Evaluation of Flexibility for a Primary Residence

Michael Pasqual
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Outline

• Motivation
• System Definition
• Model
• Uncertainties
• Design Concepts
• Decision Analysis
• Lattice Analysis
• Simulation
• Conclusions
Motivation

- Fiance and I need a home for when we get married
- Housing market is perfect
  - Low prices
  - Low mortgage rates

System Definition

- The system is our primary residence
- Demand-capacity challenge
  - Capacity: #bedrooms in home
  - Demand: #family members
- Benefits
  - My focus: shelter
  - Many others: stability, security, tax benefits, equity
- Design variables
  - My focus: #bedrooms
  - Many others: location, age, external/internal features, aesthetics, appreciation potential
Model

- **Timeline (30 years)**
  - Buy home in Year 0
  - Have children until Year 10
  - Family lives in home until Year 30
  - Sell home in Year 30

- **Home prices**
  - Constant at current median prices
  - Rationale: we are buying in near future, inflation-adjusted house prices usually steady over time

- **Benefits**
  - Occupied BR delivers $6000/year
    - Based on ½ value of a $1000/mo 1BR apartment
  - Children can share rooms if necessary
  - Unoccupied rooms deliver no value

Uncertainties

- **Family size**
  - Probability of having a child in a 2-year period is \( p = 0.8 \)
  - \#children \( N_{10} \) in 2M years is a binomial distribution:
    \[
    P(N = n) = \binom{M}{n} p^n (1 - p)^{M-n}
    \]
  - \( E[N_{10}] = 4, \sigma_{N_{10}} = 0.89 \)

- **Home/building prices**
  - Assume constant for model, but obviously uncertain too
  - Median prices rounded to nearest $50K (zillow.com)
  - Adding bedrooms cost $50K/BR (costhelper.com)
Design Concepts

- **Big House (fixed)**
  - Buy 5BR house for $500K in Year 0
  - Delivers $30K/yr at capacity
  - Sell 5BR house for $500K in Year 30
  - Note: 5BR determined to be optimal fixed design

- **Small House (flexible)**
  - Buy 3BR house for $400K in Year 0
  - Delivers $18K/yr at capacity
  - Expansion capability in Year 8: add BRs for $50K/BR
  - Sell 3BR house for $400K in Year 30

- **Condo (flexible)**
  - Buy 2BR condo for $350K in Year 0
  - Delivers $12K/yr at capacity
  - Expansion capability in Year 8: sell condo and buy a 4BR or 5BR house
    - Incur seller closing cost of $35K
  - Sell condo or 4/5BR house in Year 30

Cash Flows

<table>
<thead>
<tr>
<th>Design Concept</th>
<th>Year</th>
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<tbody>
<tr>
<td></td>
<td>0</td>
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<tr>
<td>Big House</td>
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<tr>
<td>Small House</td>
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</tr>
<tr>
<td>Condo</td>
<td>-350</td>
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</table>
Decision Tree

- 2-stage decision tree
  - Stage 1: choose design concept in Year 0, then have children for 6 years
  - Stage 2: choose how/whether to expand in Year 8, then have children for 4 more years

<table>
<thead>
<tr>
<th>1st Stage</th>
<th>2nd Stage</th>
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<tbody>
<tr>
<td>1st Decision: [HPV]</td>
<td>YI/YI (child?)</td>
</tr>
<tr>
<td>Big House</td>
<td>Y or N</td>
</tr>
<tr>
<td>Small House</td>
<td>Y or N</td>
</tr>
<tr>
<td>Condo</td>
<td>Y or N</td>
</tr>
</tbody>
</table>

Solve Decision Tree

Stage 2

- e.g., Small House – maximize E[NPV] of 2nd decision
Solve Decision Tree
Fold-back

- e.g., Small House – Stage 2 pruned and folded back

<table>
<thead>
<tr>
<th>1st Decision</th>
<th>E[NPV]</th>
<th>V2 (K)</th>
<th>V4 (K)</th>
<th>V5 (K)</th>
<th>Chance Outcomes (%)</th>
<th>2nd Decision</th>
<th>E[NPV]</th>
<th>V2 (K)</th>
<th>V4 (K)</th>
<th>V5 (K)</th>
<th>Chance Outcomes (%)</th>
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<th>NPV</th>
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Decision Analysis
Results

- Multiple Criteria
- Value-At-Risk-Gain (VARG) curve
- Conclusion
  - Small House is best by all metrics (except CAPEX)
  - Big House is second best

<table>
<thead>
<tr>
<th>Design Concept</th>
<th>Criteria</th>
<th>ROI (E[NPV] / CAPEX)</th>
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<tbody>
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## Lattice Development

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### Probability Lattice

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### Sum Check

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<th>1.00</th>
<th>1.00</th>
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</thead>
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## Lattice Analysis

- **Start with Small House**
- **Call option:** to add 2BRs for $100K
- **Is it optimal to expand in next period?**

<table>
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<tr>
<th>Years</th>
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</table>

- **Expand if...**
  - 2 children by Yr 4
  - 2 children by Yr 6
  - 3 children by Yr 10
Simulation

- Monte Carlo simulation
  - 4000 trials for each design concept

- VARG curves match those from Decision Analysis (as expected)

Conclusions

- Design concept ranking
  - Small House – defers & avoids costs
  - Big House – meets point estimate
  - Condo – hurt by seller closing costs

- Lattice analysis most helpful because it allowed expansion in any period
  - DA and simulation only allowed expansion in Yr 8

- Familiar now with mechanics of methods

- Proficient in Excel