Product Portfolio
Production Planning with Flexible Production
An Analysis Using Simulation

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Andres Garro
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Executive Summary

This document details a simulation based analysis of the value of a flexible approach to satisfying the demand for a portfolio of products by using both low-cost, long lead-time production facilities as well as high-cost, short lead-time facilities. More specifically, the analysis compared a system in which all of the production decisions are made before the selling season begins to one in which part of the production is decided before the season begins and part after some sales have been observed. These two systems were evaluated using extensive simulation (1 million trials per product). Moreover, a sensitivity analysis using response surfaces was performed to better understand the conditions most favorable for a flexible design. Overall, the results of the analysis showed that the flexible design reduced the potential losses, significantly improved the potential gains, and had an average net present value increase of over 4% for a typical set of operating conditions. Moreover, the benefit of the flexibility increases dramatically as the cost of short lead-time production goes down or the demand uncertainty goes up. Finally, as the number of products increases, the flexible design becomes progressively more attractive.
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Introduction

This paper details a simulation-based analysis performed to determine the value of flexibility in the design of a product production system. More specifically, the production system is one in which a set of products are produced in order to meet an uncertain future market demand where the demand for the products is correlated. This type of system is prevalent in consumer products and industrial goods companies. Typically, these systems are not designed. Rather, the supply chain for each particular product is optimized as if it were independent of those for the related products, so that the system is more derived than designed. This is true even in situations in which the relationship between the products is clear.

One good example is the production of wheels for automobiles. The production systems for these components are designed almost completely independently from one another. However, the demand for each wheel is very strongly correlated to that of the other wheels (in particular, when all of the wheels are designed for different versions of the same vehicle). An even better example of this type of system is the design and production of clothing to be sold at retail. As with the wheel example, production decisions are generally made for each item independently of the others. However, as would be expected, the demand for one item is often strongly linked to the demand of another if they are part of the same collection or fashion trend.

The goal of this paper is to determine the value of system flexibility — in the form of flexible, short lead-time production facilities — under varying levels of demand uncertainty. The following sections will provide further details on the system that was analyzed, the simulation methodology used, and the results that were obtained.

System Definition

At the most basic level the system described above is simply an arrangement of production facilities with different characteristics — in terms of capacity, cost and lead-time — that are arranged to meet the expected demand for a portfolio of products. Each system can be thought of as being more or less
flexible in terms of its ability to adapt to the actual product demand encountered. In other words, the most flexible system would be one that produces a unit of any product in the portfolio when a customer wants it and delivers it instantly. This ideal system is of course very difficult, if not impossible, to design, but it is certainly possible to design systems closer to this ideal with certain trade-offs. For this analysis, two different designs will be analyzed: an inflexible traditional design and a flexible design that employs short lead-time production facilities to produce more units as needed.

**Base Design**

The least flexible version of the system at hand is also the most common. This design only uses low-cost, long lead-time facilities that produce a fixed quantity of product before the start of the selling season. Also, each facility is only designed to produce one product. In this setup, each product is produced in one large batch based on the expected demand and no consideration is given to the relationships between the demand levels for different items. Thus, there is no ability to react to differences between expected and actual demand and no accounting for demand correlation between products. This system is depicted in the figure below.

![Diagram of Base Design]

**Flexible Design**

The flexible alternative to the base design just discussed uses both low-cost, long lead-time facilities as well as high-cost, short lead-time facilities. That is, for a given product, part of the production is done in a long lead-time facility and additional units can be produced in a short lead-time facility depending on the observed demand. Moreover, the decision of how much to produce at the long lead-time facility is made not only taking into account the possibility of future production, but also the relationships between the demand for different products. In practice this system has two stages. In the first stage, units are produced in low-cost, long lead-time facilities for a portion of the expected demand (e.g. two-thirds of the expected selling season). In the second stage, an additional production run is made at high-
cost, short lead-time facilities, but only for those products that exceeded their demand expectations. This system is depicted in the figure below.

Thus, the key difference is the flexibility represented by the possibility of additional production once some demand has been observed. The value of this flexibility depends on the variability in the expected demand and the relative costs of production between the long lead-time and the short lead-time facilities.

**Demand Uncertainty**

As mentioned previously, the key uncertainty that impacts the performance of the system is the demand for each product. Since this demand is observed through store sales, the distribution is best approximated using a continuous Poisson distribution. (Gelman & Hill, 2007) This type of model assumes that the sales process is a Poisson process, which is a stochastic process where observations occur continuously and independently of one another. Moreover, the demand distributions for each product will be correlated with that of other products. In general, this uncertainty is independent of the managerial decisions. This might be violated in situations where managers can implement promotions or other marketing strategies to boost demand. However, in the system at hand, it is assumed that managers are only making initial production decisions not marketing or promotion decisions.

At a high-level, the overall demand distribution will be the sum of individual demand distributions for each product. That is, a sum of a series of Poisson distributions. Over time, these distributions will get updated as the season progresses. For example, before the season begins there will be an expected
demand distribution based on the demand of similar items then as sales begin to be observed the expected demand distribution will be updated to reflect the additional information. Under these assumptions, it is possible to characterize two expected demand rate distributions that are the main sources of variation for the system: the expected demand rate before any demand has been observed and the demand rate after some demand has been observed. The latter distribution is assumed to have a significantly smaller variance relative to the former. This assumption is based on the belief that a demand forecast based on recent sales of an actual product is better than one based on historical sales of a similar product or products. The reduction is variance of the demand distribution after real sales are observed is essentially a measure of the learning that occurs when the product hits the market.

![Product Demand Rate Distributions](image)

Also, since sales must be positive, it makes sense to use a lognormal distribution to approximate the demand. The figure above shows illustrative cumulative density functions for the two demand rate distributions. As would be expected, the demands distributions have only positive expectations and very long right-hand tails, which means that some products can have unusually high demand.
Decision Rules

In order to explore the potential benefits of using the short lead-time production to take advantage of the information gained after observing some demand, two decision rules will be analyzed. The first rule will be used for the base system and the second for the flexible system. However, both decision rules will focus on deciding a production quantity to optimally satisfy the expected demand over a given period. For example, in the base system, the key decision is how much to produce in order to satisfy the expected demand over the entire selling season so that profit is maximized. Similarly, for the flexible case there will be two decisions that would have the same format, but the length of the periods will be different along with the demand distributions. That is, the system would first decide how much to produce for the first part of the selling season and then how much to produce additionally for the rest of the season. Fortunately, this type of production decision is what is commonly referred to as a newsvendor problem.

Newsvendor Formulation

The newsvendor problem deals with the key decision of how much of a particular product to produce or buy at a given time. The principal trade off in the problem is having too much or too little inventory. The consequence of having too much inventory is left over inventory at the end of the selling period that has to be sold at a loss or simply discarded. Having too little inventory, on the other hand, results in loss revenue because some portion of the customer demand for the product will not be served. Moreover, the newsvendor problem applies to situations in which the un-served demand is lost (i.e. no back-ordering), there is only one decision to make, and the product is perishable. These are reasonable assumptions for the scenarios mentioned previously. For example, in the retail environment customers rarely come to the store and order an item that they can’t find and items designed for one season are not carried over to future seasons.

As would be expected, the cost of too much inventory is not necessarily the same as the cost of too little inventory. In most cases, the lost revenue from having one less unit of a product far exceeds the extra cost of having one extra unit. However, independent of these relative cost differences, for situations that fit the criteria of the newsvendor problem it is possible to determine the quantity that minimizes the expected cost of having too little or too much inventory. To determine this quantity, it is convenient
to define the expected profit function. The value of this function depends on whether demand exceeds
the amount of product available and can be written as follows:

\[ \Pi = \begin{cases} q(r - p) & \text{if } d > q \\ (q(r - p)) + (q - d)(s - p) & \text{if } d \leq q \end{cases} \]

where \( p \) is the unit production cost, \( r \) is the unit revenue, \( s \) is the unit salvage value, \( d \) is the demand
(unknown), and \( q \) is the order quantity. Based on this profit function, it is possible to write a function for
the expected additional profit of ordering an extra unit. This function is useful because the optimal order
quantity is that for which this function becomes zero:

\[ \frac{\partial \Pi}{\partial q} = P(d > q)(r - p) + P(d \leq q)(s - p) = 0 \]
\[ (1 - P(d \leq q))(r - p) + P(d \leq q)(s - p) = 0 \]
\[ P(d \leq q)((s - p) - (r - p)) = -(r - p) \]

Therefore:

\[ P(d \leq q) = \frac{r - p}{(r - p) - (s - p)} = \frac{c}{c + h} \]

where \( c \) is the unit underage cost and \( h \) is the unit overage cost.

Thus, the optimal order quantity is that which meets the above condition. That is, one that creates a
situation in which the probability that the demand will be less than or equal to the quantity available is
equal to the critical fraction defined by the ratio between the unit underage cost and the sum of the unit
underage and overage costs. This result is reassuring because, as would be expected, the order quantity
that minimizes the total overage and underage costs is only a function of these costs and of the
anticipated demand distribution.

**Base Design**

Leveraging the newsvendor formulation discussed above, the first decision rule will only look at the
estimated demand distribution before any sales are observed and determine a production quantity to
supply the entire season. That is, the production quantity will be that which covers the critical fraction,
\( f_c \), of the demand distribution determined by the following equation:
where $R$ is the product sales price and $P^L$ is the production cost in long lead-time facilities. This assumes that the products have no salvage value. Thus, using only the expected demand distribution before any sales are observed, it is possible to make the best decision possible within the constraints of the system.

**Flexible Design**

The second decision rule uses the same logic as the calculation above to determine a long lead-time production quantity, but only for the portion of the season that cannot be supplied from short lead-time facilities. For the rest of the season, it determines and additional short lead-time production quantity that covers the critical fraction, $f^*_c$, of the demand distribution determined by the following equation:

$$f^*_c = \frac{R - P^S}{R},$$

where $P^S$ is the production cost at the short lead-time facilities. This again assumes that the products have no salvage value. This quantity is then adjusted to take into account the inventory that remains from the initial long lead-time order which produces the ultimate additional production quantity.

**Simulation**

In order to analyze the system described in the previous sections and determine the value of additional flexibility, a simulation analysis was performed. The simulation was performed using R statistical software (please see Appendix for code) and used one million trials for each set of product, system design and decision rule. For each system scenario, the performance was derived from the amount of demand that was captured and the amount of inventory that was left over at the end of the selling season. Since each item has a long lead-time cost of production, a short lead-time cost of production and a sales price, it is straightforward to calculate the total benefit from a set of decisions for different levels of observed demand. Moreover, the cash-flows can be discounted to determine a final net present value.
Thus, for the base design, the net present value of the project was calculated by applying the standard annuity formula to the monthly cash flows and subtracting the initial investment as follows:

$$\text{NPV}_{\text{Base}} = \left(\frac{\min(D, Q/T)R}{r}\right) \left(1 - \frac{1}{(1 + r)^T}\right) - Q P^L,$$

where $D$ is the actual demand rate, $Q$ is the production quantity, $T$ is the length of the season (in months), and $r$ is the monthly cost of capital. Similarly, for the flexible design, the project net present value was calculated using the following formula:

$$\text{NPV}_{\text{Flexible}} = \left(\frac{\min\left(D, \frac{Q}{T(1-E)}\right)}{r}\right) \left(1 - \frac{1}{(1 + r)^T(1-E)}\right) - Q P^L$$

$$+ \left(\frac{\min\left(\bar{D}, \frac{Q^* + \bar{Q}}{TE}\right)}{r(1 + r)^T(1-E)}\right) \left(1 - \frac{1}{(1 + r)^TE}\right) - \frac{\bar{Q} P^S}{(1 + r)^T(1-E)}$$

where $Q^*$ is the inventory left over before the end of season begins, $\bar{Q}$ is the additional production quantity for the end of the season, $\bar{D}$ is the actual demand rate for the end of the season and $E$ is the percent of the season that can be supplied from short lead-time facilities.

In order to simulate the performance of this system, the key parameters needed are as follows:

- Mean demand for each product = 5000 units/month
- Standard deviation of demand prior to observing sales = 7500 units/month
- Correlation between individual product demands ($\rho$) = 25%
- Percent reduction in standard deviation of demand prediction after observing sales = 80%
- Price of the product ($R$) = $30 per unit
- Cost of long lead-time production ($P^L$) = $10 per unit
- Cost of short lead-time production ($P^S$) = $15 per unit
- Length of selling season = 6 months
- Portion of season able to be supplied from short lead-time facilities after observing sales ($E$) = 30%
- Monthly discount rate ($r$) = 1%
The values chosen for these parameters are somewhat arbitrary, but are based on recent experience at Zara, which is a major fashion retailer. Also, with the exception of the portion of the season able to be supplied from short lead-time facilities, these parameters are fixed when the production quantities are decided. It should be noted that even though the standard deviation is larger than the mean demand rate, this will not result in negative actual demand rate values because of the use of a lognormal distribution to approximate the underlying Poisson distribution. Moreover, this type of variability in the demand for a new product is typical for the fashion industry. Using the parameters previously discussed, the demand distributions shown in Figure 1 were generated.

![Figure 1: Simulation Demand Distribution](image)

**Results**

The simulation results showed that the average net present value for the flexible system design was $325,979.20 versus $313,085.60 for the inflexible design or approximately a 4% increase. However, the net present value for the flexible design had much a much larger variability as evidenced by its standard deviation (see Table 1). Also, the expected present value of the initial capital expenditures is much lower for the flexible design. More importantly, a comparison of the value at risk and gain plots for both
decision rules reveals that the flexible design limited the downside potential and significantly increased the upside potential (see Figure 2).

Table 1: Results Metrics Summary 1

<table>
<thead>
<tr>
<th></th>
<th>E[NPV]</th>
<th>STD[NPV]</th>
<th>CAPEX</th>
<th>E[NPV]/CAPEX</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Base Design</strong></td>
<td>$313,085.60</td>
<td>$219,461.20</td>
<td>$265,680.00</td>
<td>1.18</td>
</tr>
<tr>
<td>(Inflexible)</td>
<td><strong>$325,979.20</strong></td>
<td><strong>$270,032.50</strong></td>
<td><strong>$185,976.00</strong></td>
<td><strong>1.75</strong></td>
</tr>
<tr>
<td><strong>Flexible Design</strong></td>
<td>$325,979.20</td>
<td>$270,032.50</td>
<td>$185,976.00</td>
<td>1.75</td>
</tr>
</tbody>
</table>

This occurred because the inflexible design has a fixed upper limit on the net present value. This limit occurs when the product demand is high enough for the entire inventory to be sold. This upper limit can be seen in the VARG plot where the probability goes to 100% (see Figure 2). The flexible system design on the other hand puts less inventory at risk at the beginning and only orders additional inventory for the items with unexpectedly high demand. Since the quality of the demand forecast improves after sales are observed, the probability of selling this additional inventory goes up. Thus, when the product demand is high, the flexible design is able to sell more with less risk. This “sure sale” effect makes up for
the additional cost of producing in the short lead-time facilities. In more concrete terms, the flexible design reduces the 5% VAR by over 40% and increases the 95% VAG by over 40% (see Table 2). More dramatically, the maximum net present value observed increases by over twenty five times with the flexible design (see Table 2).

Table 2: Results Metrics Summary 2

<table>
<thead>
<tr>
<th></th>
<th>P5</th>
<th>P95</th>
<th>Min(NPV)</th>
<th>Max(NPV)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Base Design</strong> (Inflexible)</td>
<td>$(91,701.71)</td>
<td>$504,191.10</td>
<td>$(258,937.60)</td>
<td>$504,191.10</td>
</tr>
<tr>
<td><strong>Flexible Design</strong></td>
<td>$(50,923.83)</td>
<td>$722,381.20</td>
<td>$(181,233.00)</td>
<td>$13,056,468.70</td>
</tr>
</tbody>
</table>

Another measure of performance is robustness or the spread of the net present value distribution. The more concentrated the outcomes, the more robust the system. In this case the inflexible design is more robust because it has a lower standard deviation, but the flexible design is safer in that it shifts the distribution of outcomes towards the positive end of the spectrum (see Figure 3).

![NPV Distributions for Different System Designs](image-url)
Sensitivity Analysis

In addition to the results using the initial parameters, an analysis of the sensitivity of the results to several key parameters was performed. The goal of the sensitivity analysis was to gain a better understanding of the situations for which the flexible system design would have a clear advantage over the base design. The key parameters that were analyzed were the coefficient of variation, the production cost ratio, the percent of the season supplied from short lead-time facilities and the number of products in the portfolio. The last two of these are self explanatory, but the first two were defined as follows:

- **Coefficient of Variation**: the ratio of the standard deviation of the expected demand to the mean demand ($\sigma/\mu$). (Weisstein, 2010) The coefficient of variation is a measure of the normalized demand uncertainty.

- **Cost Ratio**: the ratio of the cost for producing in short lead-time facilities to the cost for producing in long lead-time facilities ($P^S/P^L$).

The results of this analysis are shown using the profit improvement response surfaces shown in Figure 4, Figure 6 and Figure 7 below.

![Figure 4: Sensitivity Analysis Results for 1 Product](image-url)
In general, the sensitivity analysis shows that the value of the system flexibility is strongly dependent on the demand uncertainty. That is, as the coefficient of variation increases, the response surface shifts towards positive profit improvement. The results also show that as would be expected the value of the flexibility in the system design increases as the difference in production cost between short lead-time and long lead-time facilities decreases (the cost ratio). Moreover, there is a certain degree of interdependence between the optimal percent of the season to supply from short lead-time facilities and both the demand uncertainty and cost ratio. For example, for a single product with a coefficient of variation of 1.5 and a cost ratio of 1.5, the optimal value for the portion of the season that should be supplied from short lead-time facilities is 30% (see Figure 5).

![Optimal End of Season Length](image)

**Figure 5: Optimal End of Season Length**

Finally, due to the positive correlation between the individual product demands, an increase in the number of products increases the overall demand uncertainty. This in turn increases the value of the design flexibility in terms of being able to respond to unexpectedly high demand for individual products. Figure 6 and Figure 7 below show the response surfaces generated using a product portfolio of 5 products and 10 products, respectively. The results are consistent with those of the single product, but the response surfaces are not as smooth because a much larger number of simulation trials would have
been needed. However, running that many trials (approximately 10 million) was impractical due to time constraints.

![Figure 6: Sensitivity Analysis Results for 5 Products](image-url)
Discussion

The results above clearly show that the flexible design is better. This can be seen in the average profit figures and in the VARG plots. However, the portion of the season able to be supplied from short lead-time facilities was essentially treated as being fixed when in reality it is another decision variable. This begs the question how to find the optimal set of decision rules including the portion of the season to supply from short lead-time facilities. One approach would be to experiment with many different options until the best performing settings emerge (this is essentially what the response surface sensitivity analysis was designed to do); however, this approach is somewhat inefficient. Another option would be to build an optimization that would determine the best set of options. In this case, the decision variables would be the quantities to order. The only problem with this approach is the uncertainty around being able to represent all of the relevant objectives and constraints in a linear system, but this would be an interesting problem for future analysis.
Reflection

Where you see the most use for the Flexible approach to design?

Based on the experience in this class, including the application portfolio, the most use for a flexible approach to design is in situations where there is uncertainty as to the future value of a key design driver. In particular, a flexible approach should be used if there is uncertainty around the demand that will exist for the output of the system at the time when the system becomes operational. This lesson derives from the fact that if uncertainty is large and the cost of being wrong is also large, the additional cost of the flexible design can be easily overcome by being able to avoid large losses or take advantage of large gains. Moreover, the flexible design approach is most usable in situations where the uncertainty is large and the cost of implementing the flexibility is not excessive. For example, in this case the cost of producing in short lead-time facilities is only represented by the additional production cost. However, the cost would climb significantly higher if the flexible system required the development of new suppliers or even construction of the additional facilities.

What do you feel that you have learned from the process of doing the application?

The biggest lesson that can be gleamed from this analysis is that simulation is a great tool to get a general sense of the design space and the impact of different design choices on the results. However, it is ill-suited for fine tuning or finding the true optimal design since it involves a lot of trial and error. Thus, it is best to use simulation as a screening tool to narrow down the design space to the most promising alternatives and then to use other tools to fine-tune the design. For example, the simulation analysis performed here would be helpful in determining if the flexible design is better for a particular situation in terms of the production costs, sales prices, expected demand, etc., but additional analysis would be needed to find the best balance of initial long lead-time production and additional short lead-time production.

The other key lesson learned from this analysis — and from this class in general — is that a flexible design can be accomplished by creatively arranging traditional design elements. This was evident in many of the in-class examples including the classic parking lot construction example, but also in this particular case study. At no point was it necessary to change the manufacturing processes to be more
flexible. Rather, it was enough to think of the existing options and arrange them in such a way that they were better able to react to the key uncertainties associated with the system. This is a powerful insight because it means that even in very established industries with complex manufacturing processes it is possible to apply flexible design principles to improve the expected outcomes.
Bibliography


Appendix

# ESD.71 Application Portfolio Project ****************************
# Created by Andres Garro
# November 14, 2010
# Description: The script is a simulation based analysis of the optimal
# production quantity and allocation strategy for a portfolio of products
# under uncertain demand
#******************************************************************************

# Function Definition ****************************

sim_function <- function(N = 10000, n = 1, sigma = 1.5, rho = 0.25, gamma = 0.8, 
season = 0.3, cost_ratio = 1.5, plots = 1, i = 0.01) {

# Simulation Parameters ****************************
N <- 100000  # Number of observations in simulation
n <- 1  # Number of products in portfolio
sigma <- 2  # Coefficient of variation
rho <- 0  # Correlation coefficient
gamma <- 0.8  # Percent reduction in st. dev. after observing sales
season <- 0.3  # Portion of selling season supplied from short lead-time
cost_ratio <- 1.5  # Ratio of SLT cost to LIT cost

# Demand Parameters ****************************

K <- 10000  # Max demand rate per product (units/month)
d_mean <- runif(n-1, 0, K)
d_mean[n] <- max((n*K/2) - sum(d_mean), 1)

# Standard deviation of demand prior to observing sales
d_sd <- sigma * d_mean

# Commercial Parameters
price <- 30  # Price of products
cost_long <- 10  # Product cost in long lead-time facilities
cost_short <- cost_long * cost_ratio
season <- 6  # Length of selling season in months

# Demand Generation ****************************

# Generate matrix of correlated prior mean demand rate
d_meanlog <- log(d_mean) - 0.5*log(1 + (d_sd^2/d_mean^2))
d_sdlog <- sqrt(log(1 + (d_sd^2/d_mean^2)))
x <- rnorm(N, d_meanlog, d_sdlog)

#rm("D_prior")
for (i in 1:n) {
  d <- rnorm(n, d_meanlog[i], d_sdlog[i])
  x <- rho*X + sqrt(1 - rho^2)*d
  if (exists("D_prior")) (D_prior <- cbind(D_prior, x)) else (D_prior <- x)
}

if (n > 1) {
colnames(D_prior) <- seq(1:n)
rownames(D_Prior) <- seq(1:N)
}
# Generate posterior mean demand rate

```r
d_meanlog_post <- log(D_prior) - 0.5*log(1 + ((gamma*d_sd)^2/D_prior^2))
d_sdlog_post <- sqrt(log(1 + ((gamma*d_sd)^2/D_prior^2)))
if (n == 1) {
  D_post <- rinorm(D_prior, d_meanlog_post, d_sdlog_post)
} else {
  #rm("D_post")
  for (i in 1:n) {
    X <- rinorm(n, d_meanlog_post[,1], d_sdlog_post[,1])
    if (exists("D_post")) {D_post <- cbind(D_post, X)} else {D_post <- X}
  }
  if (n > 1) {
    colnames(D_post) <- seq(1:n)
    rownames(D_post) <- seq(1:N)
  }
}
```

# Inflexible Rule

**Inflexible Rule: determine the quantity (Q) to produce for each product based on a newsvendor calculation using the prior expected demand distribution rounded up to the nearest whole unit**

# Compute Decisions

```r
fc <- (price - cost_long)/price
Q1 <- ceiling(qlnorm(fc, d_meanlog, d_sdlog)) * T_season
```

# Compute Results

```r
#rm("P_inflex")
if (n == 1) {
  P_inflex <- (pmin(D_prior, Q1/T_season)*price/r)*(1 - (1/(1+r)*T_season)) - Q1*cost_long
} else {
  for (i in 1:n) {
    if (exists("P_inflex")) {
      P_inflex <- cbind(P_inflex, (pmin(D_prior[,1], Q1[i]/T_season)*price/r)*(1 - (1/(1+r)*T_season)) - Q1[i]*cost_long)
    } else {
      P_inflex <- (pmin(D_prior[,1], Q1[i]/T_season)*price/r)*(1 - (1/(1+r)*T_season)) - Q1[i]*cost_long
    }
  }
  if (n > 1) {P_inflex <- rowSums(P_inflex)}
}
```

# Flexible Rule

**Flexible Rule: determine the quantity (Q) to produce for the part of the season that must be supplied from LLT based on a newsvendor calculation using the prior expected demand distribution and then determine the additional quantity (Q_add) from SLT using a newsvendor calculation that uses the appropriate costs and takes into account left over inventory from Q**

# Compute Decisions

```r
fc_add <- (price - cost_short)/price
Qf <- ceiling(qlnorm(fc, d_meanlog, d_sdlog)) * T_season * (1 - R_season)
```
Q_add <- ceiling(qtnorm(frack, dmeanlog_post, dsdlog_post)) * T_season * E_season

if (n == 1) {
  Q_left <- Q - pmin(D_prior * (1 - E_season) * T_season, Q)
} else {
  for (1 in 1:n) {
    if (exists("Q_left")) {
      Q_left <- cbind(Q_left, Q[f+1] - pmin(D_prior[,1] * (1 - E_season) * T_season, Q[f+1] - Q[1]))
    } else {
      Q_left <- Q[f+1] - pmin(D_prior[,1] * (1 - E_season) * T_season, Q[f+1])
    }
  }
  Q_add <- pmax(Q_add + Q_left, 0)
}

# Compute Results
# rm("P_flex")
if (n == 1) {
  P_flex <- pmin(D_prior, Qf/((1 - E_season)*T_season)) * price/r*(1/(1+r))/(T_season*(1-E_season)) - Qf*cost_long + (pmin(D_post, Qf[1])/((1 - E_season)*T_season)) * price/r*(1/(1+r))/(T_season*(1-E_season)) - (Q_add*cost_short/((1+r)/(T_season*(1-E_season))))
} else {
  for (1 in 1:n) {
    if (exists("P_flex")) {
      P_flex <- cbind(P_flex, pmin(D_prior[,1], Qf[1]/((1 - E_season)*T_season)) * price/r*(1/(1+r))/(T_season*(1-E_season)) - Qf[1]*cost_long + (pmin(D_post[,1], Q_add[, 1])/(1/(1+r))/(T_season*(1-E_season)) - (Q_add[*c, 1])*cost_short/((1+r)/(T_season*(1-E_season))))
    } else {
      P_flex <- pmin(D_prior[,1], Qf[1]/((1 - E_season)*T_season)) * price/r*(1/(1+r))/(T_season*(1-E_season)) - Qf[1]*cost_long + (pmin(D_post[,1], Q_add[, 1])/(1/(1+r))/(T_season*(1-E_season)) - (Q_add[*c, 1])*cost_short/((1+r)/(T_season*(1-E_season))))
    }
  }
  if (n > 1) {P_flex <- rowSums(P_flex)}

# Generate Plots

if (plots) {
  png(file = "C:\Users\Andres\Documents\LGO\Fall 2010\Class\2.1\Application Portfolio\demand.png", width = 700, height = 700)
  par(mar = c(1, 1, .5, 0), xaxt = "n", yaxt = "n")
  plot(density(D_prior, bw = 500), xlim = c(0,2*K), ylim = c(0,0.0003))
  par(new = T, xaxt = "n", yaxt = "n")
  plot(density(D_post, bw = 500), xlim = c(0,2*K), ylim = c(0,0.0003), col = "blue")
  title(main = "Product Demand Rate Distributions", xlab = "Demand Rate (units/month)", ylab = "Probability")
  legend("right", legend = c("Before Season", "After Observing Sales"), col = c("black", "blue"), lty = 1)
dev.off()

png(file = "C:\Users\Andres\Documents\LGO\Fall 2010 Classes\ESD.71\Application Portfolio\varg.png", width = 700, height = 700)
par(ann = F, xaxt = "n", yaxt = "n")
plot(ecdf(P_inflex), xlim = c(min(min(P_inflex), min(P_flex)), max(max(P_inflex), max(P_flex)))*0.1, ylim = c(0,1))
par(new = T, xaxt = "s", yaxt = "s")
plot(ecdf(P_flex), xlim = c(min(min(P_inflex), min(P_flex)), max(max(P_inflex), max(P_flex)))*0.1, ylim = c(0,1), xlab = "Profit", col = "blue")
title(main = "VARG for Different System Designs", xlab = "Profit ($\%$), ylab = "Probability")
legend("right", legend = c("Inflexible", "Flexible"), col = c("black", "blue"), lty = 1)
dev.off()

png(file = "C:\Users\Andres\Documents\LGO\Fall 2010 Classes\ESD.71\Application Portfolio\profitdist.png", width = 700, height = 700)
par(ann = F, xaxt = "n", yaxt = "n")
plot(density(P_inflex), xlim = c(min(min(P_inflex), min(P_flex)), max(max(P_inflex), max(P_flex)))*0.1, ylim = c(0,0.000005))
par(new = T, xaxt = "s", yaxt = "s")
plot(density(P_flex), xlim = c(min(min(P_inflex), min(P_flex)), max(max(P_inflex), max(P_flex)))*0.1, xlab = "Profit", col = "blue", ylim = c(0,0.000005))
title(main = "NPV distributions for different system designs", xlab = "NPV ($\%$)", ylab = "Probability")
legend("right", legend = c("Inflexible", "Flexible"), col = c("black", "blue"), lty = 1)
dev.off()

Inflex <- c(mean(P_inflex), sd(P_inflex), mean(Q1)*cost_long, min(P_inflex), max(P_inflex), quantile(P_inflex, probs = c(0.05, 0.95)))
Flex <- c(mean(P_flex), sd(P_flex), mean(Q1)*cost_long, min(P_flex), max(P_flex), quantile(P_flex, probs = c(0.05, 0.95)))

Output <- rbind(Inflex, Flex)

Output
Andres Garro
Application Portfolio

# ESD.71: Application Portfolio Project
# Created by Andres Garro
# November 14, 2010
# Description: The script is a sensitivity analysis of the optimal
# production quantity and allocation strategy for a portfolio of products
# under uncertain demand with respect to magnitude of the uncertainty, the
# portion of the selling season supplied from short lead-time facilities, the
# the ratio of short lead-time production cost to long lead-time production
# cost, and the number of products.
#
# Sensitivity Parameters
# sigma <- seq(0.5, 2, 0.5)
# E <- seq(0.1, 0.9, 0.1)
# cost_ratio <- seq(1.2, 0.1)
# product <- c(1, 5, 10)
#
# Response Surface Generation
# z_inflex <- data.frame(matrix(nrow = length(E), ncol = length(cost_ratio)))
# for (n in 1:length(product)) {
#   png(file = paste("C:\Users\Andres\Documents\LGO\Fall 2010 Classes\ESD.71\Application Portfolio\sensitivity\", product[n], ".png", sep = ""), width = 960, height = 960)
#   split.screen(c(2, 2))
#   for (k in 1:length(sigma)) {
#     for (j in 1:length(cost_ratio)) {
#       cost_ratio = cost_ratio[j], sigma = sigma[k], z = product[n], N = 1000000)
#       z_inflex[1, j] <- z[1, 1]
#       z_inflex[1, j] <- z[2, 1]
#     }
#   }
#   z <- as.matrix(z_inflex)
#   nrow(z) <- as.matrix(nrow(z))
#   ncol(z)
#   # Create a function interpolating colors in the range of specified colors
#   jet.colors <- colorRampPalette(c("red", "green"))
#   # Generate the desired number of colors from this palette
#   ncol <- 100
#   color <- jet.colors(ncol)
#   # Compute the z-value at the facet centres
#   sfacet <- x[-1, -1] + x[-1, -nrz] + x[-nrz, -1] + x[-nrz, -nrz]
#   # Recode facet z-values into color indices
#   facetcol <- cut(sfacet, ncol)
#   # Add color information to plotting
#   par(las = 1, cex.axis = .75, cex.lab = 1.5, cex.main = 1.5, mai = c(0.5, 0.5, 0.5, 0.5))
#   persp(x, cost_ratio, z, ticktype = "detailed", zlim = c(-0.5, 1), col = color[facetcol], shade = 0.75, theta = 45, phi = 5, main = paste("Variation = ", sigma[k]), sub = "End of Season (%)", ylab = "Cost Ratio", xlab = "Profit Improvement (%)")
#   dev.off()


```