

Evaluation of Flexibility for a Primary Residence

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Outline

- **Motivation**
- **System Definition**
- **Model**
- **Uncertainties**
- **Design Concepts**
- **Decision Analysis**
- **Lattice Analysis**
- **Simulation**
- **Conclusions**

Motivation

- **Fiance and I need a home for when we get married**
- **Housing market is perfect**
 - Low prices
 - Low mortgage rates

System Definition

- The system is our primary residence
- **Demand-capacity challenge**
 - Capacity: #bedrooms in home
 - Demand: #family members
- **Benefits**
 - My focus: shelter
 - Many others: stability, security, tax benefits, equity
- **Design variables**
 - My focus: #bedrooms
 - Many others: location, age, external/internal features, aesthetics, appreciation potential

Model

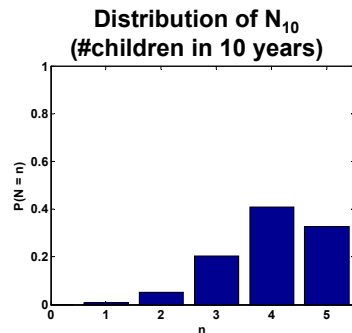
- **Timeline (30 years)**
 - Buy home in Year 0
 - Have children until Year 10
 - Family lives in home until Year 30
 - Sell home in Year 30
- **Home prices**
 - Constant at current median prices
 - Rationale: we are buying in near future, inflation-adjusted house prices usually steady over time
- **Benefits**
 - Occupied BR delivers \$6000/year
 - Based on ½ value of a \$1000/mo 1BR apartment
 - Children can share rooms if necessary
 - Unoccupied rooms deliver no value

Uncertainties

- **Family size**
 - Probability of having a child in a 2-year period is $p = 0.8$
 - #children N_{2M} in $2M$ years is a binomial distribution:

$$P(N = n) = \binom{M}{n} p^n (1-p)^{M-n}$$

- $E[N_{10}] = 4, \sigma_{N_{10}} = 0.89$



- **Home/building prices**
 - Assume constant for model, but obviously uncertain too
 - Median prices rounded to nearest \$50K (*zillow.com*)
 - Adding bedrooms cost \$50K/BR (*costhelper.com*)

Home Type	Median Price (\$K)
2BR Condo	350
3BR Condo	400
4BR House	450
5BR House	500

Design Concepts

- **Big House (fixed)**
 - Buy 5BR house for \$500K in Year 0
 - Delivers \$30K/yr at capacity
 - Sell 5BR house for \$500K in Year 30
 - *Note: 5BR determined to be optimal fixed design*

- **Small House (flexible)**
 - Buy 3BR house for \$400K in Year 0
 - Delivers \$18K/yr at capacity
 - Expansion capability in Year 8: add BRs for \$50K/BR
 - Sell 3BR house for \$400K in Year 30

- **Condo (flexible)**
 - Buy 2BR condo for \$350K in Year 0
 - Delivers \$12K/yr at capacity
 - Expansion capability in Year 8 : sell condo and buy a 4BR or 5BR house
 - Incur seller closing cost of \$35K
 - Sell condo or 4/5BR house in Year 30

Cash Flows

<u>Design Concept</u>	<u>Year</u>							
	0	1	2-3	4-5	6-7	8-9	10-29	30
Big House	-500	6	6(N ₂ +1)	6(N ₄ +1)	6(N ₆ +1)	6(N ₈ +1)	6(N ₁₀ +1)	500
Small House	-400	6	6(N ₂ +1)	6(N ₄ +1)	6(N ₆ +1)	6(N ₈ +1) -50x #BRs Added	6(N ₁₀ +1)	400 + 50 x #BRs Added
Condo	-350	6	6(N ₂ +1)	6(N ₄ +1)	6(N ₆ +1)	6(N ₈ +1) or 6(N ₈ +1) +315 - \$New Home	6(N ₁₀ +1)	300 or \$New Home

Decision Tree

- 2-stage decision tree
 - Stage 1: choose design concept in Year 0, then have children for 6 years
 - Stage 2: choose how/whether to expand in Year 8, then have children for 4 more years

1st Stage					2nd Stage							
1st Decision	E[NPV]	Y2/Y4/Y6 Outcome (child?)	Chance Outcome (N_6)	p	NPV	2nd Decision	E[NPV]	Y8/Y10 Outcome (child?)	Chance Outcome (N_{10})	p	NPV	
Start						→	N/A					
Big House	?	Y or N	0, 1, 2, or 3	?	?	→	N/A	Y or N	0, 1, 2, 3, 4, or 5	?	?	
Small House	?	Y or N	0, 1, 2, or 3	?	?	→	Add 2BR Add 1BR Nothing	Y or N	0, 1, 2, 3, 4, or 5	?	?	
Condo	?	Y or N	0, 1, 2, or 3	?	?	→	Buy 5BR Buy 4BR Nothing	Y or N	0, 1, 2, 3, 4, or 5	?	?	

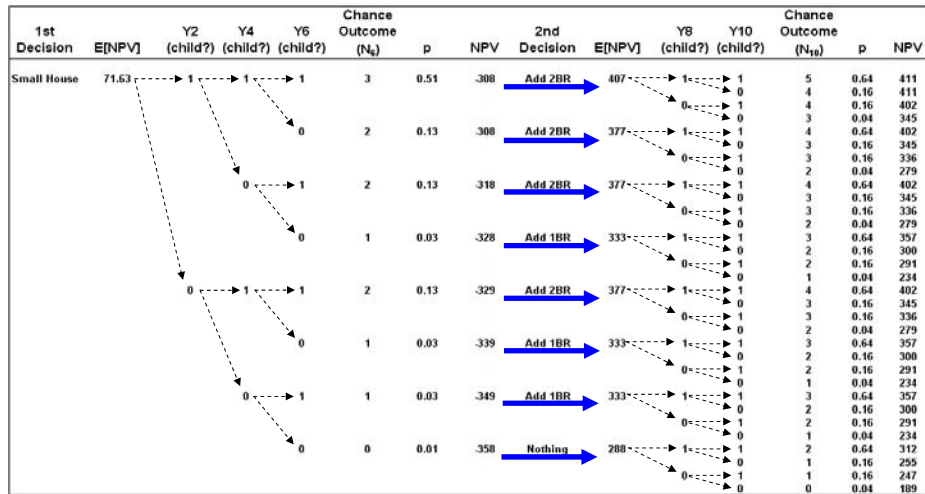
Solve Decision Tree Stage 2

- e.g., Small House – maximize E[NPV] of 2nd decision

Chance Outcome (N_6)	2nd Decision	E[NPV]	Y8 (child?)	Y10 (child?)	Chance Outcome (N_{10})	p	NPV
2	Add 2BR	377	1	1	4	0.64	402
			0	3	0.16	345	
			0	3	0.16	336	
	Add 1BR	335	1	4	0.64	324	
			0	3	0.16	366	
			0	3	0.16	357	
	Nothing	321	1	4	0.64	321	
			0	3	0.16	321	
			0	3	0.16	321	
1	Add 2BR	311	1	3	0.64	336	
			0	2	0.16	279	
			0	2	0.16	270	
	Add 1BR	333	1	3	0.64	357	
			0	2	0.16	300	
			0	2	0.16	291	
	Nothing	317	1	3	0.64	321	
			0	2	0.16	321	
			0	2	0.16	312	
			1	0.04	255		

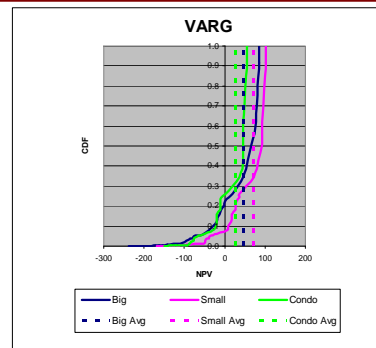
Solve Decision Tree Fold-back

- e.g., Small House – Stage 2 pruned and folded back



Decision Analysis Results

- Multiple Criteria
- Value-At-Risk-Gain (VARG) curve
- Conclusion
 - Small House is best by all metrics (except CAPEX)
 - Big House is second best



Design Concept	Criteria					
	E[NPV] (\$K)	P ₁₀ (\$K)	P ₅₀ (\$K)	P ₉₀ (\$K)	CAPEX (\$K)	ROI (E[NPV] / CAPEX)
Big House	47	-30	67	85	500	9.4%
Small House	72	-8	94	100	400	18.0%
Condo	29	-22	47	55	350	8.3%

Lattice Development

Years	0	2	4	6	8	10	12
<u>Outcomes</u>	0	1	2	3	4	5	6
<u>Lattice</u>		0	1	2	3	4	5
			0	1	2	3	4
				0	1	2	3
					0	1	2
						0	1
							0
<u>Probability</u>	1.00	0.80	0.64	0.51	0.41	0.33	0.00
<u>Lattice</u>		0.20	0.32	0.38	0.41	0.41	0.33
			0.04	0.10	0.15	0.20	0.41
				0.01	0.03	0.05	0.20
					0.00	0.01	0.05
						0.00	0.01
							0.00
<u>Sum Check</u>	1.00	1.00	1.00	1.00	1.00	1.00	1.00

#children

Lattice Analysis

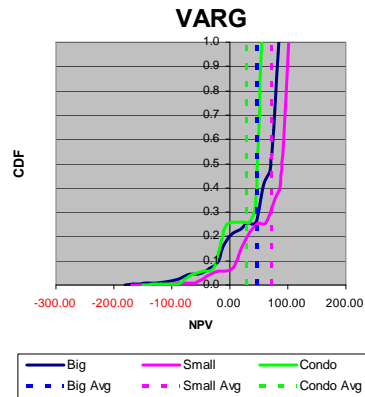
- Start with Small House
- Call option: to add 2BRs for \$100K
- Is it optimal to expand in next period?

Years	0	2	4	6	8	10	12
<u>Exercise Option?</u>	NO	NO	YES	YES	YES	YES	
		NO	NO	YES	YES	YES	
			NO	NO	NO	YES	
				NO	NO	NO	
					NO	NO	
						NO	

- Expand if...
 - 2 children by Yr 4
 - 2 children by Yr 6
 - 3 children by Yr 10

Simulation

- **Monte Carlo simulation**
 - 4000 trials for each design concept
- **VARG curves match those from Decision Analysis (as expected)**



Conclusions

- **Design concept ranking**
 - Small House – defers & avoids costs
 - Big House – meets point estimate
 - Condo – hurt by seller closing costs
- **Lattice analysis most helpful because it allowed expansion in any period**
 - DA and simulation only allowed expansion in Yr 8
- **Familiar now with mechanics of methods**
- **Proficient in Excel**