Electronic Waste Recycling
Application Portfolio for MIT ESD.71

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Abstract: Given the uncertainty of electronic scrap prices, participation in recycling systems and the future of regulation, should an electronics manufacturer build a recycling plant for their products? If so, how big? Large enough for all products which might eventually be regulated, or simply large enough to handle those pieces which can currently be recycled profitably with the ability to add capacity in the future? What is the financial value of having an option to close the plant earlier than planned? These decisions are analyzed through the use of two-stage decision analysis and binomial lattices. Most specific values used in this case are simplistic estimates; however the examples contained within this report demonstrate the power of these analytical techniques.
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Introduction: Electronics Recycling Systems
With increasing interest in the environmental sustainability of products, many electronics firms are considering what happens to their products at the product’s end-of-life. Regulations on which materials can be used in a product’s manufacture as well as regulations on what products are accepted by landfills continue to increase. With respect to electronics recycling, the most notable regulation is the European Union’s Waste Electrical and Electronic Equipment (WEEE) directive. As a result of this directive each EU member country has had to implement an electronics waste system. Anticipating the spread of similar legislation to the rest of the world, many other countries and electronics firms are also trying to figure out how to best address the growing amount of e-waste created in today’s society.

Some electronics firms have either built, or are considering building, their own recycling plant. The profitability of such a plant depends both upon the value of the materials recovered and the cost of running the operation. However, recycling may provide other benefits to the firm as well. By recycling their own products, firms can recover and reuse parts of old products, thus potentially lowering the cost of producing new products. Firms may also find that operating their own take-back or recycling system is cheaper than participating in a government run system. With more firms able to show they can responsibly handle the waste that their products create, it is less likely that the government will impose a collective system. Furthermore, firms may find that operating a recycling plant improves their corporate image.

Methodology
Several questions faced by electronics recyclers and potential electronics cyclers are addressed using various analytical techniques in this report. Two-stage decision analysis is used to answer the question: given market and regulatory uncertainty, should an electronics manufacturer build a recycling plant large enough for all products which might eventually be regulated, or simply large enough to handle those pieces which can currently be recycled profitably with the ability to add capacity in the future? Then, a binomial lattice is used to determine the financial value of having an option to close a plant earlier than planned. Most specific values used in this case are estimates derived from historical price data and cost models developed by the MIT Materials Systems Laboratory. While these examples are imperfect, they demonstrate the power of the analytical techniques applied.
Sources of Uncertainty

Price of Recovered Materials
The value returned by a recycling system is largely dependent upon the market price of the materials recovered. With respect to electronic materials, the value of the internal precious metals is particularly important. These prices are generally a function of primary metal prices and thus influenced by all of the same economic, political, and other factors which influence traditional metals markets. Materials are not easily recovered from electronic parts and therefore the level of dismantling performed by the recycler is influenced by the current value of materials. The historical prices for four such precious metals commonly found in electronic equipment are shown below. The relative difference in price between the electronic scrap and the precious metals will determine the economic viability of the recycler. The cost of collecting e-waste for recycling in the U.S. is $100-500 per ton with an average of $300/ton. The recycling process then typically costs $200-500 per ton, also with a $300/ton average value.¹

Integrated circuit boards are one particular type of electronic waste from which recyclers can extract precious metals. Recent prices of mixed scrap integrated circuits, obtained from www.recycle.net\(^2\), are shown in Figure 2. Based on the prices of these chips collected over the last 17 weeks, the average rate of growth of the scrap ICs is calculated to be 1.33%, and the standard deviation is 0.412. This growth rate and standard deviation is used to predict the future prices of electronic scrap material in the binomial lattice analysis.

**Mixed Scrap Integrated Circuits**  
(Weekly Data, July 20-November 9, 2007)

![Graph of Mixed Scrap Integrated Circuits](image)

**Figure 2: Mixed Scrap Integrated Circuit Prices**  
*Historical prices for mixed scrap integrated circuits as reported by recycle.net on Fridays between July 20, 2007 and November 9, 2007 with an exponential regression line.*

**Recovery Rate**
The amount of products returned to the company is unknown as is when they will be available for processing. Historical data also suggests that more electronic waste is brought to recyclers during the summer months than other seasons. Large special events for e-waste recycling may also produce a sudden large quantity of material for a recycler. For the purposes of this analysis it is assumed that the recycler receives a constant inflow of material for processing throughout the year.

**Regulation**
It is unknown what state or federal legislation might be introduced with respect to the handling of e-waste. The government could decide to implement a centralized recycling system thereby reducing or eliminating the supply of old products available to the private recycler. Conversely, legislation could require that all manufacturers take-back and recycle their own goods. In this case, the firm which decided to build their own system in advance of the regulation may find themselves with a competitive advantage in the industry.

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A secondary uncertainty with respect to legislation in this area is when any such legislation will be enacted. In the absence of regulations, a recycler will typically only process profitable materials. However, should a regulation be introduced requiring recycling of certain products, the volume of material given to a recycler will most likely increase substantially. For the purposes of the following analysis, it will be assumed that the recycler in question will begin obtaining three times as much material as previously upon the enactment of e-waste legislation. Therefore the recycler who has the capacity to process an increased product quantity may have a competitive advantage immediately following the enactment of legislation.
Two-Stage Decision Analysis of Alternative Designs

Conscious of the uncertainly inherent to the electronic waste market, a recycler may choose to (1) build a recycling plant with a large fixed capacity in anticipation of regulation, (2) build a smaller recycling plant with the ability to expand or (3) simply not build a plant at all and stay out of the recycling market. A two-stage decision analysis is used to determine the expected value of each alternative.

The market prices for scrap material can increase, decrease, or remain constant. Additionally, at the end of each stage, the government may regulate the recycling of material, thus increasing the total demand for capacity. For simplicity, it is assumed there is an equal likelihood that regulation will and will not occur at the end of period 1, and that should regulation occur, the demand for recycling will increase threefold. Without regulation, the demand for recycling will remain constant. The price of scrap is evaluated with a 30% chance of increasing by 45%, a 65% chance of remaining constant, and a 5% chance of decreasing by 15%. These values were estimated from the observed historical volatility in the prices of several metals found in electronic materials. See Appendix I for more details. Without regulation, it is assumed that the recycler is already recycling as much material as is available. The recycler will therefore only increase capacity when a regulation exists to increase the supply available. Realistically, the recycler might begin recycling otherwise unprofitable materials if the price of scrap increases, however, in this example we will assume that an increase in price does not increase the supply of materials available to the recycler.

If a regulation is instated, the supply of recycling is assumed to multiply by 3. In the fixed case, the recycler will build a plant with sufficient capacity to handle the full supply of material available should a regulation eventually be instated. In the flexible case, the recycler will initially build only the capacity needed to process the material available in the unregulated market. If a regulation is later created, the recycler will then build additional capacity to handle this additional material. It will cost more for capacity to be added to a plant than for the extra capacity to be included in the original design. However, if the additional capacity is never needed, the smaller plant is more cost effective.

The following assumptions are made regarding the costs of each option. These values are estimated using a recycling model developed within the MIT Material Systems Laboratory.

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With the above assumptions, the binomial lattice in Figure 3 shows that while the maximum attainable value at the end of period 2 is $39.3 million, the expected value of the fixed plan is $17.1 million. The maximum attainable value under the flexible plan is less than that of the fixed plan at $39.1 million; however, the expected value is $17.4 million, $0.3 million greater than that of the fixed plan. Therefore, given the uncertainty of material prices, the flexible plan is more likely to have a greater value than the fixed plan.
Figure 3: Decision tree comparing a fixed and flexible approach to building recycling capacity. All values are in millions of US dollars.
Binomial Lattice

A binomial lattice is used here to predict the future prices of e-waste material, along with a probability for each hypothetical price. As obtained using the Materials Systems Laboratory’s recycling cost model, an average price for the diverse mix of materials being processed by a recycler is $1.84/lb. As the volatility and growth rate for most e-waste scrap materials is largely unavailable, those observed for the previously described mixed scrap ICs are assumed to be representative of all scrap processed by the recycler in question. The following values were therefore used to calculate the lattice.

Starting Price, \( S = 1.84 \text{$/lb} \)
Growth Rate, \( v = 1.33\% \)
Volatility, \( \sigma = 2.80\% \)
Time Step, \( \Delta t = 1 \text{ week} \)

\[
\begin{align*}
\text{Up Factor, } u &= e^{\sigma \sqrt{\Delta t}} = e^{(.02797)} = 1.028 \\
\text{Down Factor, } d &= e^{-\sigma \sqrt{\Delta t}} = e^{-(.02797)} = 0.9724 \\
\text{Up Probability, } p &= .5 + .5(v/\sigma) \sqrt{\Delta t} = 0.737 \\
\end{align*}
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Probability Distribution Function (PDF) for Lattice

PDF for log of relative outcomes
Decision Analysis Using Lattice

Should the price of scrap materials drop such that the recycler’s revenue becomes less than the operating costs of the plant, the recycler may choose to shut down. Expanding upon the lattice technique previously described, the put option of closing a recycling plant is evaluated based upon the previously modeled fluctuating price of electronic waste. The recycling plant evaluated here is equivalent to the one initially built under the flexible plan described in the two-stage decision analysis. The following variables are used to define the lattice. Note that unlike the last analysis which used a 1 week time step, the following analysis uses a 4 week time step. This is the mathematically maximum time step which can be used in this scenario; larger time steps yield negative probabilities.

Obtained from the Materials Systems Lab Recycling Model
- Starting Price, \( S = \$1.84/\text{lb} \)
- Fixed Cost = $600,000 per year
- Marginal Cost per pound = $0.14/lb
- Production Capacity = 3 million pounds per year

Derived from the historical IC prices
- Growth Rate, \( v = 1.33\% \)
- Volatility, \( \sigma = 2.80\% \)
- Time Step, \( \Delta t = 4 \text{ weeks} \)
- Up Factor, \( u = 1.058 \)
- Down Factor, \( d = 0.946 \)
- Up Probability, \( p = 0.976 \)

Estimated from US Treasury Bond Interest Rates
- Discount Rate = 3.58\% annually\(^3\) (Risk Free Interest Rate) = 0.27\% every 4 weeks

Given these assumptions, the net revenues of the firm in each period are predicted with and without the ability to exercise the option of closing as a function of the uncertain sale price of the recovered materials.

As detailed on the following pages, using the above definition of the option, and a 52 week (1 year) analysis, the recycler can always earn greater revenue from operating than shutting down. Therefore, the option has no value during the first year of operation. In either case the project is worth $394,270. Given the other assumptions used in this example, the volatility of the price must be increased tenfold in order for the option to obtain small values.

\(^3\) Interest Rate of the most recent 181-day treasury bill auction reported on [http://www.treasurydirect.gov/RI/OFBill](http://www.treasurydirect.gov/RI/OFBill) as of November 20 2007.
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Net Revenue:

E [Revenues] 0 369,546 394,094 419,987 447,298 476,106 506,492 538,543 608,011 645,624 685,299 727,148 771,289

PV(E[Revenues]) 0 184,523 98,257 52,286 27,805 4,168 2,212 1,173 622 330 175 93

NPV (over 13 t) 394,270

PV(Net Revenue) 394,270 792,068 846,457 903,775 963,988 1,026,858 1,091,722 1,157,059 1,219,609 1,272,612 1,302,263 1,280,594 1,151,186 800,455

WITHOPTIONS 691,643 740,277 791,532 845,379 901,610 959,641 1,018,126 1,074,184 1,121,835 1,148,863 1,130,522 1,106,882 707,423

(check t = 13) 645,336 691,169 739,325 789,620 841,540 893,899 944,152 987,017 1,011,701 996,335 896,795 624,238

601,430 644,497 689,484 735,940 782,821 827,884 866,469 889,057 876,351 789,418 549,858

559,706 599,947 641,518 683,501 723,923 758,882 779,394 769,068 693,407 483,351

519,888 557,091 594,694 630,966 662,303 681,340 673,140 607,558 542,833

481,600 515,287 547,848 576,126 593,664 587,367 530,797 423,166

444,285 473,528 499,071 515,287 510,672 462,161 323,166

407,075 430,172 445,172 442,095 400,790 280,655

368,566 382,494 380,777 345,914 242,642

326,451 325,950 296,848 208,654

276,926 252,975 178,263

123,746 151,089 126,791
Conclusions
Both the decision analysis and binomial lattice provide non-obvious insight into operational
decisions regarding engineering projects. Given the different natures of these tools, each served
a distinct purpose in this analysis. The binomial lattice approach was particularly useful in
numerical analysis surrounding prices or other quantitative data for which historical data is
available to predict future value. For more qualitative forms of uncertainty, including regulation,
the decision analysis tree is a more appropriate tool. Furthermore, as can be implied by the
results of both types of analysis, options are most useful in scenarios where the variables in
question are likely to fall to unfavorable levels within the time period of the analysis. In the
binomial lattice example presented in this report, the likelihood that the price of the recycler’s
product price would rise was much greater than the likelihood that it would fall. Therefore, it is
unsurprising that the probability of the recycler choosing to exercise the option to close is very
low. Flexible approaches to design generally appear to be most useful in cases where the
uncertainty involved is such that the probability of meeting a certain projection, or other
favorable projections, is low.

Course Reflections
I came into this course believing that flexibility provides value in most situations containing
uncertainty; however, through the exercises contained in this report I have learned how to
quantify the value of flexibility. While both decision analysis and binomial lattices simplify
situations to evaluate only a few variables at a time they demonstrate the potential financial
benefits which can be achieved by simply incorporating the uncertainty surrounding a few
variables into the analysis. The decision tree analysis was much easier to implement than the
binomial lattice; however, through working to determine how to apply each to this specific case,
I developed a better understanding of how each tool works. I also gained an appreciation for the
difficulties associated with finding relevant historical data to use in producing future projections.

I found the individual AP assignments successfully distributed the workload for this project
across the semester. However, in order maintain consistent assumptions between the decision
tree analysis and the binomial lattice, I found myself needing to repeat the decision analysis
assignment after learning how to calculate the u, d and p variables necessary for the binomial
lattice. I therefore recommend teaching how to use historical data to derive u, d and p before the
decision analysis assignment.
### Appendix I – Historical Metal Prices

#### Average Prices

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<th></th>
<th>Palladium (NYMEX)</th>
<th>Platinum (NYMEX)</th>
<th>Gold (COMEX)</th>
<th>Silver (COMEX)</th>
<th>Steel (Dow Jones CR)</th>
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#### Percent Change in Price from Previous Year

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<th>Gold (COMEX)</th>
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<th>Steel (Dow Jones CR)</th>
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- # of samples: 21 21 17 17 3
- % of years price rose >15%: 33% 33% 24% 18% 33% average: 28%
- % of years price changed <15%: 52% 57% 71% 82% 67% average: 66%
- % of years price dropped >15%: 14% 10% 6% 0% 0% average: 6%

#### Other Statistics

- average % increase, when >15%: 49% 27% 25% 43% 88% average: 46%
- average % change when <abs(15%): 2% 0% 1% 2% -1% average: 1%
- average % decrease, when <-15%: -41% -23% -19% 0 0 average: -16%

---

4 All prices obtained from www.metalprices.com on December 4, 2007.