Evaluation of Flexibility for a Primary Residence

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Abstract

In this paper, we apply real-options analysis to the design – specifically the selection and possible expansion - of a primary residence. A primary residence can be viewed as a system that delivers benefit by providing shelter, specifically bedrooms, to family members. Due to the uncertainty in the size of the family over time, the best system design is not so obvious. We face a challenge to build capacity (i.e.., bedrooms) to meet uncertain demand (i.e., parents and children who need a place to sleep). Three design concepts are considered: (1) a fixed Big House, (2) a flexible Small House, or (3) a flexible Condo. We analyze the alternatives using decision analysis, lattice analysis, and simulation. Results from all three methods reveal that the Small House concept yields the highest expect net-present-value, in addition to other criteria.

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Introduction

The topic for this Application Portfolio (AP) is inspired by an important decision that my fiancé and I are currently facing. Namely, we are in the process of buying a home for our eventual primary residence.

Our desire to buy a primary residence presents a design challenge. The residence must have enough bedrooms (capacity) to shelter ourselves and future children (demand), but we must recognize that future demand is uncertain. This AP will explore how and why we might incorporate flexibility into our design decisions. We will analyze the system using three different methods: (1) decision analysis, (2) lattice analysis, and (3) simulation.

System Definition

The system we are designing is simply our primary residence. The system can take many forms, such as a single-family house or a condominium. We might consider buying a large residence initially, or instead starting small and then upgrading or expanding later. Later we will quantify and explore the various strategies.

The system's benefit will be to provide a place for our family to live, i.e. the two of us, and any future children. Of course, home ownership also delivers many others benefits, including stability, security, tax benefits, and the accumulation of equity.

As real estate goes, the system has numerous design variables, e.g. location, size, age, external and internal features, aesthetics, and the potential for appreciation. My analysis only focuses on a single, but significant, design variable: the *number of bedrooms*. The number of bedrooms is essentially the system's capacity, which delivers value by meeting demand (i.e., family members in need of bedrooms). Of course, it costs more money to buy or build more bedrooms, which is where flexible expansion may provide savings.

Model

Analysis of the system requires us to quantitatively model the system.

First, we assume that we will only try to have children over a 10 year period. We also suppose that the children will live in the house for 20 years beyond the end of the childbearing 10 year period. Thus, the system's total lifetime will be 30 years.

Next, we assume deterministic median home prices and average building prices, as specified in the next section ('Principal Uncertainties'). Due to the current economic environment, we will very likely buy a home within the next year. Given our time horizon, home prices are not subject to as much uncertainty as they would over many years, especially since the federal government is continuing its efforts to stabilize the housing market. Thus, we will use a constant model of home prices. Furthermore, to avoid the complexity of calculating principal-interest breakdowns of mortgage payments (particularly when we potentially sell our home), we will assume that we can completely pay for a home upfront with cash. Finally, after 30 years, we'll assume that we'll sell the home for whatever we bought it for (plus the price of added bedrooms). The reasoning here is that we would downsize or move to Florida once the children have moved out.

We also need to quantify the benefits of the system. For simplicity, we will assume that an occupied bedroom, whether it's the master bedroom or a child's bedroom, delivers an annual benefit of $6000. An average 1-bedroon apartment (in the Boston area) usually rents for $1000/month, and if we assign half that amount to the bedroom itself, the value of a bedroom is 12*$500 = $6000 per year. A vacant bedroom, i.e., one without any family member living in it, delivers no benefit. In essence, excess capacity has no value. Also, if at any point in time we have more family members than bedrooms, then children will share bedrooms as needed, but a shared bedroom will still only deliver $6000 per year. This could very well be the case in some situations. Finally, we will ignore the value of the kitchen, living room, etc., because all design concepts will provide these at roughly the same level.

Discount Rate

Quantitative analysis also requires us to establish a discount rate. In lieu of buying a home, we would likely put our monthly paychecks into a liquid and low-risk account (after funding retirement accounts and paying off other debt). As a result, we will use an annual discount rate of 4%, which I judge to be a decent estimate of the average long-term interest rate earned in a savings account (or money market).

Based on the discount rate (DR), we can calculate the present value (PV) of a future cash flow (P) with the following equation:

$$PV = \frac{P}{(1 + DR)^n}$$

To calculate the net present value (NPV) of the system after 30 years of cash flows, we use the following equation:

$$NPV = \sum_{n=0}^{30} \frac{P}{(1 + DR)^n}$$

Principal Uncertainties

We face several major uncertainties as we buy our primary residence, especially (1) our family size and (2) home prices. While both uncertainties are described below, I plan to focus only on family size in the analysis to follow.

Family size

The first uncertainty is in the size of our family, particularly depending on how many children we have in the future. Our situation is similar to that of a production facility designer who wishes to have the capacity to meet uncertain future demand. In our case, we need bedrooms (capacity) to accommodate an uncertain number of family members (demand).

Of course, the ultimate size of our family is uncertain. Firstly, we don't know how many children we want. Furthermore, we also must consider the biological reality that having children is not a guaranteed blessing.

Data

According to the Mayo Clinic, a couple usually has an 85% chance of becoming pregnant within one year, which gives a 97% chance every two years. We can also suppose a, say 0.8, probability of us trying to have another child every two years. As a result, we can roughly claim that every two years, there is a p = 0.8 probability (from 0.8*0.97) that we have another child.

Model

Given this setup, we can use a binomial distribution to model the total number of children, N, we have. We can view every two-year period as a trial with probability of success (having a child) equal to p. For a time span M years, the distribution becomes the following, for values of 0 to M:

$$P(N = n) = \binom{M}{n} p^n (1 - p)^{M-n}$$

Figure 1 shows the distribution of N for p = 0.8 over a time span of 10 years (5 trials), which has an average and standard deviation of 4 and 0.89 children, respectively. We will use this exact distribution for analyzing system design concepts.

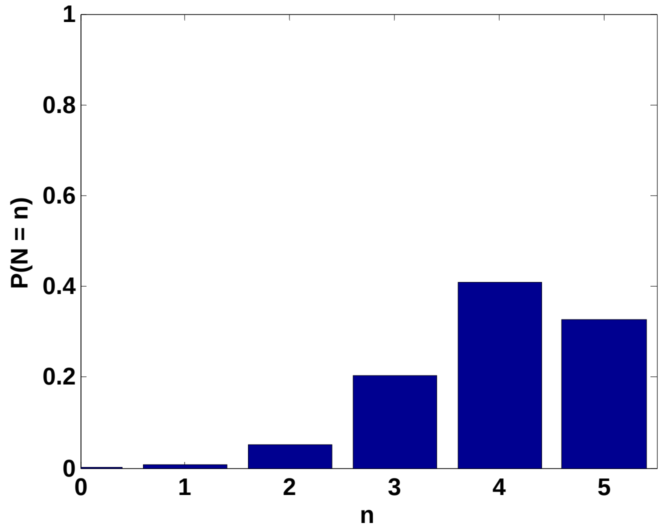

Figure 1: Binomial Distribution of Number of Children

Home Prices

The housing market theoretically presents more uncertainty for our design decisions. To analyze the uncertainty, we would need a probabilistic model that captures the price-range of houses in a particular location, as well as the direction of prices over time.

Data

The price range of homes for buyers and sellers depends significantly on the local housing market. We will consider house prices in the suburbs around Medford, Massachusetts, which is a likely area for us to house-hunt.

In developing system concepts, we might be interested in the prices of a 2-bedroom (BR) condominium, 3-BR house, 4-BR house, and 5-BR house. Using *zillow.com*, we surveyed the prices of homes sold in the last six months in the Medford, MA area. The results are found in Table 1 below.

Table 1: Home Prices in Medford, MA area

Home Type	Min ($K)	Mean ($K)	Median ($K)	Max ($K)
2 BR condo	170	345	351	540
3 BR house	207	439	425	600
4 BR house	264	457	470	575
5 BR house	350	488	502	585

For the direction of prices in the future, we turn to the history of house prices in the United States. Figure 2 from *The Business Insider*) shows national, inflation-adjusted, median house prices for last 120 years.

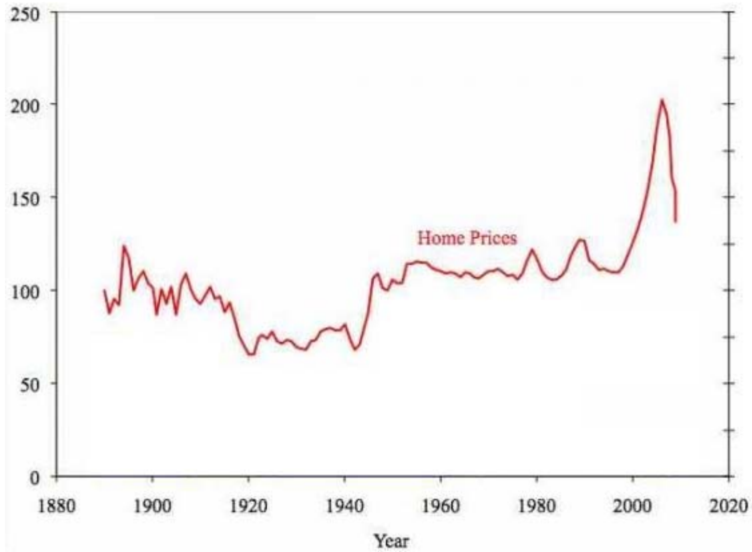

Figure 2: History of Inflation-adjusted Home Prices

House prices were relatively steady from the end of World War II to the 1990s, followed by a steep rise in prices until 2006. Since then, the current economic crisis and onslaught of unemployment and foreclosures has caused house prices to drop significantly. Today in 2009, many analysts anticipate a continued fall in prices for the next year or so, followed by a rebound and flat prices after 2012. However, the direction of future house prices obviously remains uncertain.

One last useful piece of data is the price of adding bedrooms to a house. According to *costhelper.com*, it would cost (paying a contractor) $20,000 for the addition of a basic 10'x15' bedroom. The addition of a bathroom costs around $50,000, which realistically might be necessary if our family is getting bigger.

Model

Due to the current economic environment, we will very likely buy a home within the next year. Given our time horizon, home prices are not subject to as much uncertainty as they would over many years, especially since the federal government is continuing its efforts to stabilize the housing market. Thus, we will use a constant model for home prices.

We assume that a certain type of home will cost the median price from Table 1, but rounded to the nearest $50K. This nicely sets the prices of a 3-BR, 4-BR, and 5-BR house to $400K, $450K, ad $500K (with a 2-BR condo at $350K). I will also assume that these prices will hold constant over the next 10 years, for instances of selling and buying again. This seems adequate because since 1950, US house prices have generally been flat with occasional spikes.

For simplicity, we will also assume that the price of adding to a pre-existing house is $50K per bedroom. This is a rough estimate reflecting the numbers from *costhelper.com*. Moreover, $50K is pleasingly the price difference between a 3- & 4- BR house and 4- & 5-BR house.

Design Concepts

We now quantify the following three major design concepts for the system:

1. **Big House (fixed) –** buy a 5-BR house in Year 0 for $500K. This is a fixed design. It can deliver 5 BRs of benefit ($30K/yr) at full capacity. Our family will live in the house until Year 30, at which point we sell the house for $500K.
2. **Small House (flexible) –** buy a 3-BR house in Year 0 for $400K. This design can initially only deliver 3 BRs of benefit ($18K/yr) at full capacity. However, in Year 8, we can expand the home by adding 1 or 2 bedrooms for $50K/BR. Our family will live in the house until Year 30, when we sell the house for $400K plus $50K per added BR.
3. **Condo (flexible) –** buy a 2-BR condominium in Year 0 for $350K. This design can initially only deliver 2 BRs of benefits ($12K/yr) at full capacity. In Year 8, because we cannot add bedrooms to a condo, we can expand by selling the condo and buying a 4-BR house ($450K) or 5-BR house ($500K) (plus a seller's closing cost of $35K), according to some decision rule. Our family will live in the condo/house until Year 30, when we sell the condo/house for $350K (condo), $450K (4-BR house), or $500K (5-BR house).

In the flexible design concepts (i.e., Small House and Condo), we employ a decision rule to expand in Year 8. For the Small House, we decide how many bedrooms to add; for the Condo, we decide whether to sell the condo and buy a larger home. The Condo concept is similar to the "starter-home" strategy sometimes recommended to first-time home buyers.

Cash Flows

Let's look at the cash flow of each design concept, as shown in Table 2 in $K. In Year 0, each design concept incurs the cost of the home. Only the master bedroom is occupied (for $6000/yr) in Year 1, because no children are possible yet. However, in Years 2-3, 4-5, 6-7, 8-9, and 10-29, a benefit of $6K per BR is delivered by the master bedroom and bedrooms occupied by N_2, N_4, N_6, N_8, and N_{10} children, respectively. In Year 8, we potentially incur expansions costs for the Small House ($50K per added BR) and Condo (selling and buying) concepts. In Year 30, we sell the home at median market price.

Table 2: Cash Flow ($K) for each design concept

Design Concept	**Year**							
	0	1	2-3	4-5	6-7	8-9	10-29	30
Big House	-500	6	$6(N_2+1)$	$6(N_4+1)$	$6(N_6+1)$	$6(N_8+1)$	$6(N_{10}+1)$	500
Small House	-400	6	$6(N_2+1)$	$6(N_4+1)$	$6(N_6+1)$	$6(N_8+1)$ -50x #BRs Added	$6N_{10}+1)$	400 + 50x # BRs Added
Condo	-350	6	$6(N_2+1)$	$6(N_4+1)$	$6(N_6+1)$	$6(N_8+1)$ or $6(N_8+1)$ + 315 - $New Home	$6(N_{10}+1)$	350 or $New Home

Rationale for Fixed Design

Under the fixed or Big House concept, we would buy a 5-BR house. To determine that a 5-BR house was the optimal fixed design, I ran the model for fixed designs of varying size, i.e., a house with 3, 4, 5, or 6 bedrooms.

Figure 3 shows the results, which indicate that a 5-BR house (in red) is the optimal fixed design, with E[NPV] = $47K.

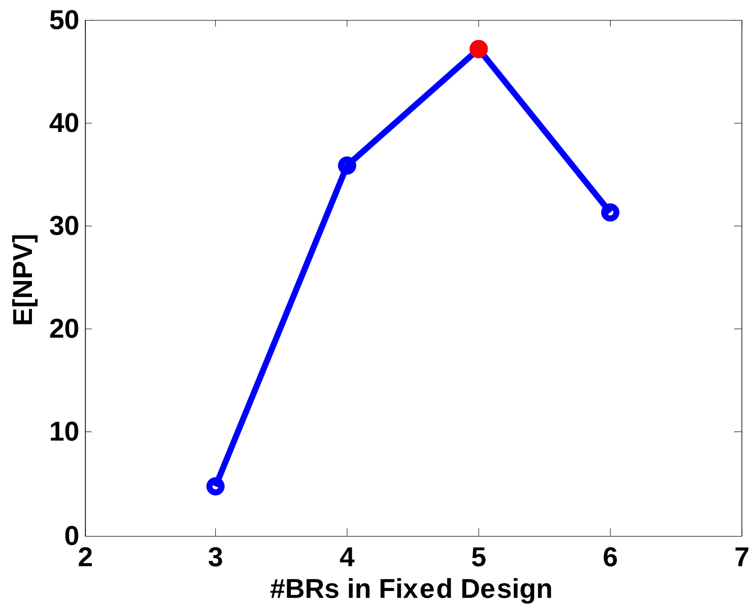

Figure 3: E[NPV] for fixed designs of varying size

Decision Analysis

We now proceed with decision analysis. We can set up a 2-stage decision tree, where each stage requires a design decision followed by chance outcomes. In Stage 1, our decision is to choose a design concept. Then for the first 6 years of the system, there are three opportunities (Years 2, 4, 6) to have children. In Stage 2, we make a decision to adjust the flexible designs (Small House and Condo) if desired. Depending on N_6, (the number of children born by Year 6), in Year 8 we might add 0, 1, or 2 BRs to the Small House, or sell the Condo and buy a 4- or 5-BR house. Meanwhile, the Big House concept is fixed and cannot be altered. After the possible expansion, there are two more opportunities (Years 8, 10) to have children.

Figure 4 shows the 2-stage decision tree, as described above, but condensed for display purposes. Once we run the decision tree through the model, we will be able to fill in the "?s" with probabilities, NPVs, and E[NPV]s.

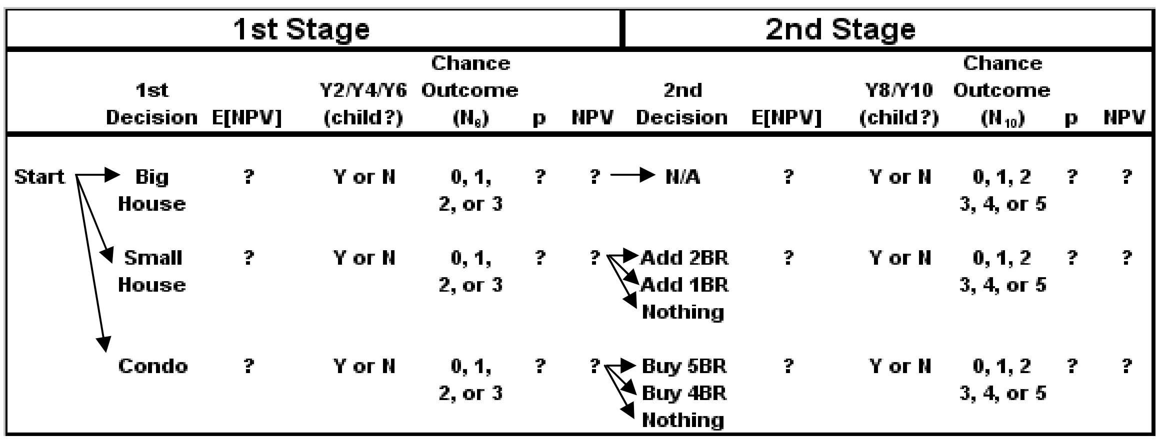

Figure 4: 2-Stage Decision Tree

According to decision analysis, the optimal design concept will be the one with the highest expect NPV, after averaging over all chance outcomes after each decision is made. We will solve the decision tree by first solving Stage 2 and then folding back the results to solve Stage 1.

Stage 2

We must first solve Stage 2 of the decision tree. In other words, we must find the optimal 2^{nd} decision, given each possible chance outcome N_6 from Stage 1. For the Big House concept, there is no 2^{nd} decision, because it is a fixed design. For the Small House concept, we decide whether to add 1 BR, 2 BRs, or do nothing. For the Condo concept, we decide whether to sell the condo and buy a 4- or 5-BR house, or do nothing.

For each design concept, we first calculate the E[NPV] of each chance outcome N_6 and 2nd decision choice. Then we find which decision yielded the highest E[NPV] for each chance outcome N_6.

Figure 5, Figure 6, and Figure 7 show Stage 2 for the Big House, Small House, and Condo concepts, respectively. In each figure, the arrows of the optimal 2nd decision are bolded and highlighted in blue.

As indicated in the figures, the optimal 2nd decisions are:

- **Big House**: n/a
- **Small House**: Add 2 BR, if N_6 = 2 or 3
Add 1 BR, if N_6 = 1
Nothing, if N_6 = 0
- **Condo**: Sell condo and buy a 5-BR house, if N_6 = 2 or 3
Sell condo and buy a 4-BR house, if N_6 = 1
Nothing, if N_6 = 0

Big House – Stage 2

Chance Outcome (N_6)	2nd Decision	E[NPV]	Y8 (child?)	Y10 (child?)	Chance Outcome (N_{10})	p	NPV
3	N/A	480	1	1	5	0.64	484
				0	4	0.16	484
			0	1	4	0.16	475
				0	3	0.04	418
2	N/A	450	1	1	4	0.64	475
				0	3	0.16	418
			0	1	3	0.16	409
				0	2	0.04	352
1	N/A	384	1	1	3	0.64	409
				0	2	0.16	352
			0	1	2	0.16	343
				0	1	0.04	286
0	N/A	319	1	1	2	0.64	343
				0	1	0.16	286
			0	1	1	0.16	277
				0	0	0.04	220

Figure 5: Stage 2 of Big House Concept

Small House – Stage 2

Chance Outcome (N_6)	2nd Decision	E[NPV]	Y8 (child?)	Y10 (child?)	Chance Outcome (N_{10})	p	NPV
3	Add 2BR	407	1	1	5	0.64	411
				0	4	0.16	411
			0	1	4	0.16	402
				0	3	0.04	345
	Add 1BR	366	1	1	5	0.64	366
				0	4	0.16	366
			0	1	4	0.16	366
				0	3	0.04	366
	Nothing	321	1	1	5	0.64	321
				0	4	0.16	321
			0	1	4	0.16	321
				0	3	0.04	321
2	Add 2BR	377	1	1	4	0.64	402
				0	3	0.16	345
			0	1	3	0.16	336
				0	2	0.04	279
	Add 1BR	335	1	1	4	0.64	324
				0	3	0.16	366
			0	1	3	0.16	357
				0	2	0.04	300
	Nothing	321	1	1	4	0.64	321
				0	3	0.16	321
			0	1	3	0.16	321
				0	2	0.04	321
1	Add 2BR	311	1	1	3	0.64	336
				0	2	0.16	279
			0	1	2	0.16	270
				0	1	0.04	213
	Add 1BR	333	1	1	3	0.64	357
				0	2	0.16	300
			0	1	2	0.16	291
				0	1	0.04	234
	Nothing	317	1	1	3	0.64	321
				0	2	0.16	321
			0	1	2	0.16	312
				0	1	0.04	255
0	Add 2BR	246	1	1	2	0.64	270
				0	1	0.16	213
			0	1	1	0.16	204
				0	0	0.04	147
	Add 1BR	267	1	1	2	0.64	291
				0	1	0.16	234
			0	1	1	0.16	225
				0	0	0.04	168
	Nothing	288	1	1	2	0.64	312
				0	1	0.16	255
			0	1	1	0.16	247
				0	0	0.04	189

Figure 6: Stage 2 of Small House Concept

Condo – Stage 2

Chance Outcome (N_6)	2nd Decision	E[NPV]	Y8 (child?)	Y10 (child?)	Chance Outcome (N_{10})	p	NPV
3	Buy 5BR	329	1	1	5	0.64	333
				0	4	0.16	333
			0	1	4	0.16	324
				0	3	0.04	267
	Buy 4BR	288	1	1	5	0.64	288
				0	4	0.16	288
			0	1	4	0.16	288
				0	3	0.04	288
	Nothing	224	1	1	5	0.64	224
				0	4	0.16	224
			0	1	4	0.16	224
				0	3	0.04	224
2	Buy 5BR	300	1	1	4	0.64	324
				0	3	0.16	267
			0	1	3	0.16	259
				0	2	0.04	201
	Buy 4BR	257	1	1	4	0.64	246
				0	3	0.16	288
			0	1	3	0.16	280
				0	2	0.04	222
	Nothing	224	1	1	4	0.64	224
				0	3	0.16	224
			0	1	3	0.16	224
				0	2	0.04	224
1	Buy 5BR	234	1	1	3	0.64	259
				0	2	0.16	201
			0	1	2	0.16	193
				0	1	0.04	135
	Buy 4BR	255	1	1	3	0.64	280
				0	2	0.16	222
			0	1	2	0.16	214
				0	1	0.04	156
	Nothing	224	1	1	3	0.64	224
				0	2	0.16	224
			0	1	2	0.16	224
				0	1	0.04	224
0	Buy 5BR	168	1	1	2	0.64	193
				0	1	0.16	135
			0	1	1	0.16	127
				0	0	0.04	69
	Buy 4BR	189	1	1	2	0.64	214
				0	1	0.16	156
			0	1	1	0.16	148
				0	0	0.04	91
	Nothing	220	1	1	2	0.64	224
				0	1	0.16	224
			0	1	1	0.16	216
				0	0	0.04	158

Figure 7: Stage 2 of Condo Concept

Stage 1 (Folding Back)

Now we must solve Stage 1 by folding back the results of Stage 2. . In other words, we must find the optimal 1st decision, given that we are going to make the optimal 2nd decision later.

Figure 8, Figure 9, and Figure 10 show the decision tree for the Big House, Small House, and Condo concept, respectively. Note that the 2nd decision tree has been pruned to show the optimal choice for each N_6. This was done to condense the tree for display purposes.

Note also that each concept has 32 possible NPVs, which are weighted by the probabilities to give the expected NPV, or E[NPV]. For example, take one of the paths for the Small House concept. If we have a child in Year 6 but not in Years 2 or 4, then N_6 = 1. In that case, our optimal 2nd decision rule is to add 1 BR to the house to gives us 4 total BRs. If we then have 2 more children (i.e., N_{10} = 3), then all 4 BRs are occupied for the next 30 years.

Big House - Fold-back

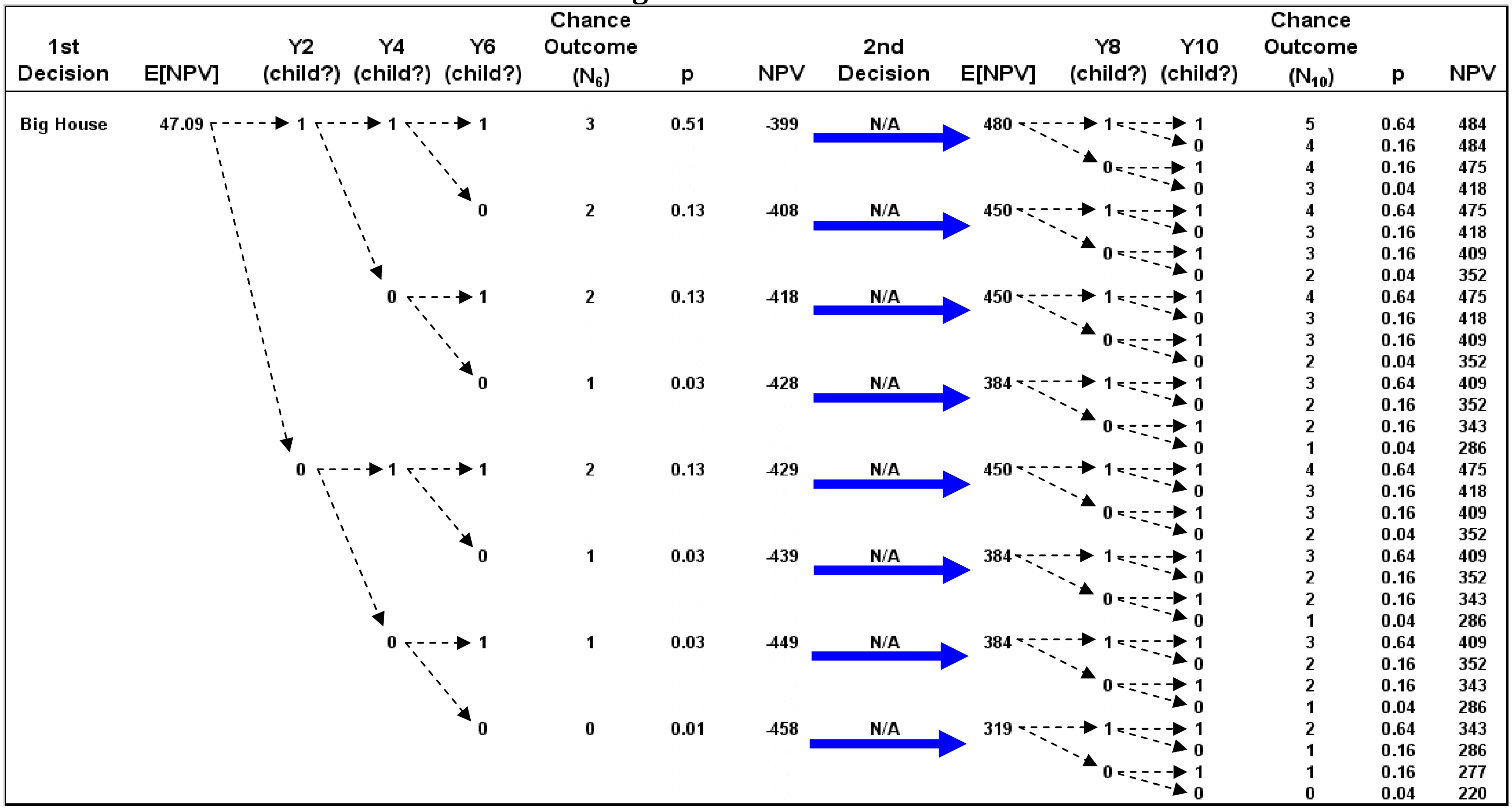

Figure 8: Fold-back for Big House Concept

Small House - Fold-back

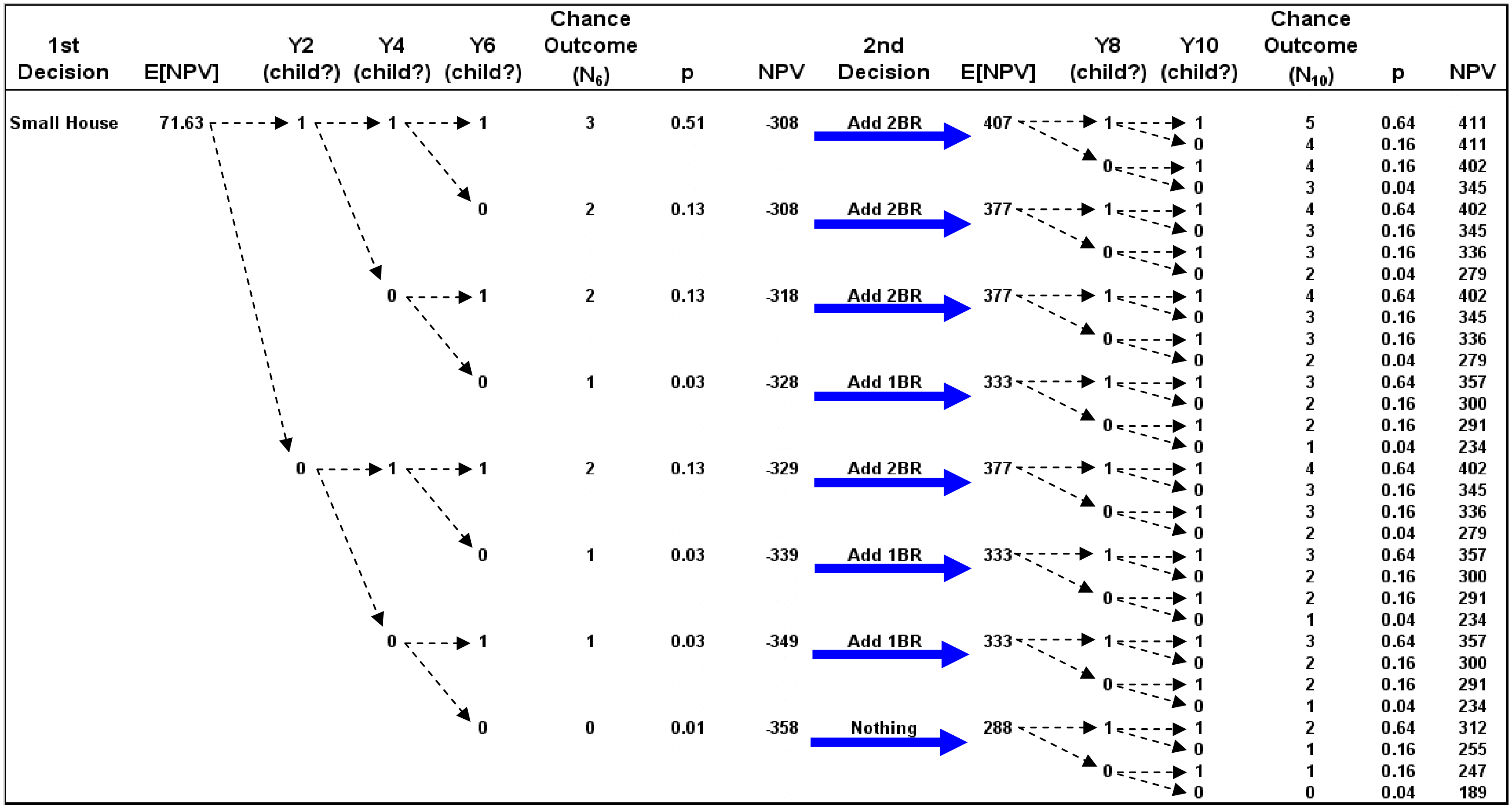

Figure 9: Fold-back for Small House Concept

Condo – Fold-back

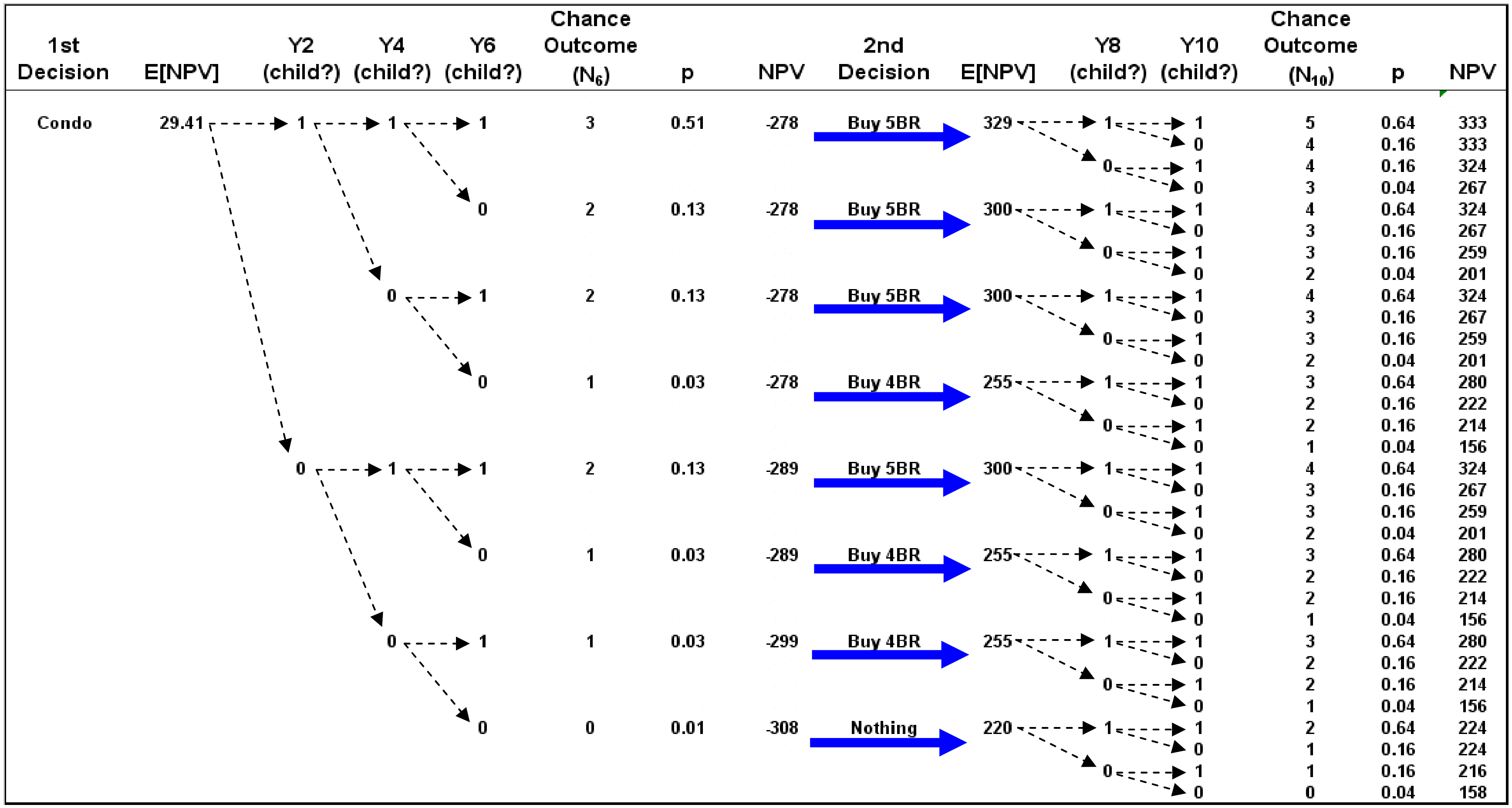

Figure 10: Fold-back for Condo Concept

Evaluation, Target Curves, Multiple Criteria

From the populated decision trees in Figure 8, Figure 9, and Figure 10, we can calculate multiple criteria, as shown in Table 3, to inform our design decision.

Table 3: Decision Analysis – Multiple Criteria

Design Concept	Criteria					
	E[NPV} ($K)	P_{10} ($K)	P_{50} ($K)	P_{90} ($K)	CAPEX ($K)	ROI (E[NPV] / CAPEX)
Big House	47	-30	67	85	500	9.4%
Small House	72	-8	94	100	400	18.0%
Condo	29	-22	47	55	350	8.3%

We conclude from these metrics that the optimal design strategy is the Small House concept. The Small House yields the highest E[NPV], P_{10}, P_{50}, P_{90}, and return on investment (ROI), while requiring the second most upfront cost. Meanwhile, the Big House is the second best, which beats the Condo on every metric except P_{10} and CAPEX.

It should be noted that the Condo concept does offer the best worst-case-scenario, which is a loss of $150K compared to worst-case losses of $169K and $238K for the Small House and Big House concepts, respectively.

These results are intuitive. The Big House concept attempts to accommodate the expected number of children (E[N_{10}] = 4), but still overestimates in almost 30% of cases. Meanwhile, the Small House concept performs better by deferring the costs of eventually-needed capacity and often avoiding the cost of unnecessary capacity. The Condo concept also defers costs, but is still heavily disadvantaged by the high closing costs associated with selling and re-buying a home.

To get a better visualization of the results, we can also look at the target or Value-At Risk-Gain (VARG) curve. A VARG curve is the cumulative distribution of NPV. Figure 11 shows the VARG curve for each design concept. The curves reinforce the conclusion that the Small House concept is the best. The Small House's VARG curve looks like a positively-shifted version of the other two VARG curves. Interestingly, the Condo concept's VARG curve starts out to the right of the Big House concept's curve, but quickly crosses over, which explains why the Condo concept yields the worst results over all.

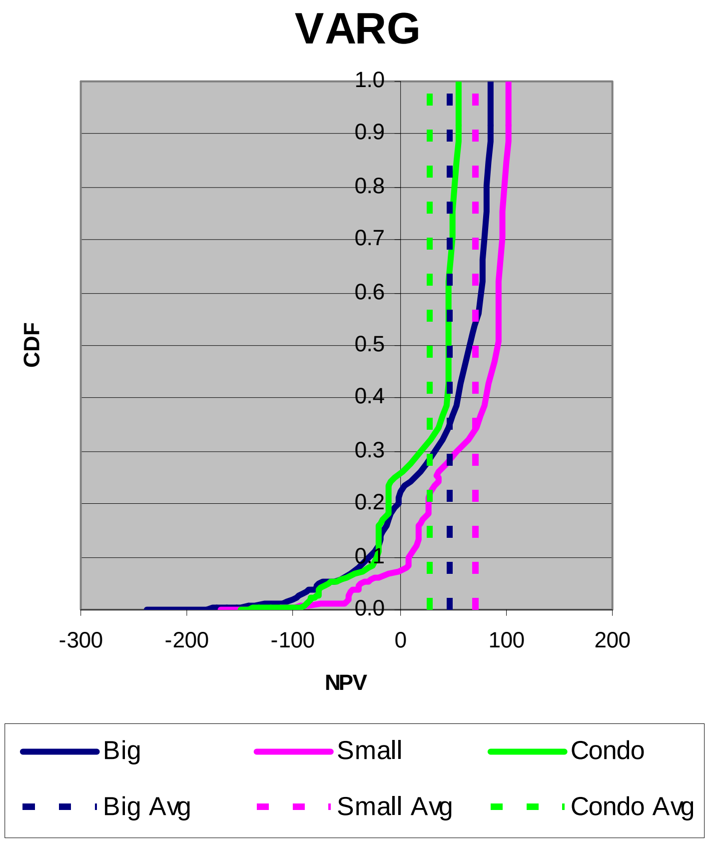

Figure 11: Decision Analysis – VARG Curves

Lattice Analysis

Now we proceed with lattice analysis.

First, we develop a lattice depicting the development of the size of our family. Based on this lattice, we can perform decision analysis to determine when it is optimal to exercise a particular option. For this analysis, we will assume the Small House design concept as a baseline and then judge when it is optimal to exercise the call option to add 2 BRs. The premise here differs slightly from the previous section ("Decision Analysis"), in which a decision rule was employed at Year 8 to dictate whether to add 0, 1, or 2 BRs to the Small House.

Lattice Development

The addition of children to our family is characterized by a binomial distribution. Every two-years, there is a $p = 0.8$ chance that another child will be born. This allows a simple lattice development, as shown in Table 4.

Table 4: Outcome and Probability Lattice

Years	0	2	4	6	8	10	12
Outcomes Lattice	0	1	2	3	4	5	6
		0	1	2	3	4	5
			0	1	2	3	4
				0	1	2	3
					0	1	2
						0	1
							0
Probability Lattice	1.00	0.80	0.64	0.51	0.41	0.33	0.00
		0.20	0.32	0.38	0.41	0.41	0.33
			0.04	0.10	0.15	0.20	0.41
				0.01	0.03	0.05	0.20
					0.00	0.01	0.05
						0.00	0.01
							0.00
Sum Check	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Each column, or stage, of the lattices corresponds to a two-year period. Starting from any cell in the Outcome Lattice, we jump horizontally right if another child is born in the next period, and diagonally down-right if not. Meanwhile, the Probability Lattice shows the probability of having the number of children indicated in the corresponding cell of the Outcome Lattice. Note that the probability of 6 children in the 6th stage (Year 12) is zero because we will only try to have children through the 5th stage (Year 10).

Decision Analysis from Lattice

With the lattice developed, we can now perform a decision analysis. Let's suppose as a baseline that we are using the Small House design concept, which starts as a 3-BR house. We would like to know at what cells in the lattice it is advantageous to exercise a call option, i.e., to add 2 BRs to the house in the next stage at a cost of $100K ($50K per BR).

First, we must map the Outcome and Probability Lattices into Cash Flow Lattices. Table 5 shows the lattices for the Small House (3 BRs) without exercising the call option. The top lattice contains undiscounted cash flows for each cell (i.e. stage and number of children). More accurately, because each stage represents two years, each cell shows two years of cash flow (with the second year discounted by one year before adding to the first year). The bottom lattice contains the expected net-present-value ENPV cash flow for each cell. These values are calculated by discounting and summing the two possible cash flows of the next stage (weighted by probabilities) to the cash flow of the current cell, all from the top lattice.

Table 5: Cash Flow Lattices for Fixed Small House

Years	0	2	4	6	8	10	12
Cash Flow ($K)	-388	24	35	35	35	35	434
		12	24	35	35	35	434
			12	24	35	35	434
				12	24	35	434
					12	24	355
						12	276
							276
ENPV Cash Flow ($K)	5	431	443	441	439	437	434
		402	429	441	439	437	434
			395	425	439	437	434
				376	412	437	434
					322	352	355
						267	276
							276

Table 6 shows the Cash Flows Lattices for the Small House while exercising the call option to add 2 BRs at every cell. The top lattice contains the undiscounted cash flows, while the bottom lattice contains the ENPV cash flows. The calculations are the same as those for Table 5, except there are more bedrooms.

Table 6: Cash Flow Lattices for Expanded Small House

Years	0	2	4	6	8	10	12
Cash Flow ($K)	-388	24	35	47	59	59	642
		12	24	35	47	59	642
			12	24	35	47	563
				12	24	35	484
					12	24	405
						12	326
							326
ENPV Cash Flow ($K)	147	595	631	654	662	652	642
		516	565	605	634	652	642
			466	510	544	567	563
				415	454	483	484
					364	398	405
						313	326
							326

Using Table 5 and Table 6, we can determine the ENPV Cash Flow Lattice for the Small House concept with flexibility, as shown in Table 7. Starting at Stage 5, each cell was calculated by comparing the ENPV of exercising versus not exercising the call option in the next stage. The ENPV calculation also took into account the $100K of exercising the options.

Table 7: ENPV Cash Flow Lattice for Small House with Flexibility

Years	0	2	4	6	8	10	12
ENPV Cash Flow ($K)	72	511	538	550	546	536	434
		445	484	513	530	536	434
			406	439	459	463	434
				376	412	437	434
					322	352	355
						267	276
							276

Falling out of the Table 7 calculation is a binary "Yes-or-No" lattice indicating whether it is best to exercise the option if we find ourselves in a particular sell. Table 8 shows these results, which say that it is only advisable to exercise the call option in the next stage if we already have at least 2 children by Year 4, 2 children by Year 6, or 3 children by Year 10. Otherwise, we are better of staying with the current size of the house. This result is intuitive because the Small House only has enough BRs for two children to have their own BRs. Thus, if we have 2 children in the early years, then there is a high probability (0.8 every 2 years) we'll need more BRs eventually. Also, if we ever have 3 or more children, then we are guaranteed that at least one of the additional BRs would be used of built.

Table 8: Yes-or-No Lattice

Years	0	2	4	6	8	10	12
Exercise Option?	NO	NO	YES	YES	YES	YES	
		NO	NO	YES	YES	YES	
			NO	NO	NO	YES	
				NO	NO	NO	
					NO	NO	
						NO	

Finally, we can comment on the value of the flexibility to exercise the call option. The value of the flexibility is the difference in expected NPV between the flexible Small House design and the best fixed design. Thus, we calculate the value of the flexibility by subtracting the ENPV of the Big House design (see previous section, "Decision Analysis") from the top-left cell of Table 7. Following this procedure, the value of the call option is $72K minus $47K, or $25,000.

Simulation

Finally, we proceed with analysis by simulation

We can use Monte Carlo simulations to determine the distribution of possible outcomes from the three design concepts. A simulation was developed in Excel® to calculate the NPV of the system, given a random outcome for the number of children. The simulation was run 4000 times for each.

Evaluation, Target Curves, Multiple Criteria

From the Monte Carlo simulations, we can calculate multiple criteria, as shown in Table 9, to inform our design decision.

Table 9: Simulation – Multiple Criteria

Design Concept	**Criteria**					
	E[NPV} ($K)	**P_{10} ($K)**	**P_{50} ($K)**	**P_{90} ($K)**	**CAPEX ($K)**	**ROI (E[NPV] / CAPEX)**
Big House	48	-29	75	84	500	9.4%
Small House	71	-15	94	102	400	18.0%
Condo	29	-19	47	55	350	8.3%

Once again, we conclude from these metrics that the optimal design strategy is the Small House concept. The Small House yields the highest E[NPV], P_{10}, P_{50}, P_{90}, and return on investment (ROI), while requiring the second most upfront cost. Meanwhile, the Big House is the second best, which beats the Condo on every metric except P_{10} and CAPEX.

We can also generate VARG curves again from the simulation data. Figure 12 shows the VARG curve for each design concept. Not surprisingly, these VARG curves closely resemble the VARG curves made in Decision Analysis. It's obvious again why the Small House concept is the best, because its VARG curve is to the right of the others.

The absolute NPVs shown here (and elsewhere) should be taken with a grain of salt, since the NPV model does not quantify all the benefits of the system, such as the kitchen, living room, bathrooms, etc. These were excluded from the model because all design concepts were assumed to deliver equal value in these areas. Thus, more important are the VARG curves' relative to one another, which ultimately reveal the Small House concept to be the best.

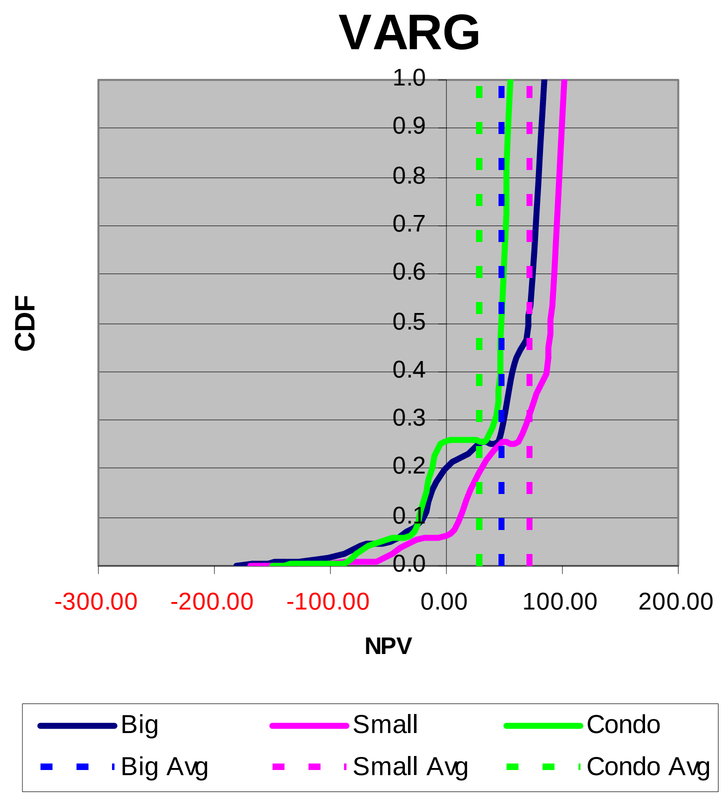

Figure 12: Simulation - VARG Curves

Conclusion

This paper discussed real-options analysis for the design of a primary residence. Three methods of analysis – decision analysis, lattice analysis, and simulation – revealed that the flexible approach to design can indeed add value to our system. Before considering flexibility, we would traditionally design to a point estimate of demand. That is, we would choose the Big House alternative, which is a fixed design with enough bedrooms (5) for the parents and the expected number of children (4). However, since the number of children is actually uncertain, we can increase our expected NPV by starting with fewer bedrooms (less capacity) and expanding as necessary. Thus, by deferring the cost of necessary capacity and avoiding the cost of excess capacity, the flexible Small House performs better than the fixed Big House design. The Condo concept's flexibility appears to have been overshadowed by its high closing cost when selling the condo.

All three methods of analysis proved useful in analyzing this system. Lattice analysis seemed most applicable because it supposed we could expand the Small House design at any stage in the first 10 years of the system. By contrast, decision analysis and simulation only let us expand in Year 8. Admittedly, the expected NPV from lattice analysis was barely higher than that of decision analysis ($72K as opposed to $71.6K). However, lattice analysis still has the advantage of easily telling us whether it's optimal to expand in any situation, given the year and current number of children.

This AP has been a very educational and enjoyable experience. The modeling of a primary residence as a system with costs and benefits is not the most intuitive exercise or obvious application for real-options analysis. However, the topic was extremely apropos to my life, making the exercise extremely fulfilling. Furthermore, as a capacity-demand scenario, the system was perfect for applying all three methods of analysis taught in class. From completing this AP, I am now quite comfortable with the mechanics and essence of these methods, and much more proficient in Excel®.