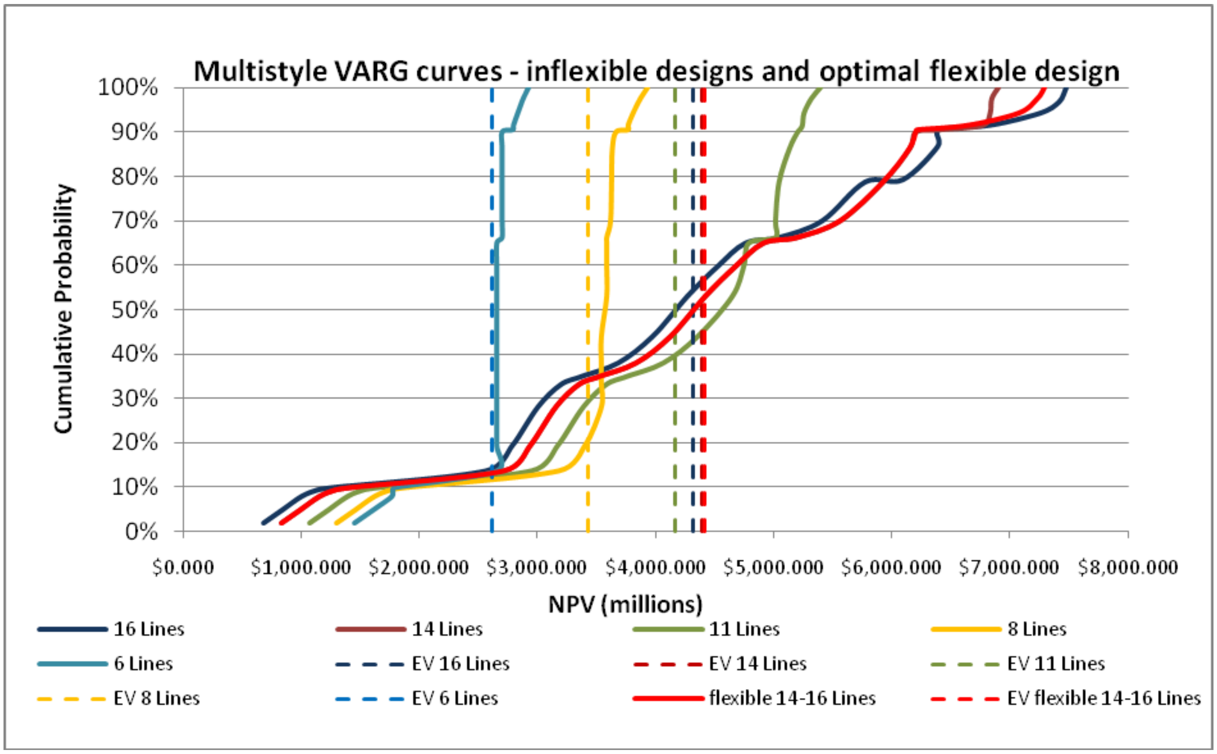


Figure 17: VARG curves for multistyle inflexible designs (6,8,11,14 or 16) and for multistyle flexible decision making design (14 lines expandable to 16 if needed). Expected net present values are shown as dashed vertical lines.

Multiple Criteria Evaluation

Based on the VARG curves derived from the decision tree analysis one can set up a multiple criteria chart (see Figure 18) that can be used to compare all designs. Criteria range from expected net present values (ENPV) to profitability given by expected return on capital expenditure (ENPV/CapEx).

		SUV		**Small Car**		**Multi-style**		**SingleStyle SUV+Small Car**	
	Capacity Decision making flexibility	**Inflexible**	**Flexible**	**Inflexible**	**Flexible**	**Inflexible**	**Flexible**	**Inflexible**	**Flexible**
ENPV ($bn)		1.756	1.760	3.425	3.440	4.391	4.420	5.181	5.200
P10 ($bn)		0.708	0.708	2.084	2.084	1.426	1.426	2.793	2.793
P90 ($bn)		2.411	2.394	4.610	4.606	6.207	6.207	7.021	7.001
Max NPV ($bn)		2.741	2.910	4.781	5.025	6.904	7.298	7.522	7.935
Min NPV ($bn)		0.510	0.510	1.694	1.694	0.837	0.837	2.204	2.204
CapEX ($bn)*	Actual	0.38	-	0.43	-	1.085		0.808	
	Min	-	0.38	-	0.43	-	1.085	-	0.808
	Max**	-	0.43	-	0.47	'	1.211	-	0.902
ENPV/CapEX (%)	Actual	459		804		405		641	
	Max	-	460	-	808	-	407	-	643
	Min	-	406	-	734	-	365	-	577

* min and max CapEX values represent the investment if no expansion took place and investment if expansion did take place

** max CapEx takes the NPV of the second year expansion investment into account discounted by two years

Figure 18: Multiple criteria table comparing flexible and inflexible designs across all assembly systems.

As discussed earlier, having flexibility in the production system by being able to produce two products on the same line is not as profitable as having two separate assembly systems for the two products. This conclusion is supported by all evaluation criteria in Figure 18 if one compares the multistyle assembly with the sum of single style assemblies. From an expected net present value perspective, however, flexibility in capacity decision making adds value particularly in the single style small car assembly. It does so by increasing the gains from the upside via the expansion option. This is represented by the increase in maximum attainable net present value for all systems. It is also interesting to note that in terms of return on investments, the flexible options do not appear to be much more profitable than the inflexible options for any assembly system. ENPV/CapEx values for the flexible systems are given as a range depending on whether or not the expansion option is exercised and therefore further investment is required. In all cases the maximum ENPV/CapEx is only slightly higher than the inflexible case. This is due to the fact that the gain in expected net present value is small compared to the capital expenditure needed to achieve it.

In summary, from an ENPV perspective the decision tree analysis suggests that the flexible capacity decision making system designs are better than the inflexible systems. From a return on investment point of view, however, the increase in ENPV does not justify the higher investments required. In all cases, production flexibility proved to be unprofitable. Given the demand projections used in this analysis, it is more profitable to have two separate assembly lines.

Lattice Analysis Setup

In the following lattice analysis only one source of flexibility was considered, namely capacity decision making flexibility. Production flexibility given by multi style assembly lines was left out due to the fact that the decision tree analysis indicates it has a lower ENPV than the sum of single style ENPVS under all scenarios and is therefore not an appropriate system design. That is to say that the decision tree analysis indicates that it is more profitable to produce the two vehicles separately than on the same multi style assembly line. In order to run a lattice analysis on the assembly systems subject to uncertain vehicle demand, the following adjustments have to be done to the established model implemented in the decision tree analysis. The first adjustment is to assume path independence, the assumption that the future growth rates are independent of previous period demand conditions. Hence, instead of using a set of possible growth rates and associated probabilities depending on actualized first year demand as in the case of the decision tree analysis, an expected growth rate was calculated with its associated standard deviation for each product demand. These would apply to all periods under analysis. The second adjustment involves dropping the projected first year demand distribution and uncertainties and instead combining that uncertainty with the uncertainty in growth rate. This involves deriving a starting demand at t_0 off of which the model would run. In order to do this, the projected expected first year demand was traced back one year to present using calculated expected annual growth rates of -9% and 0% for SUV and small car demand respectively. Hence, SUV sales are set at 140k and small car sales at 300k at point zero. The annual standard deviations were derived to be 3% for SUV and 2.5% for small car demand. Given expected values for growth rates and the associated standard deviations one can now derive the parameters (u,d,p) needed to model demand uncertainty in the binomial lattice model where:

$$u = e^{\sigma\sqrt{\Delta t}} \qquad d = \frac{1}{u} \qquad p = 0.5 + 0.5 \cdot \left(\frac{\nu}{\sigma}\right)\sqrt{\Delta t}$$

Annual expected growth rates and standard deviations resulted in negative probability values, so the time period increment had to be reduced to a month, whilst adjusted monthly expected growth rates and standard deviations. Figure 19 below shows the obtained monthly growth rates and standard deviations as well as the obtained values for u,d and p.

a)	Growth rates		Standard Deviations	
	g(a)	**g(m)**	**sdev(a)**	**sdec(m)**
SUV	-0.09	-0.00783	0.03	0.0087
Small Car	0	0	0.025	0.0072

b)

	SUV	**Small Car**
u	1.00870	1.00724
d	0.99138	0.99281
p	0.04803	0.50000

c)

	SUV	**Small Car**
u	1.1095	1.0905
d	0.9012	0.9170

Figure 19: a) Annual and monthly expected growth rates and associated standard deviations. **b)** calculated monthly u,d,p values for binomial lattice model. **c)** adjusted yearly u and d values.

The u,d and p parameters derived apply for monthly increments. This is a major limitation of the binomial lattice analysis because it results in additional simplifications and approximations that are necessary to limit the size of the analysis. More specifically, in this lattice analysis, the monthly values of u and d were adjusted to cover annual increments for a total of 6 periods/years (see Figure 19c). These were adjusted crudely by considering twelve consecutive high growth (up) periods as an annual up state and twelve consecutive low growth (down) periods as a the annual down state. This does not consider in between states and their respective probabilities for that would inflate the lattice model and make it less workable.

Note that a six year period was under investigation in the lattice model whereas only two years were investigated in the decision tree analysis. The reason is that a six year decision tree analysis is impractical and difficult to evaluate. The results of the six year lattice analysis are not comparable to the previous results of the decision tree analysis but nevertheless provide helpful insights on flexible versus inflexible system evaluation.

Figure 20 and 21 show the lattice model demand development of SUV and small car demand along with the associated probability distributions.

OUTCOME LATTICE

Year	0	1	2	3	4	5	6
(000s)	140.00	155.33	172.34	191.22	212.16	235.39	261.17
		126.18	140.00	155.33	172.34	191.22	212.16
			113.73	126.18	140.00	155.33	172.34
				102.50	113.73	126.18	140.00
					92.38	102.50	113.73
						83.26	92.38
							75.05

PROBABILITY LATTICE

Year	0	1	2	3	4	5	6
	1.00	0.05	0.00	0.00	0.00	0.00	0.00
		0.95	0.09	0.01	0.00	0.00	0.00
			0.91	0.13	0.01	0.00	0.00
				0.86	0.17	0.02	0.00
					0.82	0.20	0.03
						0.78	0.23
							0.74

Figure 20: SUV demand modeled as a binomial lattice and resulting probability distribution.

OUTCOME LATTICE

Year	0	1	2	3	4	5	6
(000s)	300.00	327.14	356.73	389.00	424.19	462.57	504.41
		275.11	300.00	327.14	356.73	389.00	424.19
			252.29	275.11	300.00	327.14	356.73
				231.36	252.29	275.11	300.00
					212.17	231.36	252.29
						194.57	212.17
							178.42

PROBABILITY LATTICE

Year	0	1	2	3	4	5	6
	1.00	0.50	0.25	0.13	0.06	0.03	0.02
		0.50	0.50	0.38	0.25	0.16	0.09
			0.25	0.38	0.38	0.31	0.23
				0.13	0.25	0.31	0.31
					0.06	0.16	0.23
						0.03	0.09
							0.02

Figure 21: Small car demand modeled as a binomial lattice and resulting probability distribution.

Lattice Analysis

The demands and associated probabilities were used in conjunction with profit function to obtain weighted payoff lattices for each design option that can be used to come up with an expected net present value from the six year period cash flow under investigation. Figure 22 shows the ENPV obtained for all inflexible and flexible designs of each assembly system and identifies what the ENPV maximizing designs are. The two dimensions in the tables represent production capacity decisions (in # lines) as decision 1 and decision 2 undertaken in periods one and two respectively. Inflexible assembly system would therefore have D1=D2 whereas a flexible assembly system allows an upgrade of capacity. The lattice model limits the flexibility to a one time capacity upgrade decision that can be undertaken any time during the six periods in the analysis. In Figure 22, the optimal inflexible and flexible design decisions for both types of vehicle assemblies are color coded. Yellow corresponds to the optimal inflexible strategies and red corresponds to the optimal decision pairs in the flexible strategies. The optimal, ENPV maximizing, flexible strategys' VARG curve will be shown alongside the inflexible strategy VARG curves.

SUV ENPV ($bn)		D2 (# lines)				
		2	4	5	6	7
D1 (# lines)	2	1.413	2.079	1.572	1.423	1.414
	4	-	2.0791	2.0794	2.0792	2.0791
	5	-	-	1.57196	1.572	1.572
	6	-	-	-	0.975	0.975
	7	-	-	-	-	0.375

S.C. ENPV ($bn)		D2				
		4	5	6	8	9
D1	4	4.531	5.599	6.259	5.885	5.413
	5	-	5.599	6.259	6.087	5.904
	6	-	-	6.259	6.407	6.355
	8	-	-	-	5.885	5.890
	9	-	-	-	-	5.413

Figure 22: ENPVs for flexible and inflexible capacity decision making scenarios for SUV and Small Car Assembly systems. Inflexible designs have a green border. The value maximizing inflexible decisions are highlighted in orange, while the pink highlighted cells represent the value maximizing flexible decision set.

In the flexible decision making scenarios dynamic programming was implemented to derive the expected net present value of a decision pair as well as a strategy of when to exercise flexibility identified by the chosen design pair. The flexible strategies are shown in Figure 23 alongside the demand projections. An example showing a more detailed explanation of how dynamic programming was implemented is provided in Appendix C.

SUV demand projections and strategy for exercising Call Option

Year	0	1	2	3	4	5	6
Demand (000s)	140	155.3321	172.3433	191.2175	212.1587	235.3933	261.1724
		126.1813	140	155.3321	172.3433	191.2175	212.1587
			113.7265	126.1813	140	155.3321	172.3433
				102.5011	113.7265	126.1813	140
					92.38367	102.5011	113.7265
						83.26491	92.38367
							75.04622
Optimal strategy:	Excercise CALL	NO	NO	YES	YES	YES	YES
4 lines to 5 lines	OPTION ?		NO	NO	NO	YES	YES
				NO	NO	NO	YES
					NO	NO	NO
						NO	NO
							NO

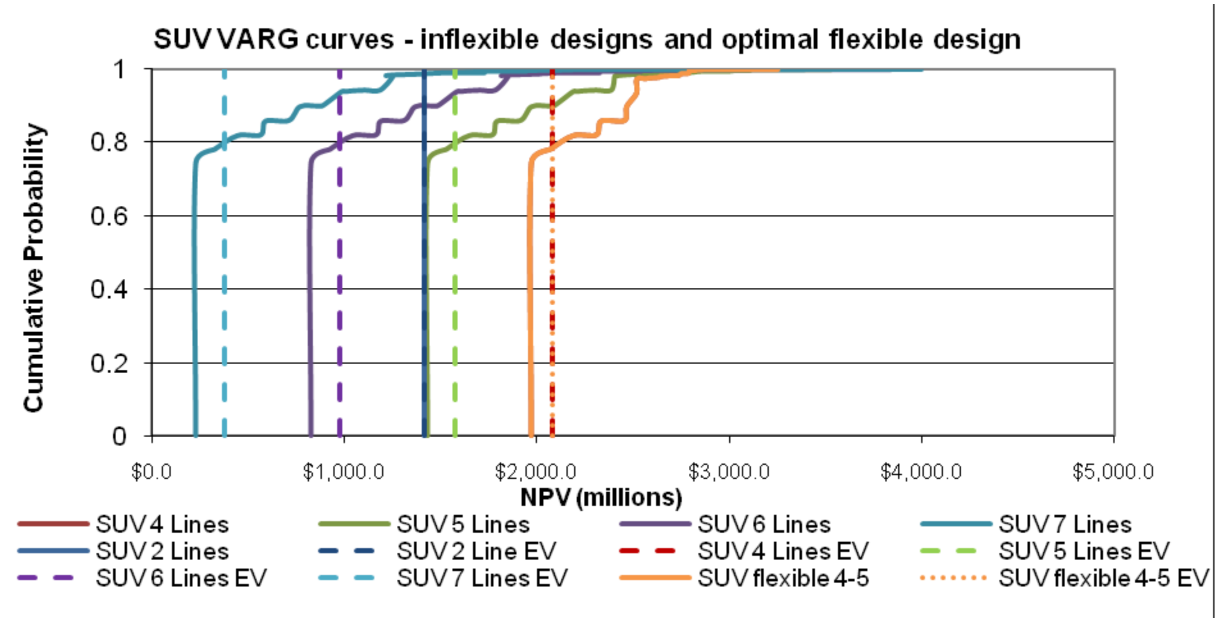
Small Car demand projections and strategy for exercising Call Option

Year	0	1	2	3	4	5	6
Demand (000s)	300	327.139	356.733	389.0042	424.1947	462.5687	504.4142
		275.1125	300	327.139	356.733	389.0042	424.1947
			252.2895	275.1125	300	327.139	356.733
				231.36	252.2895	275.1125	300
					212.1667	231.36	252.2895
						194.5657	212.1667
							178.4248
Optimal strategy:	Excercise CALL	NO	NO	NO	YES	YES	YES
6 lines to 8 lines	OPTION ?		NO	NO	NO	NO	YES
				NO	NO	NO	NO
					NO	NO	NO
						NO	NO
							NO

Figure 23: Projected SUV and small car demands and strategy for exercising call option of optimal flexible strategy based on observed demand.

Not surprisingly, when examining a six year cash flow period using the binomial lattice model and dynamic programming one can see that the results differ from the decision tree analysis approach. All decisions are shifted down one or two plant sizes. The decreasing projected demand and the longer time frame seem to suggest smaller capacities are more profitable in the long run. The optimal fixed decisions for SUV and small car assembly is to build 4 and 6 lines of capacity respectively instead of 6 and 8 lines. Moreover, in the flexible capacity decision making scenario the optimal strategies are to invest in 4 and 6 lines initially just as in the inflexible scenario and then to exercise the call option and upgrade to 5 or 8 lines respectively if required at any stage as identified by the strategies in Figure 23.

The range of possible NPVs derived from the lattice analysis allows us to set up target curves representing the cumulative probabilities of outcomes. Figure 24 illustrates the cumulative distribution (target) curves and outcomes for all inflexible designs as well as of the optimal flexible design identified in Figure 22. Expected net present values are represented by dashed vertical lines. As one can see, the flexible design shifts the VARG curves and the ENPV to the right, representing a more profitable opportunity on average. The value of flexibility in the case of SUV assembly is $249,623 versus $148,647,667 in the small car assembly system. This is derived by subtracting ENPV of optimum inflexible design from the ENPV of optimum flexible design.



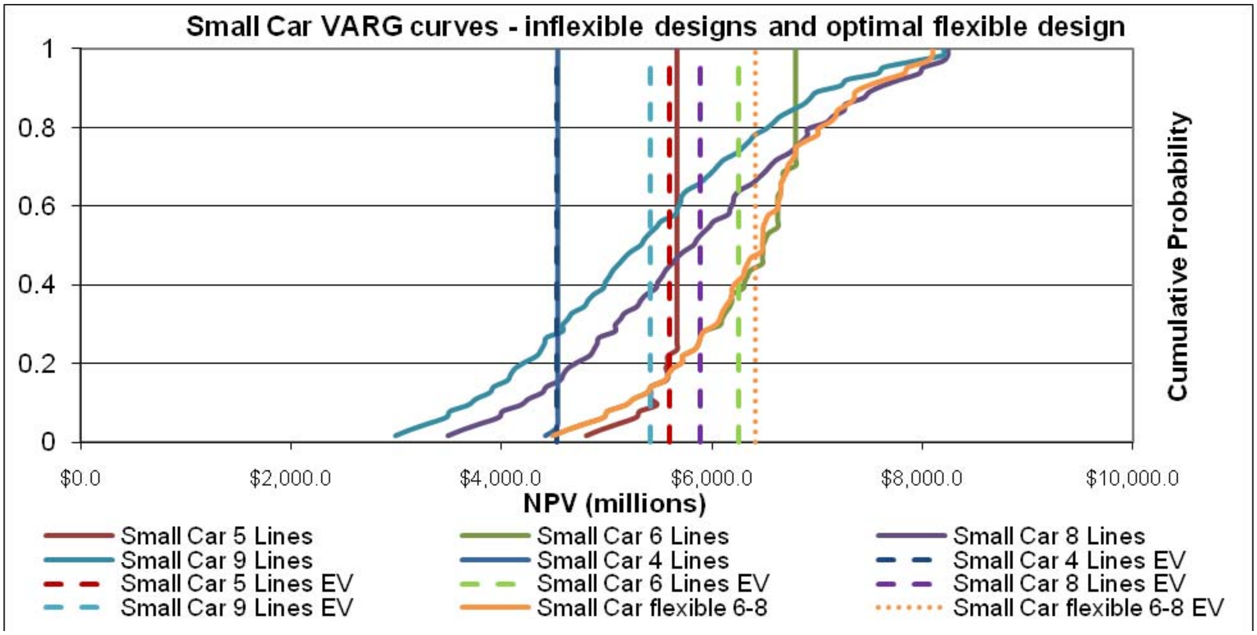


Figure 24: Cumulative Distribution functions for SUV and Small Car assembly designs identify flexible design as having highest ENPV. These figures were generated from binomial lattice data.

Multiple Criteria Evaluation

Just as was done in the decision tree analysis, a multiple criteria table (Figure 25) was set up based on data obtained from the binomial lattice analysis. Multi style production flexibility was discarded from this analysis due to the inappropriateness demonstrated in the decision tree analysis.

		SUV		Small Car		Multi-style	
	Capacity Decision making flexibility	**Inflexible**	**Flexible**	**Inflexible**	**Flexible**	**Inflexible**	**Flexible**
ENPV ($bn)		2.079	2.079	6.259	6.407	Decision tree analysis has shown that production flexibility is not profitable in all criteria (see Figure 18). Therefore, no further analysis was conducted	
P10 ($bn)		1.973	1.973	5.258	5.258		
P90 ($bn)		2.467	2.467	6.799	7.479		
Max NPV ($bn)		2.826	3.251	6.799	8.101		
Min NPV ($bn)		1.973	1.973	4.492	4.492		
CapEX ($bn) *	Actual	2.42	-	3.02	-		
	Min	-	2.42	-	3.02		
	Max**	-	2.87	-	3.71		
ENPV/CapEX (%)	Actual	85.95		207.06			
	Max	-	85.96	-	211.98		
	Min	-	72.42	-	172.81		

* min and max CapEX values represent the investment if no expansion took place and investment if expansion did take place

** max CapEx takes the NPV of the expansion investment into account discounted using the earliest implementation as given by the dynamic programming results in Figure 23 where the earliest implementation date for SUV assembly is 3 years and for small cars 4 years.

Figure 25: Table of multiple criteria used for evaluation of inflexible and flexible system designs.

Similar conclusions can be drawn from the lattice analysis as the decision tree analysis. When comparing flexible with inflexible designs it is apparent that capacity decision making flexibility increases ENPV in both single style assemblies. From an ENPV perspective therefore system designs with capacity decision making flexibility are more profitable than inflexible systems. This analysis also shows that the increase in ENPV is attributable to an increase in upside gains This is demonstrated by an increase in Max NPV in the SUV system and an increase in both P90 and Max NPV in the small car system. In fact, the most value added is in the small car assembly system where the increase in ENPV is from $6.259 to $6.407 billion. Therefore it demonstrates the highest value of flexibility.

Figure 25 also shows important evaluation parameters for returns on investment. For flexibly assembly systems, a min and max ENPV/CapEx are reported because it is unknown if expansion investments are undertaken. In both single style assembly systems, flexibility does not increase the return on investment ratio significantly. The maximum ENPV/CapEx values for the flexible systems are only slightly higher than their inflexible counterparts' although it is higher for the small car assembly than the SUV. This is most likely due to the fact that increase in profits is not as high as increases in investments necessary for expansion. Therefore values for returns on investment suggest that the inflexible systems perform better.

Conclusion

This application portfolio has demonstrated that flexible design can or cannot be beneficial in system design depending on the system itself and the criteria used for evaluation. In this case of vehicle assembly lines subject to the demand scenarios and uncertainties described in this model, production flexibility was not a value adding design component, primarily because demand of the two products was not sufficiently negatively correlated. In another demand scenario, production flexibility may be helpful. Flexibility in adjusting production capacities, however, seemed to add value to the systems from an expected net present value perspective. ENPV was higher in cases where one had the ability to increase capacity after observing previous year demands. Nevertheless, the demand projections and uncertainties were such that this flexibility was more beneficial for the small car assembly system than the SUV assembly system. This is due to the fact that the small car demand projection showed a potential of increase whereas SUV demand growth probability was insignificant. From a return on investment perspective where the measure was expected new present value over capital expenditure (ENPV/CapEx), flexibility does not seem very profitable as the increase in expected net present value does not compare to the much larger investments required. Therefore, there is a need to conduct a detailed analysis of the system to determine the optimal use of flexibility and more than one criterion should be used for evaluation.

In this application portfolio, the binomial lattice analysis seemed to be the easiest to implement despite the abundance of approximations and simplifications that were necessary for the set up. The fact that only certain time intervals would result in non negative probabilities overcomplicated the analysis. Another limitation of the set up is that only one expansion option is included in each lattice analysis. Nevertheless, the lattice model results allowed the construction of target/VARG curves and provided an abundance of parameters useful for comparing designs. The decision tree analysis was a more accurate approach to modeling the systems in this portfolio but had its limitations as well. For one, a set of discrete probabilities had to be derived in such a way that the overall demand and growth distributions resembled the projections. Moreover, a six year time period analysis would have blown up the calculations in the decision tree model making it very impractical. Although the lattice model involved crude approximations and is less accurate than the decision tree model, it proved to be more practical and easy to use for this portfolio.

This application portfolio has helped me apply and understand the evaluation techniques learned in class. As mentioned above, there were some limitations in the evaluation tools that made my analysis irritating at times for it forced me to make several simplifications to the model. This is particularly the case with the binomial lattice model evaluation where there was a discrepancy between the time increments I desired and the time increments possible. Yet, part of the analysis process is to learn how to make simplifications and approximations, which is also a message of this course, namely that full detail analysis is often times not feasible and even impractical. It takes skill to know what analysis is most practical for which case. Finally, this application portfolio has helped me really understand another major concept in the course; knowing when it is that flexibility is desirable. As this portfolio showed, production flexibility was not suitable in this case, whereas capacity flexibility was and even that depended on demand projections. A careful analysis of the existing system allows for determining when to apply flexibility. Overall, this course was very useful in that it had the proper balance of theory communicated through lectures and practical work applied to this application portfolio.

Appendix A
Decision Tree Analysis example for inflexible capacity decision making systems

This example will illustrate how the decision tree analysis was carried out for a system with a fixed capacity decision done in period 1. For illustration purposes, the analysis is applied to an SUV system where the decision is to build the largest facility. Figure 26 below is a representation of the decision tree where only the decision branch corresponding to building largest plant size is expanded followed by an expansion of the scenario where very high demand is actualized in the first year.

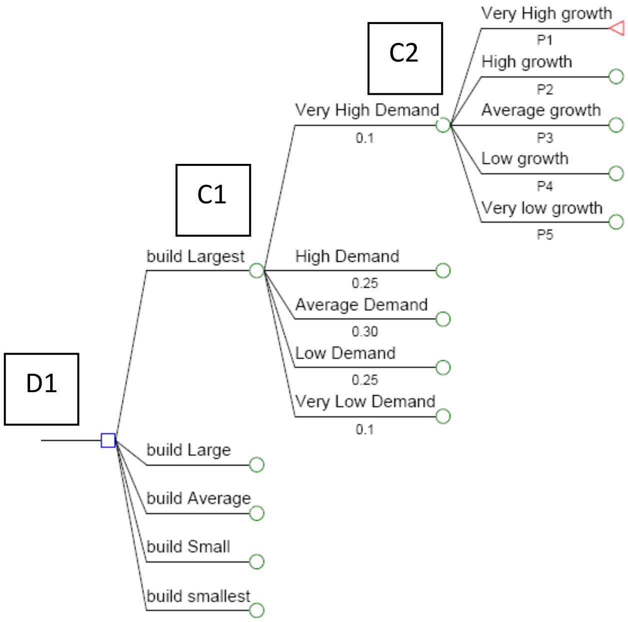


Figure 26: Partially filled decision tree depicting an assembly system with inflexible capacity decision making, where capacity decisions are fixed in the first period. There are a total of 125 payoffs.

In this example the decision is to build the largest assembly line, consisting of 7 lines. This decision node is followed by two chance nodes. The first chance node represents the first year demand whereas the second chance node represents the subsequent growth rate.

Decision analysis is conducted by rolling back the branches, end to start. If, as in this example, we are only considering one decision and finding its ENPV, then there are a total of 5^2=25 outcomes representing all the different 1st year demand-growth rate combinations possible. Starting from the end, there are five possible growth rates, each with its own probability from the set P1-P5 shown in Figure 26, where the probability will depend on actualized first year demand. P1-P5 for the different growth rates given first year demand was very high are different than P1-P5 if first year demand was high. Meanwhile, there are five possible first year demands each with its own probability of occurring. These are shown on the individual branches coming off the first decision node.

An excel spreadsheet was used for the decision tree analysis. The payoffs at the end of each path along the decision tree are derived by first calculating the first and second term demands. Demands were found by travelling along decision tree to the payoff node and on the way subjecting the first year demand to second year growth rates. Figure 27 below shows the possible SUV first and second year demands based on first year demands and the following growth rate possibilities. Demands are then used to find the net present value of profit by using the equation for profit and system specific parameters identified in the section: **System Design Parameters.** The profits are discounting by one or two years accordingly and summed to give a total NPV value for each path. Figure 28 below shows what these possible NPV values are (25 total) for a system where 7 lines of production are present. Note that in both tables the demands and growth rates have numerical values while in the decision tree these are ranked from very high to very low to avoid confusion. First year demands correspond to the first chance nodes and subsequent growth rates correspond to the second chance nodes.

	GROWTH RATE				
SUV DEMAND PROJECTIONS	**-0.30**	**-0.20**	**-0.10**	**0.00**	**0.10**
First Year Demand	**SECOND YEAR DEMANDS**				
60,000	42,000	48,000	54,000	60,000	66,000
100,000	70,000	80,000	90,000	100,000	110,000
130,000	91,000	104,000	117,000	130,000	143,000
160,000	112,000	128,000	144,000	160,000	176,000
200,000	140,000	160,000	180,000	200,000	220,000

Figure 27: First and second year SUV demand projections. There are 25 possible results depending on first year demand and subsequent growth rate combinations.

	GROWTH RATE				
7 Assembly Lines	**-0.30**	**-0.20**	**-0.10**	**0.00**	**0.10**
First Year Demand	**NPV of PAYOFFS ($)**				
60,000	448,332,154	497,918,931	547,505,708	597,092,485	646,679,262
100,000	1,043,373,477	1,126,018,105	1,208,662,733	1,291,307,361	1,373,951,989
130,000	1,489,654,468	1,597,092,485	1,704,530,501	1,811,968,518	1,919,406,534
160,000	1,935,935,460	2,068,166,865	2,200,398,270	2,332,629,675	2,464,861,080
200,000	2,530,976,782	2,696,266,039	2,861,555,295	3,026,844,551	3,109,489,179

Figure 28: Total NPV of 7 line SUV assembly system subject to different first year demand- growth rate combinations.

The NPV of payoffs given in Figure 28 are rolled back one chance node by multiplying their value with the probability of that growth occurring given first year demands (from Figure 6). Figure 29 shows the P*NPV transform of Figure 28.

	GROWTH RATE				
7 Lines	**-0.30**	**-0.20**	**-0.10**	**0.00**	**0.10**
First Year Demand	**P*NPV ($)**				
60,000	89,666,431	149,375,679	164,251,712	89,563,873	32,333,963
100,000	156,506,021	253,354,074	423,031,956	258,261,472	103,046,399
130,000	148,965,447	319,418,497	511,359,150	407,692,917	335,896,144
160,000	96,796,773	310,225,030	770,139,394	699,788,902	369,729,162
200,000	126,548,839	269,626,604	786,927,706	983,724,479	777,372,295

Figure 29: P*NPV transform of Figure 28 where the probabilities of growth given first year demands is derived from Figure 6.

Again, each row in the tables above corresponds to being at some first year demand chance node. Summing up the corresponding P*NPV values across all subsequent growth rates (i.e. all values across that row) gives us the ENPV of being at that node, the first chance node in the decision tree. These ENPVs are shown below in Figure 30.

7 Lines SUV Inflexible Assembly System		
First Year Demand	**P(D)**	**ENPV ($)**
Very Low	0.10	$525,191,658
Low	0.25	$1,194,199,923
Average	0.30	$1,723,332,154
Good	0.25	$2,246,679,262
Very Good	0.10	$2,944,199,923

Figure 30: ENPV of being at chance nodes corresponding to very low to very high first year SUV demands. Probabilities of those first year demands occurring are highlighted.

The decision tree can now be rolled back to the decision node by multiplying these ENPVs of being at a certain first year demand by the probability of that demand occurring, where the probabilities are highlighted in Figure 30. Again, the sum of these multiplications will give us the ENPV of deciding to build a 7 line inflexible SUV assembly system. This ENPV is evaluated to $1,724,158,601.

A similar analysis can be conducted for all different possible assembly plant capacity decisions and the ENPV maximizing inflexible capacity decision can be identified. In the case of SUV assembly, this decision is to build 6 lines with an ENPV of $1,756,057,475.

VARG curves can also be constructed from this analysis by using all possible payoffs and correlating them to the probability of them occurring. This probability is equal to the product of probability of first year demand and probability of a certain growth rate given that first year demand. The VARG curve is then the cumulative probability of the NPVs ranked by value. Figure 31 represents the data used to obtain the VARG curve for a 7 line inflexible capacity decision making SUV assembly system.

7 Lines inflexible SUV

P	NPV ($)	Cumulative P
2%	$448.332	2%
3%	$497.919	5%
3%	$547.506	8%
2%	$597.092	10%
0%	$646.679	10%
4%	$1,043.373	14%
6%	$1,126.018	19%
9%	$1,208.663	28%
5%	$1,291.307	33%
2%	$1,373.952	35%
3%	$1,489.654	38%
6%	$1,597.092	44%
9%	$1,704.531	53%
7%	$1,811.969	60%
5%	$1,919.407	65%
1%	$1,935.935	66%
4%	$2,068.167	70%
9%	$2,200.398	79%
8%	$2,332.630	86%
4%	$2,464.861	90%
1%	$2,530.977	91%
1%	$2,696.266	92%
3%	$2,861.555	94%
3%	$3,026.845	98%
3%	$3,109.489	100%
100%		

Figure 31: VARG data for 7 line inflexible capacity decision making SUV assembly system

Appendix B
Decision Tree Analysis example for flexible capacity decision making systems

This example will illustrate how the decision tree analysis was carried out for a system with a flexible capacity decision making capability. That is to say that capacity decisions are made in period one and can be upgraded in period two depending on what is observed in period one. Figure 32 below is a representation of the flexible system decision tree. For illustration purposes, we shall focus on one branch of the tree just as we in Appendix A. This analysis is applied to an SUV system where the decision is to build 6 lines in period one with a possible upgrade to 7 lines in period two so the branch we are looking at corresponds to the one where decision one is to build 6 lines for assembly (i.e build large). Given that the system only permits upgrades in capacity or remaining at the same capacity and given that there are five possible capacity designs, there are a total of 5+4+3+2+1=15 possible capacity decision sets {D1,D2}.

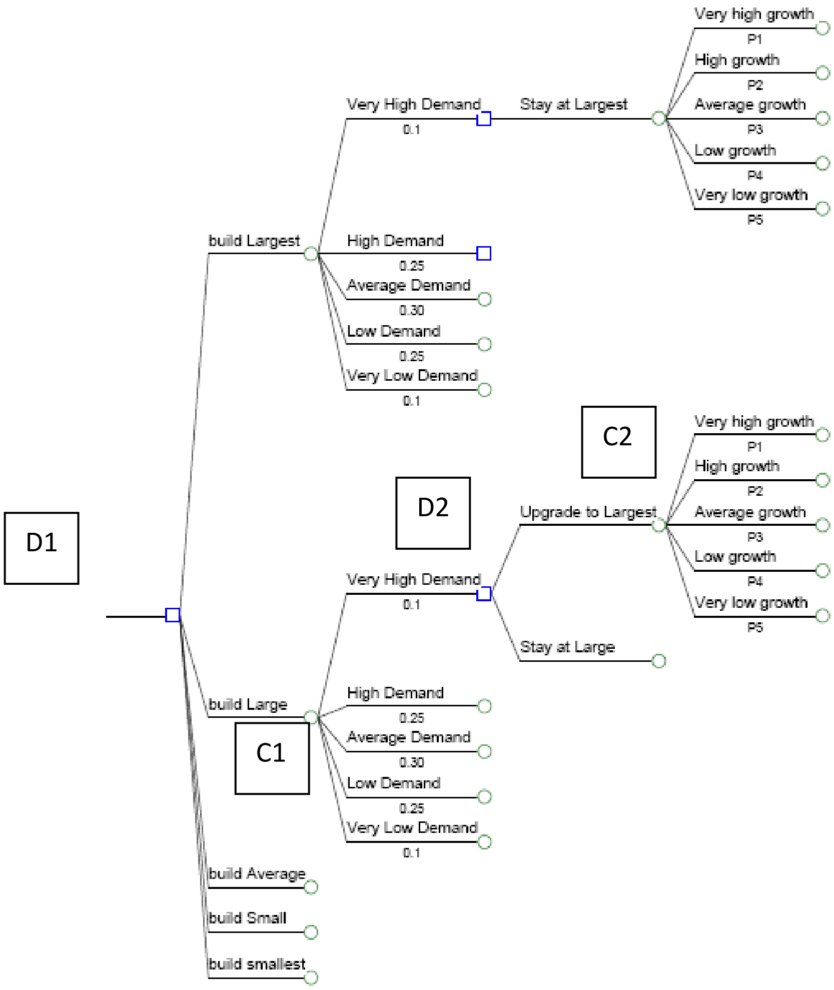


Figure 32: Partially filled decision tree depicting an assembly system with flexible capacity decision making, where capacity decisions are made in the first period and second period upgrades are possible. There are two chance (circle) and two decision nodes (square) resulting in a total of 375 payoffs

In this decision tree there are two decision nodes and two chance nodes and a total of 375 different payoffs. Given that there are 15 possible decision sets, each decision set consisting of decision one and decision two {D1,D2} will lead to 25 possible payoffs. Note that the payoffs are now a more complex function of separate capacity investments incurred in year one and year two. If D2 is an upgrade of D1 the cost incurred in year two is the difference in investments for the two capacities depreciated accordingly.

The rollback mechanism for evaluating the flexible system is also slightly different than the one outlined in Appendix A for the inflexible system. In the first step, the payoffs corresponding to decision set {D1,D2} are rolled back to the second chance node (C2) by multiplying the payoffs with the probability of growth rates which depend on first year demand. Summing these values of P*NPV up gives us the expected net present value of pursuing capacity decision two (D2) given capacity decision one (D1) and observed first year demand. Figure 33 shows the possible payoffs for decision sets {6,6} and {6,7} depending on first year demand and subsequent growth. Meanwhile, Figure 34 shows the P*NPV values as well as the expected net present values of choosing D2 (6 or 7) for different first year demands given that the first year capacity decision (D1) is 6.

D1	**D2**	**GROWTH RATE**				
6	**6**	**-0.30**	**-0.20**	**-0.10**	**0.00**	**0.10**
First Year Demand						
60,000		509,983,095	559,569,872	609,156,649	658,743,426	708,330,203
100,000		1,105,024,418	1,187,669,046	1,270,313,674	1,352,958,302	1,435,602,930
130,000		1,551,305,409	1,658,743,426	1,766,181,442	1,873,619,459	1,981,057,475
160,000		1,997,586,401	2,129,817,806	2,262,049,211	2,394,280,616	2,526,512,021
200,000		2,410,809,542	2,576,098,798	2,741,388,054	2,741,388,054	2,741,388,054

D1	**D2**	**GROWTH RATE**				
6	**7**	**-0.30**	**-0.20**	**-0.10**	**0.00**	**0.10**
First Year Demand						
60,000		431,038,726	480,625,503	530,212,280	579,799,057	629,385,834
100,000		1,026,080,048	1,108,724,677	1,191,369,305	1,274,013,933	1,356,658,561
130,000		1,472,361,040	1,579,799,057	1,687,237,073	1,794,675,090	1,902,113,106
160,000		1,918,642,032	2,050,873,437	2,183,104,842	2,315,336,247	2,447,567,652
200,000		2,331,865,172	2,497,154,429	2,662,443,685	2,827,732,941	2,910,377,569

Figure 33: SUV assembly system NPV of payoffs for the capacity decisions sets {6,6} and {6,7} given first year demands and subsequent growth rates.

D1	D2	GROWTH RATE					
6	6	-0.30	-0.20	-0.10	0.00	0.10	
First Year Demand		P*NPV ($)					ENPV ($)
Very Low		101,996,619	167,870,962	182,746,995	98,811,514	35,416,510	586,842,599
Low		165,753,663	267,225,535	444,609,786	270,591,660	107,670,220	1,255,850,864
Average		155,130,541	331,748,685	529,854,433	421,564,378	346,685,058	1,784,983,095
High		99,879,320	319,472,671	791,717,224	718,284,185	378,976,803	2,308,330,203
Very High		120,540,477	257,609,880	753,881,715	890,951,118	685,347,014	2,708,330,203

D1	D2	GROWTH RATE					
6	7	-0.30	-0.20	-0.10	0.00	0.10	
First Year Demand		P*NPV ($)					ENPV ($)
Very Low		86,207,745	144,187,651	159,063,684	86,969,859	31,469,292	507,898,230
Low		153,912,007	249,463,052	416,979,257	254,802,787	101,749,392	1,176,906,495
Average		147,236,104	315,959,811	506,171,122	403,801,895	332,869,794	1,706,038,726
High		95,932,102	307,631,016	764,086,695	694,600,874	367,135,148	2,229,385,834
Very High		116,593,259	249,715,443	732,172,013	919,013,206	727,594,392	2,745,088,313

Figure 34: P*NPV transform of Figure 33 as well as ENPV values for being at decision node 2 (D2=6 or 7) given first year demands and D1=6.

Now what we are at the second chance node (C2) we can roll back to the second decision node (D2) by identifying the optimal second year decision (D2) given observed first year demands. In essence, we are pruning the decision tree by eliminating the unprofitable D2s given D1=6 in this example. This is done by comparing the ENPVs of all decision sets {D1,D2} branching off of D1=6 for each first year demand scenario and choosing the value maximizing D2 in each case. Looking at Figure 34 on can tell that D2=6 gives the highest ENPV for observed first year demands ranging from very low (60k) to high (160k). If the observed demand is very high (200k) D2=7 representing an upgrade of capacity seems to be more profitable than D2=6. Hence an optimum flexible strategy for when D1=6 is formed: keep capacity at 6 lines unless first year demand is very high in which case expand to 7 lines. This is shown in Figure 35 alongside the corresponding ENPV. But is this the optimal capacity decision set, i.e. the optimal flexible design?

First Year Demand	D1	D2	ENPV ($)
Very Low	6	6	586,842,599
Low	6	6	1,255,850,864
Average	6	6	1,784,983,095
High	6	6	2,308,330,203
Very High	6	7	2,745,088,313

Figure 35: Optimal flexible capacity decision strategy based on observed first year demand with expected net present values. This is for the case where the first period capacity decision is 6 lines.

To check this the next step would be to roll back one more time to chance node one (C1) and compare the optimal decision sets for each D1 in order to get the optimal D1,D2 combination, i.e. the optimal flexible design strategy. In this example we only consider the possible decision sets branching off of D1=6 but a similar approach can be done for the other D1s.

The ENPV at D1=6 is equal to the sum of ENPVs identified in Figure 35 multiplied by the probability of first year demands. This is shown in Figure 36 below.

First Year Demand	D1	D2	ENPV	Probability	P*EV
Very Low	6	6	586,842,599	0.10	58684259.95
Low	6	6	1,255,850,864	0.25	313962716
Average	6	6	1,784,983,095	0.30	535494928.6
High	6	6	2,308,330,203	0.25	577082550.7
Very High	6	7	2,745,088,313	0.10	274508831.3
				SUM	1759733287

Figure 36: ENPV at D1=6 is given by the sum of the products of first year demand probability and expected net present value of optimal {D1,D2} decision sets.

This obtained ENPV, $1,759,733,287, for being at D1=6 is compared to all ENPVs of other D1s in the system to obtain the value maximizing flexible strategy. In the case of single style SUV assembly, the optimal flexible strategy happens to be that in Figure 36, namely to build 6 lines in period one and expand to 7 lines if very high first year demand I observed.

Just as in the inflexible decision tree analysis a VARG curve can be constructed for the optimal flexible strategies. The payoffs are different but the probabilities of the demands are identical. Figure 37 is the data used to obtain the VARG curve for a flexible SUV assembly system where the first year capacity decision is 6 lines with the option of expanding to 7 in the second year.

6-7 Lines flexible SUV

P	NPV	CumP
2%	509.9831	2%
3%	559.5699	5%
3%	609.1566	8%
2%	658.7434	10%
0%	708.3302	10%
4%	1105.024	14%
6%	1187.669	19%
9%	1270.314	28%
5%	1352.958	33%
2%	1435.603	35%
3%	1551.305	38%
6%	1658.743	44%
9%	1766.181	53%
7%	1873.619	60%
5%	1981.057	65%
1%	1997.586	66%
4%	2129.818	70%
9%	2262.049	79%
4%	2331.865	86%
8%	2394.281	90%
1%	2410.81	91%
1%	2497.154	92%
3%	2662.444	94%
3%	2827.733	98%
3%	2910.378	100%
100%		

Figure 37: VARG data for optimal flexible SUV assembly system where the strategy is to build 6 lines with the option of expanding to 7.

Appendix C
Dynamic Programming Implementation

Dynamic programming was implemented to derive the expected net present value of flexible assembly systems. Using the same terminology as Appendix B, this example will demonstrate how this was done to the SUV system where the decision set is {4,5}; that is the first capacity decision is to build 4 lines with an option of expanding to 5 lines if needed. The Lattice Analysis method is limited in that it can only consider one option at a time. Therefore the 15 possible decision sets {D1.D2} identified for the decision analysis have to be explored separately. The other limitation is that although we are considering a six year time period only one expansion option can be exercised.

The lattice analysis was done using an excel spreadsheet program originally constructed to represent a financial call option but modified to accommodate the cash flows associated with this model.

The first step is to obtain the projected demands. The up and down growth parameters are applied to initial demand at time zero in a multiplicative way demonstrated in Figure 38, where initial demand is normalized to 1. Using the up and down (u,d) parameters for SUV demand from Figure 19c as well as initial demand of 140k, one can derive the 6 year SUV demand projection shown in figure 39 below. In period 1 the possible demands are 140k*u = 155.33k and 140k*d=126.18k. In period 2 we have three possible demands; 155.33k*u, 155.33*d=126.18k*u=140k and 126.18k*d=113.73k and so forth. Path independence applies and the order of operations in multiplying by growth factor does not matter such that at each period N there are N+1 possible demands as shown in Figure 38.

Period	**0**	**1**	**2**	**3**	**4**	**5**	**6**
Demand	1.00	u	uu	uuu	**u^4**	u^5	u^6
		d	ud	udu	u^3d	u^4d	u^5d
			dd	udd	u^2d^2	u^3d^2	u^4d^2
				d^3	ud^3	u^2d^3	u^3d^3
					d^4	ud^4	u^2d^4
						d^5	ud^5
							d^6

Figure 38: u and d growth rates are applied in a multiplicative manner to derive demand projections. Here initial demand is normalized to 1.

SUV demand projections

Year	0	1	2	3	4	5	6
Demand (000s)	140	155.3321	172.3433	191.2175	212.1587	235.3933	261.1724
		126.1813	140	155.3321	172.3433	191.2175	212.1587
			113.7265	126.1813	140	155.3321	172.3433
				102.5011	113.7265	126.1813	140
					92.38367	102.5011	113.7265
						83.26491	92.38367
							75.04622

Figure 39: SUV 6 year demand projection using binomial lattice analysis

Just as the growth factors were multiplied to get a demand projection, probabilities can be multiplied to get the probabilities of being at each demand in the demand projection. Here the probability associated with the up state growth rate is p=0.05 and the down state growth rate 1-p=0.95. Hence 0.05 would replace u in Figure 38 while 0.95 would replace d. The resulting probabilities of being at each demand state from Figure 39 is shown in Figure 40.

Period	**0**	**1**	**2**	**3**	**4**	**5**	**6**
Probabilities:	1.00	0.05	0.00	0.000	0.000	0.000	0.000
		0.95	0.09	0.007	0.000	0.000	0.000
			0.91	0.131	0.013	0.001	0.000
				0.863	0.166	0.020	0.002
					0.821	0.197	0.028
						0.782	0.225
							0.744

Figure 40: Probabilities associated with SUV 6 year demand projection (Figure 39).

The next step is to obtain the cash flow and discounted cash flow from the demand projections. This is first done using D1=4 capacity decision. The cash flows and discounted cash flows are shown in Figure 41 and 42. Note that cash flows are profits obtained by using the profit equation explained in the section System Design Parameters and capital costs are depreciated for six years.

Period	**0**	**1**	**2**	**3**	**4**	**5**	**6**
4 Line Capacity							
Cash Flow:	0	648,942,283	648,942,283	648,942,283	648,942,283	648,942,283	648,942,283
		648,942,283	648,942,283	648,942,283	648,942,283	648,942,283	648,942,283
			586,207,149	648,942,283	648,942,283	648,942,283	648,942,283
				473,953,023	586,207,149	648,942,283	648,942,283
					372,778,980	473,953,023	586,207,149
						281,591,356	372,778,980
							199,404,439
PV Cash Flow:	0	589,947,530	536,315,936	487,559,942	443,236,311	402,942,101	366,311,001
		589,947,530	536,315,936	487,559,942	443,236,311	402,942,101	366,311,001
			484,468,718	487,559,942	443,236,311	402,942,101	366,311,001
				356,087,921	400,387,371	402,942,101	366,311,001
					254,613,059	294,287,538	330,898,653
						174,846,078	210,424,016
							112,558,607

Figure 41: Undiscounted and discounted cash flow projections for SUV assembly system with capacity = 4 lines.

To obtain the ENPV of the system with capacity inflexible and set at 4 lines, the undiscounted Cash Flows in Figure 41 are first multiplied by the probability lattice in Figure 40 to get probability weighted cash flows for each year. These annual values are then summed before being discounted appropriately. The sum of all discounted annual net present value cash flows is the ENPV of the system. This is shown in Figure 42.

Period	**0**	**1**	**2**	**3**	**4**	**5**	**6**
Probability Weighted Cash Flow	0	31,166,064	1,496,780	71,884	3,452	166	8
		617,776,219	59,338,568	4,274,686	273,728	16,433	947
			531,252,931	84,733,167	8,138,780	651,454	46,930
				408,893,909	97,154,367	12,913,179	1,240,335
					306,162,414	93,472,023	16,656,916
						220,163,282	83,985,463
							148,417,635

	0	**1**	**2**	**3**	**4**	**5**	**6**
E [Cash Flow]		648,942,283	592,088,279	497,973,645	411,732,741	327,216,537	250,348,234
PV(E[Cash Flow])		589,947,530	489,329,156	374,134,970	281,219,002	203,175,725	141,315,051
ENPV over 6 years	**2,079,121,434**						

Figure 42: Probability weighted cash flows and ENPV calculations for SUV assembly system with capacity set at 4 lines.

Before we can implement dynamic programming to analyze the flexible system consisting of decision set {D1,D2}, we need to get the cash flow and ENPV of the expanded assembly plant. The expanded assembly line system cash flows are shown in Figure 43. The ENPV is calculated in the same method as Figure 42 to be $1,571,957,593. This ENPV assumes we invest in the expanded plant right from the start.

Period	**0**	**1**	**2**	**3**	**4**	**5**	**6**
Cash Flow EXPANDED PLANT	0	811,177,854	811,177,854	811,177,854	811,177,854	811,177,854	811,177,854
		572,990,365	711,177,854	811,177,854	811,177,854	811,177,854	811,177,854
NOT dynamic programming approach (check current year)			448,442,721	572,990,365	711,177,854	811,177,854	811,177,854
				336,188,595	448,442,721	572,990,365	711,177,854
					235,014,552	336,188,595	448,442,721
						143,826,928	235,014,552
							61,640,010

Figure 43: Cash flow of SUV assembly plant with capacity decision set at 5 lines.

One can now implement dynamic programming to find the ENPV of having the flexible capacity decision set {D1,D2}={4,5}. This is done back to front starting at year 6 and ending at year 0. The resulting lattice is shown in Figure 44. At year 6 the cash flows are set to be those of the initial capacity decision (see Figure 41). Working backwards for any given demand projection at the end of year 5, we choose the maximum expected present value between the expanded plant and the non-expand plant. These are calculated by multiplying the cash flows of the two possible subsequent year six states (u,d) by the probability of up and down growth rates occurring and summing these probability weighted cash flows up. We then discount the max value by one year to bring it back to present value at the end of year 5, and add the cash flows received for year 5 depending on specific the demand projection for year 5. This is done iteratively always checking the maximum expected value of the next year until we reach year zero. The ENPV at this point is the expected net present value of the flexible design with option of expanding the plant. In Figure 44 it is shown in red and is equal to $2,079,371,057, higher than its inflexible counterpart ENPV calculated in Figure 42.

Note that whenever the maximum expected present value is that of the expended plant we note that down as an expansion decision that must be taken if we arrive at that year 5 demand projection. This process of dynamic programming progresses back and captures the profit maximizing flexible strategy. Whenever expanding the assembly plant proves to have a higher expected net present value in the future the expansion option is implemented. The strategy depends on what the observed demand is at any given point of time and is shown in the lattice in Figure 45.

Period	**0**	**1**	**2**	**3**	**4**	**5**	**6**
ENPV (Cash Flow)	**2,079,371,057**	2,833,483,026	2,776,182,176	2,601,403,296	2,056,771,616	1,386,376,696	648,942,283
WITH FLEXIBILITY TO EXPAND		2,259,754,301	2,384,167,966	2,233,427,765	1,981,874,541	1,386,376,696	648,942,283
Dynamic programming approach (check next year)			1,741,004,700	1,892,368,562	1,730,879,771	1,299,833,599	648,942,283
				1,238,893,553	1,349,450,403	1,184,596,886	648,942,283
					815,805,799	822,161,268	586,207,149
						470,437,642	372,778,980
							199,404,439

Figure 44: Lattice showing dynamic programming approach to evaluate ENPV of flexible design with decision set {D1,D2}={4,5}. The resulting ENPV is shown in red.

Period	**0**	**1**	**2**	**3**	**4**	**5**	**6**
Exercise CALL	NO	NO	**YES**	**YES**	**YES**	**YES**	=>
OPTION ?		NO	NO	NO	**YES**	**YES**	
			NO	NO	NO	**YES**	
				NO	NO	NO	
					NO	NO	
						NO	

Figure 45: Strategy for implementing expansion decision (to plant capacity of 5 lines from 4 lines originally) depending on current observed demand.

This dynamic programming approach is applied to all 15 possible flexible decision sets in each assembly systems {D1,D2}.The optimal flexible strategy is then identified based on ENPV.

The final payoffs taking all six years into account will depend on the path taken across the lattice to reach a possible final demand state. If the path crosses into a demand state that calls for capacity expansion the cash flow is taken from the expanded plants cash flow instead of the unexpanded plant cash flow. All possible paths with the resulting total payoffs can be enumerated alongside their probability of occurring. For each final demand state the probability of any path leading to it is equal. The resulting possible payoffs and the number of paths leading to the final demand states are different however. Figure 46 below is the enumeration of paths and payoffs for the flexible design {4,5} used in this example. The enumeration tree is helpful in constructing VARG curves by using NPV of payoffs alongside their probability to derive cumulative probability of payoffs. To see how this is done refer to Appendix B.

Final State	u^6	u^5*d	u^4*d^2	u^3*d^3	u^2*d^4	u*d^5	ud^6
Number of Paths	1	6	15	20	15	6	1
P(PATH)	1.22703E-08	2.43223E-07	4.82118E-06	9.55659E-05	0.001894315	0.037549266	0.74430457
ENUMERATION OF PATHS (NPV)	$3,251,325,161	$3,251,325,161	$3,251,325,161	$3,194,877,768	$2,830,373,598	$2,518,922,333	$1,972,521,914
		$3,251,325,161	$3,251,325,161	$3,194,877,768	$2,790,900,473	$2,518,922,333	
		$3,251,325,161	$3,251,325,161	$3,126,576,422	$2,790,900,473	$2,467,075,115	
		$3,251,325,161	$3,251,325,161	$2,978,680,731	$2,748,051,533	$2,335,603,095	
		$3,018,626,096	$3,251,325,161	$2,962,178,703	$2,639,396,970	$2,189,828,783	
		$3,018,626,096	$3,183,023,815	$2,861,443,178	$2,790,900,473	$2,070,387,323	
			$3,018,626,096	$2,826,312,820	$2,790,900,473		
			$3,018,626,096	$2,861,443,178	$2,748,051,533		
			$2,917,890,571	$2,826,312,820	$2,639,396,970		
			$2,917,890,571	$2,783,463,880	$2,739,053,255		
			$3,018,626,096	$2,962,178,703	$2,696,204,315		
			$3,018,626,096	$2,861,443,178	$2,587,549,752		
			$2,917,890,571	$2,826,312,820	$2,564,732,294		
			$2,917,890,571	$2,861,443,178	$2,456,077,732		
			$2,866,043,353	$2,826,312,820	$2,310,303,421		
				$2,783,463,880			
				$2,809,595,960			
				$2,774,465,602			
				$2,731,616,662			
				$2,600,144,642			

Figure 46: Enumeration of possible payoffs alongside their probability for SUV assembly system flexible design with decision set {4,5}.