

## **Introducing Flexibility into an Aircraft Assembly Plant**

**Abstract:** This application portfolio analyzes the impact of uncertainty on the profits obtained in aircraft assembly. The primary source of uncertainty is the demand of aircraft in any given year due to the current state of the economy, and political influences. A simplified model has been created to relate these parameters to the overall demand of aircraft, which then translates into the profits obtained by an aircraft manufacturing company. The aircraft manufacturing company has the option to build and operate a facility of fixed size, or a facility that has the option to grow larger if necessary.

This paper outlines the uncertainties and flexibility options for the aircraft manufacturer. Then the two designs are analyzed using both a decision tree and binomial lattice to establish a value of the flexibility. Finally a hybrid decision tree with lattice is outlined and analyzed. From all three analyses, the flexible facility is more profitable than the fixed facility, allowing the aircraft manufacturer to realize higher potential gains even though he has a possibility of experiencing lower minimums.

## Table of Contents

|     |  |    |
|-----|--|----|
| 1   | System Description .....                               | 3  |
| 1.2 | Design Levels.....                                     | 3  |
| 2   | Uncertainties .....                                    | 4  |
| 2.1 | Uncertainty Number One: The Economy .....              | 4  |
| 2.2 | Uncertainty number Two: Wartime production spikes..... | 6  |
| 3   | Two-stage decision analysis .....                      | 8  |
| 4   | Lattice Development of Economic Uncertainty .....      | 10 |
| 5   | Decision Analysis using Lattice.....                   | 14 |
| 6   | Simplified Hybrid Decision Analysis – Lattice .....    | 17 |
| 7   | Discussion .....                                       | 20 |

# 1 System Description

Each year the United States government approves procurement funds available for each of the Department of Defense branches in the National Defense Authorization Act. In fiscal year 2008, the authorized funds for procurement purposes totaled \$18.5 billion for the Army, \$35.7 billion for the Navy and Marine Corps, \$33.8 billion for the Air Force, and \$3.9 billion for joint activities or rapid acquisitions. This gives the United States up to \$91.9 billion on procurements out of a total budget of \$623 billion for military expenditures [1]. Contractors to DoD organizations therefore have much to gain from a relationship with these organizations, yet there is also uncertainty in the total number of planes that will be demanded each year due to uncertainties primarily in the economy and the current political state. The question therefore, is how is flexibility able to help a contractor plan for these uncertainties.

In particular I will be looking at the tanker acquisition as it is a current topic of interest. 50 years ago, Boeing began producing the KC-135 tanker. In total it delivered 600 aircraft. The Air Force first purchased 29 of the fleet in 1954, with the first aircraft flying in 1956 and the last in 1965<sup>1</sup>. 732 KC-135 tankers have been produced, with 57 additional airframes being cancelled<sup>2</sup>. 600<sup>1</sup> of which have been delivered to the US Air Force, with 505 still being flown today<sup>3</sup>. The other tankers are being operated by France, Singapore, Pakistan and Turkey<sup>3</sup>. For the purposes of this exercise I will focus on the number of aircraft required by the US as the majority of previous tankers have been delivered here.

## 1.2 Design Levels

The aircraft manufacturer will therefore produce a baseline of 10 aircraft per year over the next 10 years (the tanker contract calls for 100 aircraft, versus the 600 previously desired). Because this is a highly specialized aircraft, a new assembly facility has to be built and staffed. This facility can either be built to support the size of the contract of 10 aircraft per year, or can be built with additional flexibility to allow for future expansion. The aircraft manufacturer may want this additional capability if demand rises, yet demand can also fall as will be illustrated. In addition the flexibility will cost the aircraft manufacturer both initially and for each aircraft produced as additional space, equipment and personnel will need to be purchased. This type of flexibility is an American call option “in-project” where if the outcome looks favorable at the end of any given year, the aircraft manufacturer can decide to exercise the option to expand. For simplicity in the decision lattice example, I used the European model where the aircraft manufacturer could only use this flexibility at the end of year 5.

### *Fixed:*

The fixed facility has a capacity to produce 10 aircraft per year. It utilizes 50 acres of land and 10 assembly lines. Each acre of land costs \$2 million over a 5 year period and each assembly line costs \$10 million over a 5 year period. Per year this is equivalent to

\$40 million in fixed costs. There is a marginal cost per aircraft of \$2.5 million for personnel, equipment, raw materials, and any pre-fabricated parts.

*Flexible:*

The flexible facility has an initial capacity of 10 aircraft per year, which can be increased to 12 aircraft per year yielding 60 aircraft in a 5 year period. The facility can also be decreased to 8 aircraft per year yielding 40 aircraft in a 5 year period. There is an initial \$30 million to purchase extra land, and building space. If the aircraft manufacturer decides to expand there is an additional \$5 million to purchase the assembly line machinery necessary giving a fixed cost of \$45 million. There is a marginal cost per aircraft of \$3 million for personnel equipment raw materials and any pre-fabricated parts. If the aircraft manufacturer decides to downsize to the 8 aircraft capacity he will raise a profit of \$3 million per year for the space and equipment that he can lend out.

In addition, for any year that demand cannot be met with supply, the aircraft manufacturer will lose \$10 million from loss of a secondary contract for both the fixed and flexible cases. The aircraft can be sold at \$10 million each. A summarization of all costs and profits can be seen in Table 1.

**Table 1: Costs and Profits for DoD Aircraft Manufacturer (in US Millions)**

|                             |           | Flexibility | Fixed                               |
|-----------------------------|-----------|-------------|-------------------------------------|
| Fixed Costs                 | Year 1    | \$75        | \$40                                |
|                             | Year 2-10 | \$45        | \$40                                |
|                             |           |             | per year                            |
| Marginal Cost per Plane     |           | \$2.50      | \$3<br>(if increased facility size) |
| Low supply fee              |           | \$2         | \$2                                 |
| Price per Plane             |           | \$10        | \$10                                |
| Profit from decreasing size |           | \$3         | \$0                                 |

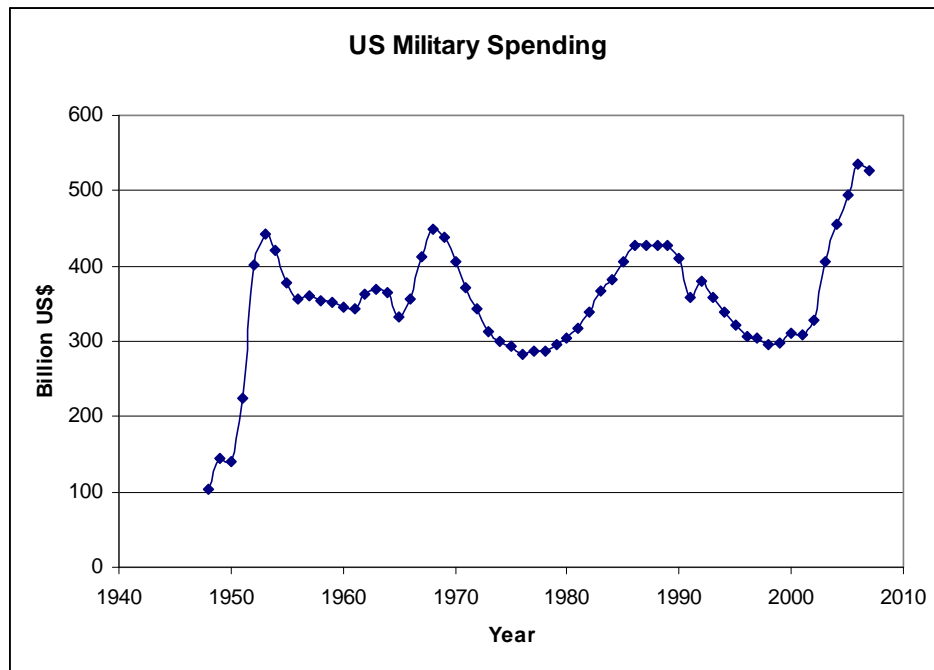
## 2 Uncertainties

As mentioned, there are uncertainties that can either increase or decrease the demand of aircraft over the 10 year period. Demand is primarily influenced by economic changes (recession, boom), and political changes (war, new president). Other uncertainties that are not being analyzed that could effect either the demand of the aircraft, or the profit margin include technological advances, regulations, environmental changes, wage fluctuations, and changes to the prices of raw materials. Each of these uncertainties are arguably important, yet are a smaller influence than economic and political changes.

### 2.1 Uncertainty Number One: The Economy

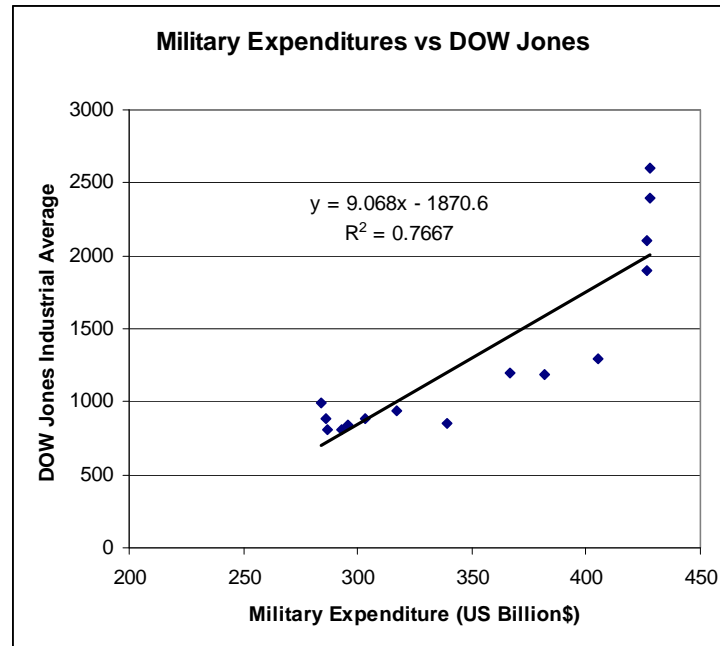
Though the US has a vast array of assets in order to invest in its aircraft, the government must curtail spending if the economy goes into a recession. It also has the ability to procure additional units if there is an economy boom. This can be seen in the Figure 1, showing the US Military Spending for a given year according to infoplease<sup>4</sup>. After the Vietnam War ended, there was concern over inflation causing the US market to be fairly stagnant, 5 then in the early 80s; Ronald Reagan was able to turn the market around through the deregulation movement. At this point the market rebounded as characterized by the Dow Jones Industrial Average<sup>6</sup>. This same financial trend can be seen in the history of US Military Spending shown below (adjusted to 2001 US\$) where there is a dip after the early 70s and then a rise after the early 80s.

**Figure 1: US Military Spending**



Though there is no exact correlation between how many airplanes would be requested or canceled in relation to the current state of the US economy, we know that the previous tanker cost around \$10 million each. We also know that in times of peace, the economy will have a much larger effect on military spending than times of war (times of war will always have higher spending). For times of peace, therefore I would like to relate the Dow Jones Industrial Average to the number of aircraft that are added or canceled. Taking an approximate average Dow Jones Industrial Average for 1975-1989 I found the trend between Military Expenditure and the Industrial Average as shown in Figure 2.

**Figure 2: Relationship between Military Expenditure and the Dow Jones Industrial Average**



I will presume that if total military expenditures dropped by 1,000 points for any given year, the military will cancel the order of 10 aircraft the following year saving itself 1 million on the tanker project alone. Similarly if it grows by 1,000 points, it will request 10 additional aircrafts. Thus all that is needed is the probability that the DOW will change by 1,000 points. 6 out of 14 of the sample peace years (1975-1989) increased more than 1,000 points in the year, 4 out of 14 of the sample years decreased by greater than 100 points, while 4 out of 14 of the same years remained within 100 points.

Hence there is an average chance of 3/7 that 10 additional aircraft from the contractual agreement will be required during any given year and a 2/7 chance that 10 fewer aircraft will be required. A Gaussian distribution with a standard deviation of 1/7 will be used as these estimates are fairly approximate.

## 2.2 Uncertainty number Two: Wartime production spikes

A second major uncertainty is political changes. When the United States is involved in a war, it will require a greater number of military aircraft due to increased military activity and/or a decreased number of current aircraft that can fly due to being hit.

We can estimate the length of wars, and how often wars are fought by looking at historical data of recent wars the United States become involved in. World War II lasted from 1939 to 1945, the Korean war from 1950-1953, and the Vietnam War from 1959-1975. The US then engaged in the Gulf War from 1990-1991, and then the war on Terror from 2001-present. On average wars last 6.6 years with a standard deviation of 5.77

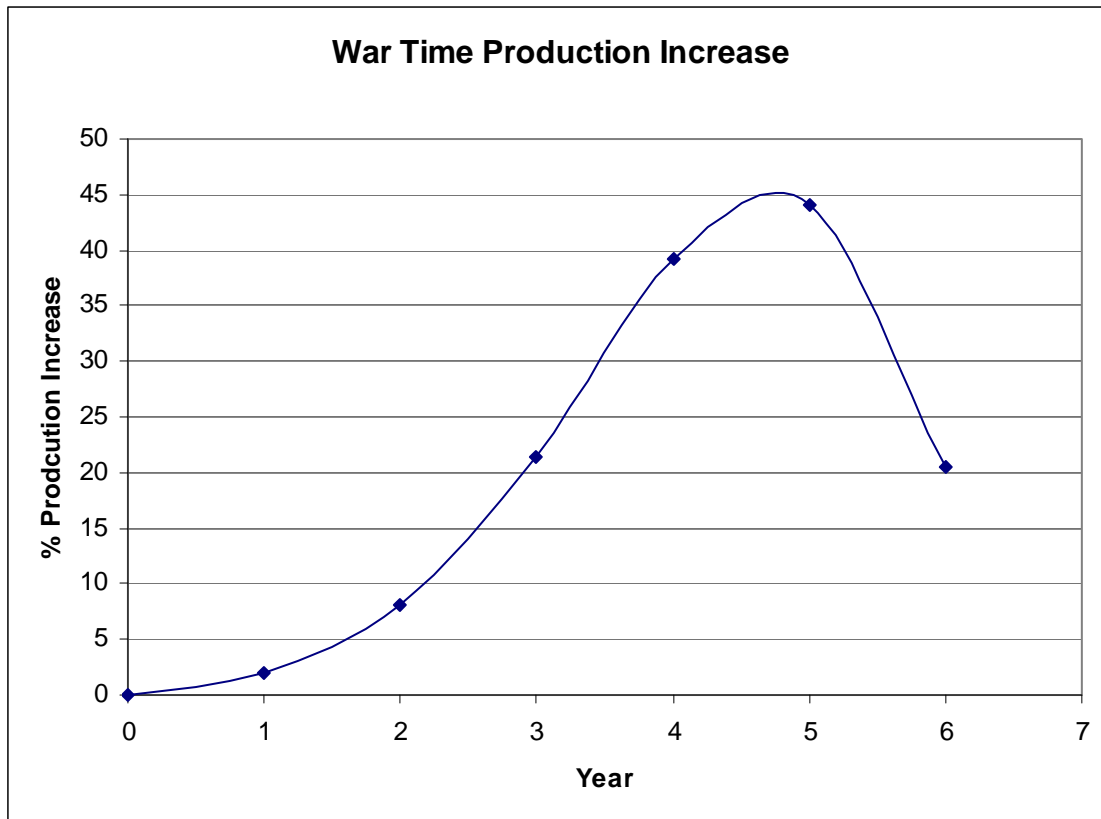
years, and time between wars is on average 11.4 years with a standard deviation of 6.66 year. I will assume a normal Gaussian distribution with those values.

Using the trend from World War II as shown in Table 2, the production of aircraft will increase about a year after the conflict begins, and then decrease again after the conflict has ended as shown in the table and graph below<sup>7</sup>. In the years after a war, I will assume that the post war production level remains somewhat constant (no additional affect from war, but other factors will influence this level as will be explained later). The trend line from this data as shown in Figure 3, gives a base for how overall aircraft production will vary over the time span of the war given an initial production. Given there is uncertainty within the trend as well as how long the war will actually last, I will assume that the actual weighted production increase will have a Gaussian distribution with a standard deviation equal to 10% of the stated value (the mean).

**Table 2: Total Aircraft Production Levels during WWII**

| Year                  | 0    | 1    | 2     | 3     | 4     | 5     | 6     |
|-----------------------|------|------|-------|-------|-------|-------|-------|
| Production Total      | 2141 | 6186 | 19433 | 47836 | 85898 | 96318 | 46001 |
| % Production Increase | 0    | 1.88 | 8.07  | 21.34 | 39.12 | 43.98 | 20.48 |

**Figure 3: Trend of Aircraft War Time Production Increases**



To simplify this uncertainty I analyzed what occurred during the 5 year period that war was occurring. Here I saw that if a war occurred, demand increased by 20%, or for our particular instance demand would increase by 10 aircraft over the 5 year period. Similarly, I estimated that for each 5 year period war would be occurring with a 2/5 probability versus a 3/5 chance of no war occurring.

Analyzing the two uncertainties together, I found the probability distribution as illustrated in Table 3. This distribution was used in the decision analysis to be outlined in the next section.

**Table 3: Demand Probability Distribution**

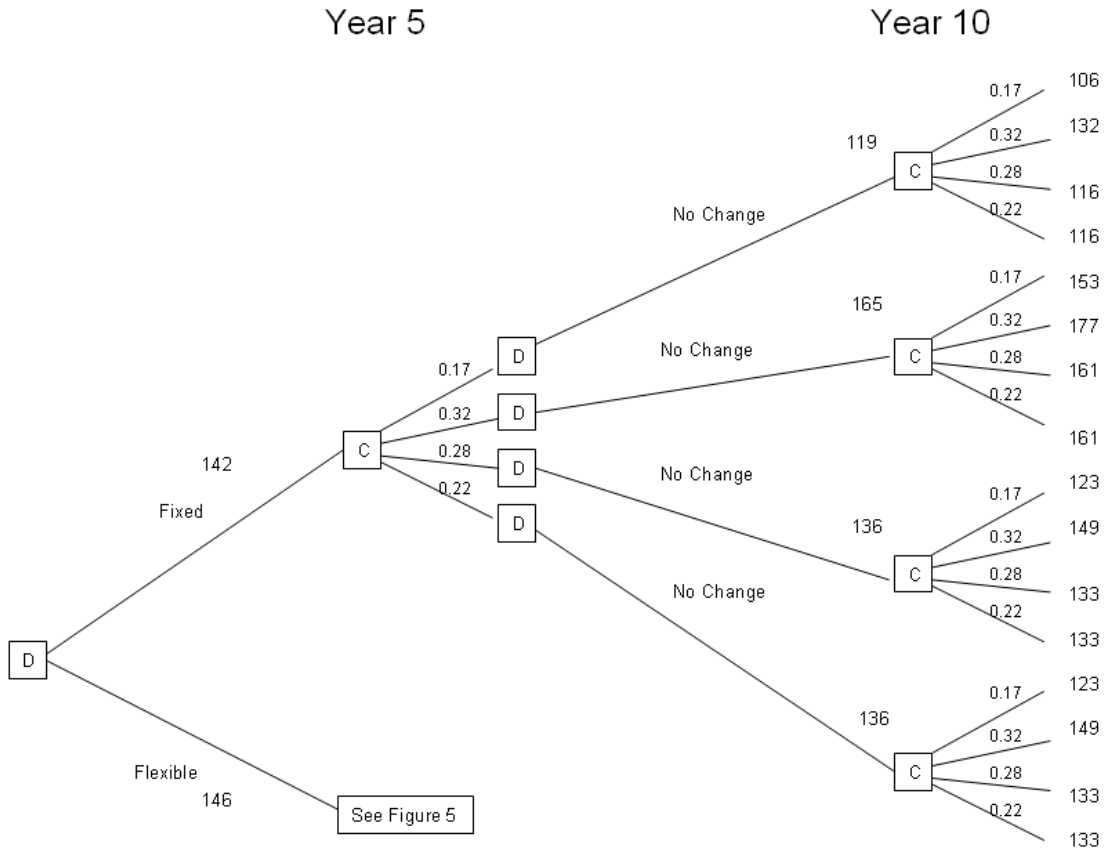
| #Aircraft | Probability |
|-----------|-------------|
| 40        | 0.24        |
| 50        | 0.28        |
| 60        | 0.32        |
| 70        | 0.16        |

### 3 Two-stage decision analysis

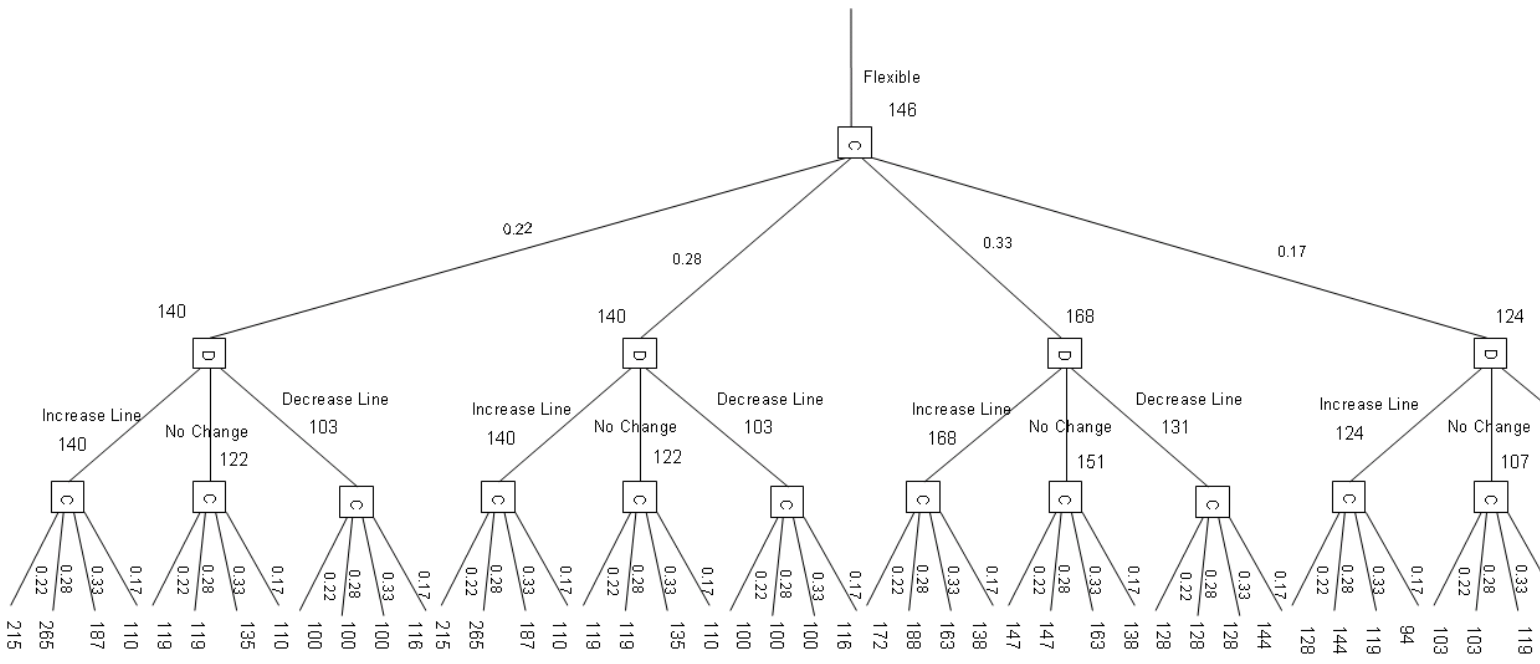
The decision analysis was set up using the probabilities of the uncertainties, and the cost and profit figures from Table 1 and 3. Starting at time  $t=0$ , the aircraft manufacturer can decide to build the fixed facility or the flexible facility. After 5 years there is a chance that the demand will have either increased or decreased from its original 50 aircraft. At this time if the aircraft manufacturer decided to choose the flexible facility, he can choose to either keep the facility at its current size, increase capacity to 60 aircraft or decrease capacity to 40 aircraft. If the aircraft manufacturer decided to choose the flexible facility than at  $t=5$  he has no decisions he can make. After the next 5 year period, at  $t=10$ , there is another chance that demand for that period will have either increased or decreased. This structure can be seen in Figure 4 and 5, which illustrates the chance and decision nodes over the two periods.



**Figure 4: Two-stage Decision Analysis Structure**



**Figure 5: Flexible Branch of Two-stage Decision Analysis Structure**

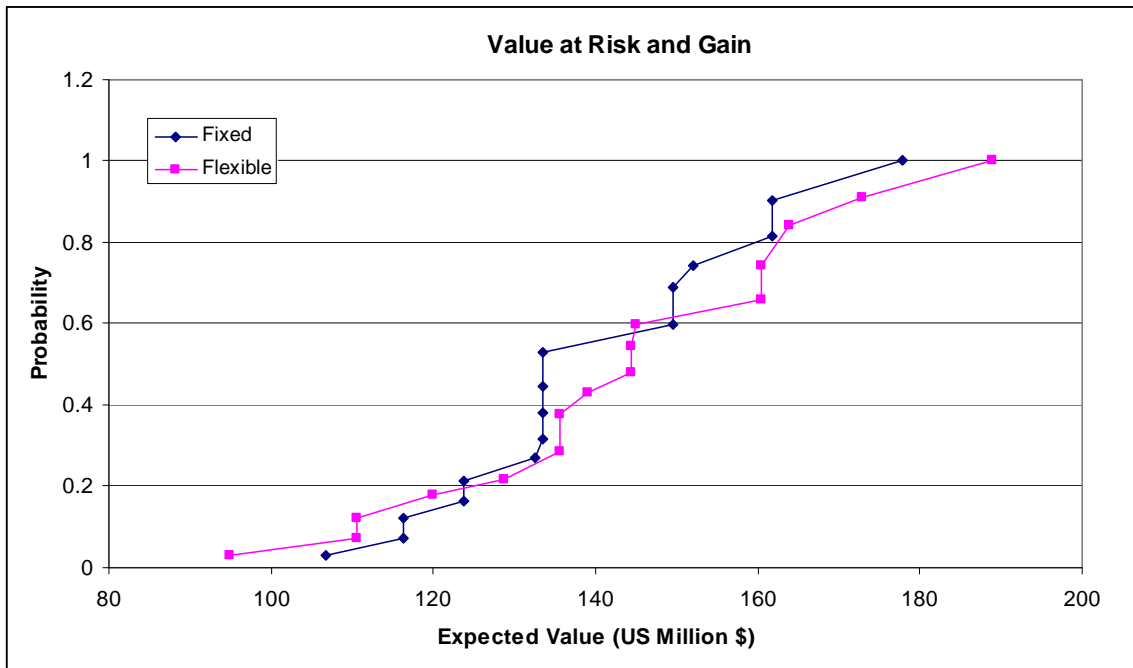


Using this decision structure, I found that the flexible design had a higher expected present value than the fixed design as well as a higher maximum expected value and benefit to cost ratio as shown in Table 4. However, the fixed design had a higher minimum expected value. Analyzing the VARG chart in Figure 6, the flexible design allows the aircraft manufacturer to take advantage of the upper potential but the cost of the flexibility extends the downside risk of the operation. It is likely that this lower minimum value for flexibility is due to the cost of flexibility. This cost is not as apparent in the maximum or expected value as the increased capacity allows for higher sales, allowing for the cost of flexibility to be absorbed.

**Table 4: Multiple Criteria Table (in million where applicable)**

|                | Flexible | Fixed |
|----------------|----------|-------|
| Expected Value | \$146    | \$142 |
| Min            | \$95     | \$106 |
| Max            | \$188    | \$177 |
| B/C            | 1.52     | 1.5   |

**Figure 6: Value at Risk and Gain for the Fixed and Flexible Design**

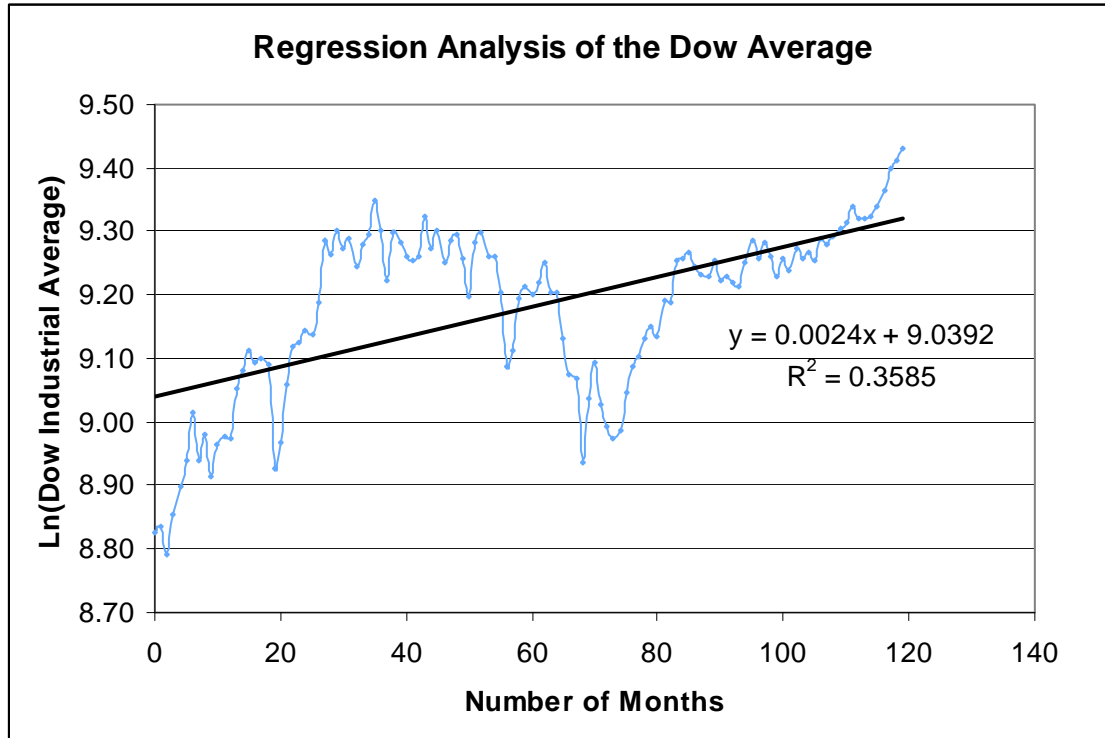


## 4 Lattice Development of Economic Uncertainty

The main uncertainty in this scenario is the current state of the economy. Every day the Dow can either increase or decrease. Analyzing the historical trends, one can observe

that similarity the Dow can either increase or decrease each year. Currently (November, 18 2008), the DOW is at approximately 8,000 points. In the long term I would expect to see a slow upward trend, much like the one that was seen between 1997 and 2007. In 1997, the Dow averaged at around 9000 and in 2007 it averaged at 12000. Using the monthly data from January 1997-January 2007<sup>6</sup>, I plotted the natural log of the Dow Industrial Average for that month with the number of months since January 1997 as shown in Figure 7.

**Figure 7: Natural Log of the Dow Jones Industrial Average**



By plotting a trend line on top of this data I found a Beta of 0.24 indicating that per month there is 0.24% increase per month. Translated yearly, this gives me a 2.88% increase per year which I will approximate to 2.5%. The standard deviation for any given year is approximately 1000 points (12.5%). Given  $r$  and  $\sigma$  I was able to find  $u$ ,  $d$  and  $p$  given the equations below:

$$u = e^{\sigma\sqrt{v^*t}}$$

$$d = e^{-\sigma\sqrt{v^*t}}$$

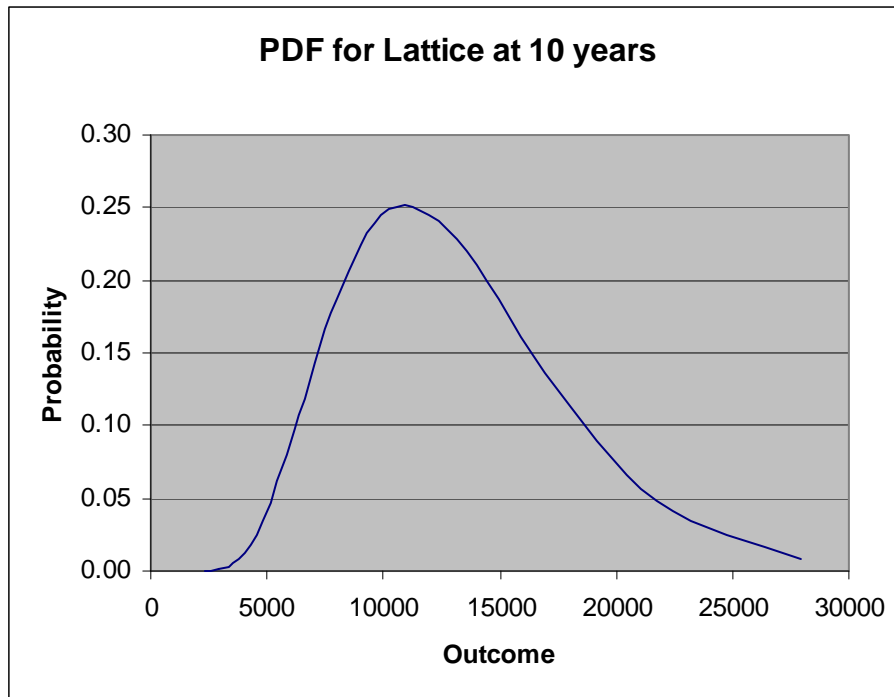
$$p = 0.5 + 0.5(v/\sigma)\sqrt{v^*t}$$

Using these values I found  $u=1.133$ ,  $d=0.882$ , and  $p=0.615$ . The discount rate of 12% was chosen as an approximation. From my knowledge, the aviation industry uses a discount rate between 10% and 15% in general. With these parameters I have all necessary variables for the lattice analysis as summarized in Table 5.

**Table 5 Summary Parameters for Lattice Analysis**

|                                      |        |
|--------------------------------------|--------|
| Variable                             |        |
| Initial Dow Jones Industrial Average | 8,000  |
| Discount Rate                        | 12%    |
| Time per Period                      | 1 year |
| Number of Periods                    | 10     |
| Growth Rate per period               | 2.5%   |
| Volatility                           | 12.5%  |
| u                                    | 1.131  |
| d                                    | 0.882  |
| p                                    | 0.615  |

The resultant PDF for the lattice is shown in Figure 8. After 10 years, the average Dow Jones will be 11450, which given the historical trend of the Dow is reasonable.

**Figure 8: PDF for the Dow Jones Industrial Average at 10 years**

Given these values I then calculated the expected Dow after 10 years to reevaluate the flexibility of the aircraft manufacturer. More pertinent to our aircraft manufacturer is how the Dow fluctuations will effect his production. To approximate the relationship between the Dow and demand, I created blocks of increasing point increments where 8000-10000 points corresponded to a demand of 10 aircraft, 10000-14000 corresponds to a demand of 12 aircraft, 14000-20000 14 aircraft and 20000+ to 16 aircraft per year. Similarly if the Dow drops, 5000-8000 corresponds to a demand of 8 aircraft and less

than 50000 corresponds to a demand of 6 aircraft. Unlike the two-stage decision analysis, the lattice analysis is calculated on a yearly basis. Calculated over the five year period, however, the demand rates match. For example the demand rate of 10 aircraft per year corresponds to the demand rate of 50 aircraft over 5 years. To show this blocking in action, Table 6 illustrates the different outcomes that could occur at year 10 along with the resultant demand and probability.

Though there have been modifications to the model, to keep consistency the fixed facility can only support a demand of 10 aircraft per year (50 per 5 year period), and the flexible facility can increase production to 12 aircraft per year (60 per 5 year period). Additionally, the penalty for not meeting demand remains at \$10 million per 5 years, or \$2 million per year, regardless of the supply-demand differential.

**Table 6: Dow Jones Industrial Average and Demand at Year 10**

| Outcome | Resultant Demand | Probability |
|---------|------------------|-------------|
| 27923   | 16               | 0.01        |
| 21746   | 16               | 0.05        |
| 16936   | 14               | 0.14        |
| 13190   | 12               | 0.23        |
| 10272   | 10               | 0.25        |
| 8000    | 10               | 0.19        |
| 6230    | 8                | 0.10        |
| 4852    | 6                | 0.03        |
| 3779    | 6                | 0.01        |
| 2943    | 6                | 0.00        |
| 2292    | 6                | 0.00        |

Using this new model, I then transformed the outcome lattice to the demand lattice. From this we can find the yearly demand for all scenarios, along with the probability that each scenario will occur as shown in Table 7.

**Table 7: Outcome, Probability and Demand Lattice to Model Economic Uncertainty**

| OUTCOME LATTICE     |      |      |       |       |       |       |       |       |       |       |       |
|---------------------|------|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Time                | 0    | 1    | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    |
|                     | 8000 | 9065 | 10272 | 11640 | 13190 | 14946 | 16936 | 19191 | 21746 | 24642 | 27923 |
|                     |      | 7060 | 8000  | 9065  | 10272 | 11640 | 13190 | 14946 | 16936 | 19191 | 21746 |
|                     |      |      | 6230  | 7060  | 8000  | 9065  | 10272 | 11640 | 13190 | 14946 | 16936 |
|                     |      |      |       | 5498  | 6230  | 7060  | 8000  | 9065  | 10272 | 11640 | 13190 |
|                     |      |      |       |       | 4852  | 5498  | 6230  | 7060  | 8000  | 9065  | 10272 |
|                     |      |      |       |       |       | 4282  | 4852  | 5498  | 6230  | 7060  | 8000  |
|                     |      |      |       |       |       |       | 3779  | 4282  | 4852  | 5498  | 6230  |
|                     |      |      |       |       |       |       |       | 3335  | 3779  | 4282  | 4852  |
|                     |      |      |       |       |       |       |       |       | 2943  | 3335  | 3779  |
|                     |      |      |       |       |       |       |       |       |       | 2597  | 2943  |
|                     |      |      |       |       |       |       |       |       |       |       | 2292  |
| PROBABILITY LATTICE |      |      |       |       |       |       |       |       |       |       |       |
| Time                | 0    | 1    | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    |

|                       |      |      |      |      |      |      |      |      |      |      |      |
|-----------------------|------|------|------|------|------|------|------|------|------|------|------|
|                       | 1.00 | 0.62 | 0.38 | 0.23 | 0.14 | 0.09 | 0.05 | 0.03 | 0.02 | 0.01 | 0.01 |
|                       |      | 0.38 | 0.47 | 0.44 | 0.36 | 0.28 | 0.20 | 0.15 | 0.10 | 0.07 | 0.05 |
|                       |      |      | 0.15 | 0.27 | 0.34 | 0.34 | 0.32 | 0.27 | 0.22 | 0.18 | 0.14 |
|                       |      |      |      | 0.06 | 0.14 | 0.22 | 0.27 | 0.29 | 0.28 | 0.26 | 0.23 |
|                       |      |      |      |      | 0.02 | 0.07 | 0.12 | 0.18 | 0.22 | 0.24 | 0.25 |
|                       |      |      |      |      |      | 0.01 | 0.03 | 0.07 | 0.11 | 0.15 | 0.19 |
|                       |      |      |      |      |      |      | 0.00 | 0.01 | 0.03 | 0.06 | 0.10 |
|                       |      |      |      |      |      |      |      | 0.00 | 0.01 | 0.02 | 0.03 |
|                       |      |      |      |      |      |      |      |      | 0.00 | 0.00 | 0.01 |
|                       |      |      |      |      |      |      |      |      |      | 0.00 | 0.00 |
|                       |      |      |      |      |      |      |      |      |      |      | 0.00 |
| <b>DEMAND LATTICE</b> |      |      |      |      |      |      |      |      |      |      |      |
| Time                  | 0    | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   |
|                       | 10   | 10   | 12   | 12   | 12   | 14   | 14   | 14   | 16   | 16   | 16   |
|                       |      | 8    | 10   | 10   | 12   | 12   | 12   | 14   | 14   | 14   | 16   |
|                       |      |      | 8    | 8    | 10   | 10   | 12   | 12   | 12   | 14   | 14   |
|                       |      |      |      | 8    | 8    | 8    | 10   | 12   | 12   | 12   | 12   |
|                       |      |      |      |      | 6    | 8    | 8    | 8    | 10   | 10   | 10   |
|                       |      |      |      |      |      | 6    | 6    | 8    | 8    | 8    | 10   |
|                       |      |      |      |      |      |      | 6    | 6    | 6    | 8    | 8    |
|                       |      |      |      |      |      |      |      | 6    | 6    | 6    | 6    |
|                       |      |      |      |      |      |      |      |      | 6    | 6    | 6    |
|                       |      |      |      |      |      |      |      |      |      | 6    | 6    |
|                       |      |      |      |      |      |      |      |      |      |      | 6    |

## 5 Decision Analysis using Lattice

Using the DOW Jones Indicator I was now able to analyze the value of flexibility given the economic uncertainty. Here, if the DOW grows past a certain point, increasing demand past a certain degree, then it will be advantageous for the aircraft manufacturer to utilize his flexibility and expand the capacity from 10 to 12 aircraft a year. For this evaluation I am not considering that the aircraft manufacturer can go back to the 10 aircraft capacity or decrease capacity to 8 aircraft.

Using this demand matrix as illustrated in Table 7 from the previous section, I found the cash flow and weighted cash flow for each scenario. Using this I found that I had an ENPV of 114 Million over 6 years and 153 Million over 10 years. From here I focused the analysis on the first 6 years looking at the introduction of flexibility.

I calculated the cash flow of the expanded plan then used the dynamic programming set up in the Binomial Tree Model 2008 to evaluate when it was beneficial to utilize the flexibility. The three resultant ENPV lattices are as shown in Table 8 where the ENPV is in million US\$.

**Table 8: ENPV for the Fixed Small and Expanded Facility, and the Flexible Facility**

| Year | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|------|---|---|---|---|---|---|---|
|------|---|---|---|---|---|---|---|

|                              |     |     |     |     |    |    |    |
|------------------------------|-----|-----|-----|-----|----|----|----|
| <b>ENPV (Cash Flow)</b>      | 114 | 136 | 108 | 87  | 67 | 47 | 25 |
| <b>NO FLEXIBILITY</b>        |     | 116 | 121 | 104 | 72 | 47 | 25 |
| <b>SMALL FACILITY</b>        |     |     | 87  | 85  | 84 | 61 | 25 |
| Dynamic programming approach |     |     |     | 59  | 56 | 46 | 35 |
| (check next year)            |     |     |     |     | 26 | 33 | 20 |
|                              |     |     |     |     |    | 9  | 5  |
|                              |     |     |     |     |    |    | 5  |

|                              |     |     |     |     |    |    |    |
|------------------------------|-----|-----|-----|-----|----|----|----|
| <b>ENPV (Cash Flow)</b>      | 118 | 142 | 117 | 98  | 80 | 59 | 34 |
| <b>EXPANDED FACILITY</b>     |     | 118 | 125 | 111 | 84 | 64 | 44 |
| Dynamic programming approach |     |     | 86  | 86  | 87 | 69 | 44 |
| (check next year)            |     |     |     | 57  | 53 | 42 | 30 |
|                              |     |     |     |     | 23 | 29 | 16 |
|                              |     |     |     |     |    | 7  | 2  |
|                              |     |     |     |     |    |    | 2  |

|                                   |     |     |     |     |    |    |    |
|-----------------------------------|-----|-----|-----|-----|----|----|----|
| <b>ENPV (Cash Flow)</b>           | 118 | 142 | 117 | 98  | 80 | 59 | 25 |
| <b>WITH FLEXIBILITY TO EXPAND</b> |     | 118 | 125 | 111 | 84 | 64 | 25 |
| Dynamic programming approach      |     |     | 87  | 86  | 87 | 69 | 25 |
| (check next year)                 |     |     |     | 59  | 56 | 46 | 35 |
|                                   |     |     |     |     | 26 | 33 | 20 |
|                                   |     |     |     |     |    | 9  | 5  |
|                                   |     |     |     |     |    |    | 5  |

Each cell in the lattice is found recursively. To find the ENPV for the small plant and expanded plant case is relatively simple. Each cell in year 6 simply equals the cash flow during year 6 for that cell. The cells in year 5 equal the cash flow for year 5 plus the expected cash flow for all potential future values multiplied by the probability that that value could occur discounted to year 5's present value. For example, to find the 3<sup>rd</sup> cell in the fixed plant year 5 (value of 61 million), you take that cell's cash flow (35 million) plus the discounted up possibility  $(25 \cdot 0.4) / (1 + .12)$  plus the discounted down probability  $(35 \cdot 0.6) / (1 + .12)$ .

The ENPV with flexibility to expand uses the same methodology but in addition, each cell is analyzed to determine if the flexible or fixed plant would give a higher expected present value. Depending on which is higher, that option is utilized. For example the 1<sup>st</sup> cell of year 5 (value of \$59 million) is found by analyzing if the flexible plant yields a higher ENPV than the fixed plant. It can be found that the fixed facility will yield an ENPV of \$47 which is the cash flow at that cell, 25, plus discounted up possibility  $(25 \cdot 0.6) / (1 + .12)$  plus the discounted down probability  $(25 \cdot .4) / (1 + .12)$ . Whereas the flexible facility will yield an ENPV of \$59 which is the cash flow at that cell, 22, plus discounted up possibility  $(34 \cdot 0.6) / (1 + .12)$  plus the discounted down probability  $(44 \cdot 0.4) / (1 + .12)$ . Because the flexible facility yields a higher ENPV, the flexible facility should be chosen, and thus the value at that cell is \$59 million.

The model can thus determine which option is higher, which can in turn be recorded. As shown in Table 9, it can be seen that in the first cell of year 5, the aircraft manufacturer should exercise the call option, consistent to what was found above. In fact, the aircraft manufacturer should expand the facility immediately. Overall the value of this flexibility is 4 million.

**Table 9: Exercise Call to Option**

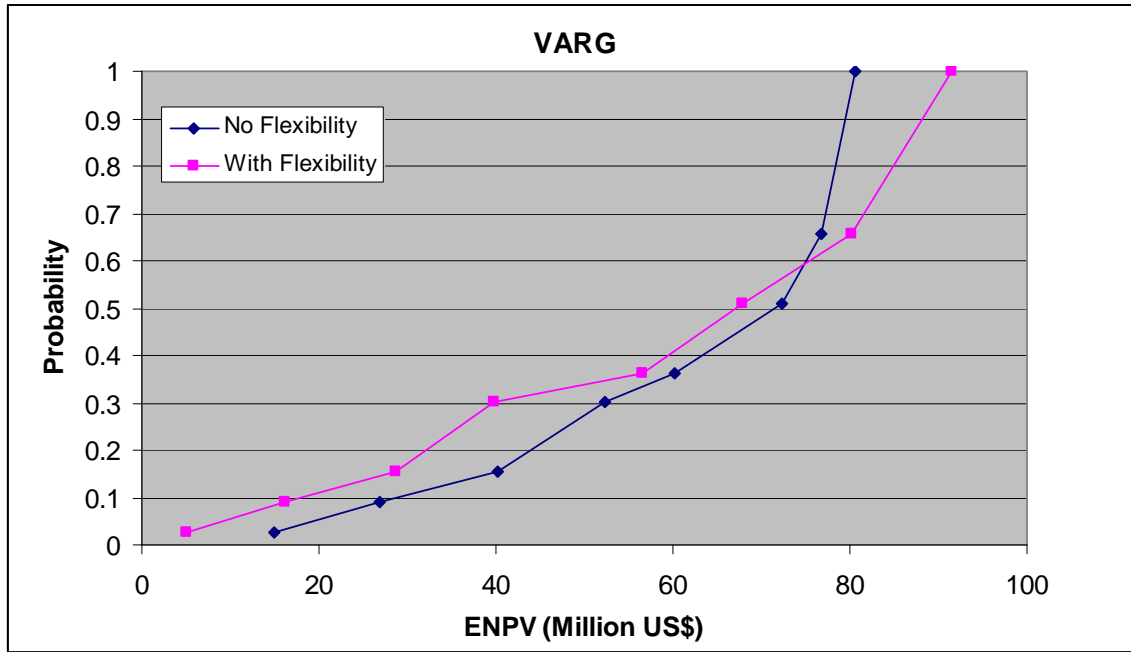
|                                  |            |            |            |            |            |            |
|----------------------------------|------------|------------|------------|------------|------------|------------|
| <b>Exercise CALL<br/>OPTION?</b> | <b>YES</b> | <b>YES</b> | <b>YES</b> | <b>YES</b> | <b>YES</b> | <b>YES</b> |
|                                  |            | <b>YES</b> | <b>YES</b> | <b>YES</b> | <b>YES</b> | <b>YES</b> |
|                                  |            |            | <b>NO</b>  | <b>YES</b> | <b>YES</b> | <b>YES</b> |
|                                  |            |            |            | <b>NO</b>  | <b>NO</b>  | <b>NO</b>  |
|                                  |            |            |            |            | <b>NO</b>  | <b>NO</b>  |
|                                  |            |            |            |            |            | <b>NO</b>  |
|                                  |            |            |            |            |            | <b>NO</b>  |

To analyze how the fixed and flexible facilities differed further I created a VARG chart up to year 3 as shown in Figure 9. This VARG was found by analyzing each of the nine distinct paths through the lattice to year 3. These paths are the combinations of the up and down possibilities of the Dow Jones. One such path is the probability that for years one through three, the Dow Jones increases. The probability of such an occurrence is  $0.6^3$ , or 0.216. The ENPV for this path in the fixed facility is \$70, which is found by taking the discounted value of the cash flows in the 1<sup>st</sup> cell of years one through three, or  $35/(1.12)+25/(1.12)^2+25/(1.12)^3$ . Similarly, the flexible facility’s ENPV of \$91 for this path can be found by taking the discounted value of the cash flows in the 1<sup>st</sup> cell of years one through three of the expanded facility. If the aircraft manufacturer waited to exercise the call to option, then the ENPV would be found by analyzing the discounted cash flow in the 1<sup>st</sup> cell of whichever facility Table 9 indicated to be used. As it is, the aircraft manufacturer should expand immediately, thus only the expanded facility’s cash flow is used.

Here it can be seen that the flexibility primarily allows the company to take advantage of upside gain, maximum of \$91 million with flexibility compared to \$70 million fixed. However, there is a greater downside risk, min = \$4 million with flexibility compared to \$14 million fixed. I would assume a similar VARG would be seen if the entire 10 year period was analyzed. By analyzing the VARG and the multiple criteria as illustrated in Table 10, it is beneficial for the company to utilize flexibility, and expand immediately.



**Figure 9: Value at Risk and Gain for the Fixed and Flexible Facilities**



**Table 10: Multiple Criteria Table (in million where applicable)**

|                          | Flexible | Fixed |
|--------------------------|----------|-------|
| Expected Value (year 10) | \$159    | \$153 |
| Expected Value (year 4)  | \$68     | \$64  |
| Min                      | \$4      | \$14  |
| Max                      | \$91     | \$70  |
| B/C                      | 1.51     | 1.69  |

The overall expected value has increased from the two-stage decision analysis for both the flexible and fixed cases which were \$146 and \$142 respectively, however, the overall trends from the lattice method match those of the two-stage decision analysis method. It is likely that the increase here was caused from a shift in call option strategies (American versus European) and by the omission of political uncertainty.

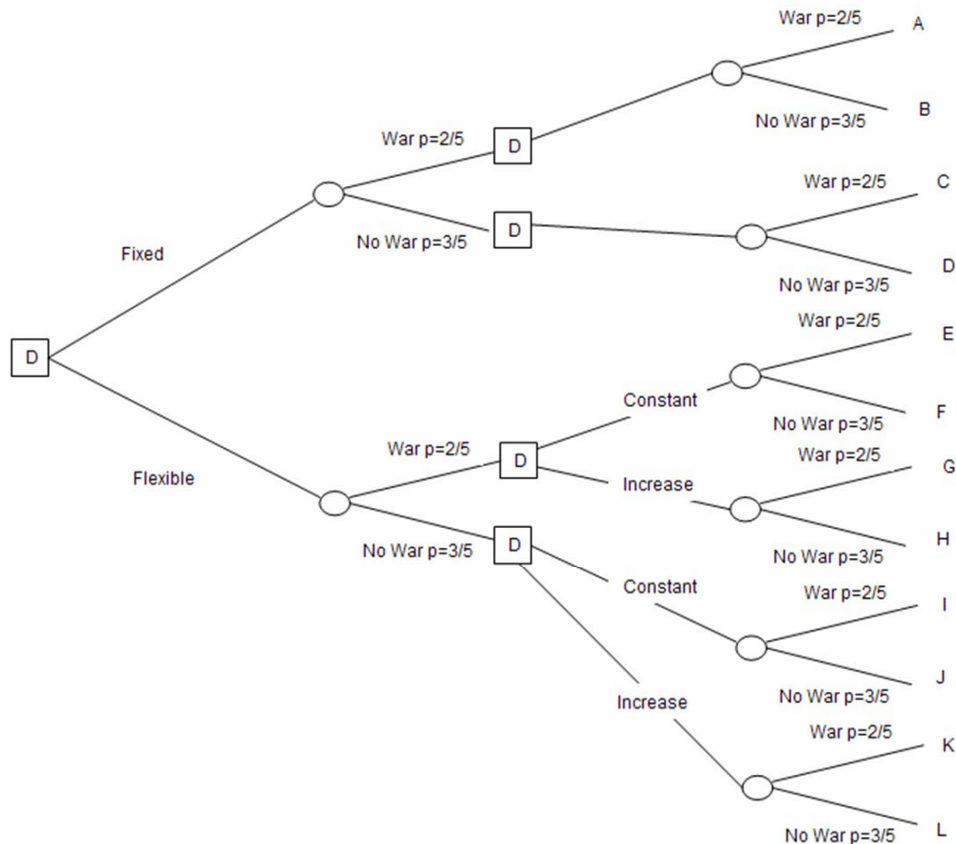
## 6 Simplified Hybrid Decision Analysis – Lattice

From here I wanted to attempt to use a modified version of the hybrid lattice and decision analysis approach presented in class. I felt that my particular case study was built in a similar manner to the example, and through a little manipulation could be quite useful.

I modified my decision tree so that the chance event is the occurrence of war, and the underlying uncertainty modeled by the lattice is the current state of the economy as shown below. This gave me 12 scenarios to evaluate as shown in Figure 10. Much like

the example, I calculated the effective outcome lattice and the effective instant value lattice. These lattices were found using the model established in my decision analysis lattice where in the flexibility case, the aircraft manufacturer could decide to expand at any point he deems instead of only at the end of year 5.

**Figure 10: Hybrid Decision Analysis Structure**



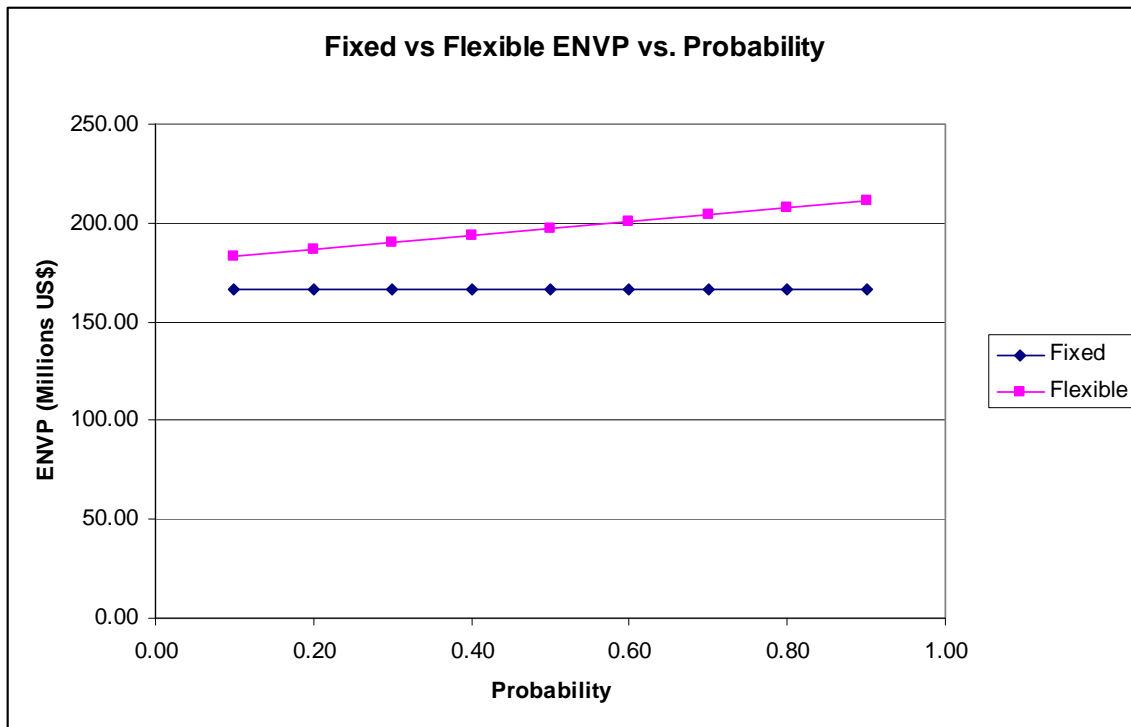
I then combined the effective instant value lattices resulting from a particular chance node using the probability that war would occur. In this manner I was able to obtain the effective instant value lattice for A+B, C+D, and so forth. Using the probability lattice from the previous section I then obtained the weighted case flow, and resultant expected cash flow using a discount value of 12%. This gave me the expected net present value at each of the chance nodes. For example, A+B had an expected present value of \$166.62 million, E+F \$105.07 million and so forth.

From here I found the best path in the case of the flexible design. To find the best path, I compared the expected value of keeping the capacity at the same level, and of increasing capacity to 12 airplanes a year. Using this decision I then calculated the overall expected value for the fixed and flexible circumstance by multiplying the resultant expected value of the decision nodes to their corresponding chance probabilities. I found that the aircraft manufacturer should expand if there is or is not a war.

Once all decision nodes were solved, I took the expected net present value of each decision resulting from a chance node and using the chance probabilities found the expected net present value of that chance node. This is albeit a simplification of what I could have completed, and prevents me from analyzing the min, max, and VARG for each case, but can nonetheless be very informative. From this analysis I found that the expected net present value of the fixed facility is \$166.62 million and the flexible facility is \$192.75. The resultant value of flexibility is \$27.13 million, nearly 3 times the initial cost of the flexibility (\$10 million).

One uncertainty that was not modeled as thoroughly previously was the probability that a war could occur. I conducted a quick sensitivity analysis to analyze if the fixed facility would become more advantageous given a different assumption. A similar analysis could be conducted varying the parameters of the lattice structure for the economic uncertainty. I found that independent of the probability that war would occur, the flexible facility would always yield a higher ENPV than the fixed facility as can be seen in Figure 11. However, as the probability decreases, the gap between the two values decreases. Future studies could analyze how variations in the economic uncertainty parameters influence the two designs.

**Figure 11: Sensitivity of ENPV to the Probability that War Would Occur in the 5 Year Period**



## 7 Discussion

Overall, the flexible facility proves to be more profitable than a fixed facility given uncertainties in the economy and political state of the world. Both designs yield a profit for the aircraft manufacturer, but dependent upon the model, flexibility adds an addition \$4 to \$23 million US dollars of value. This value exists due to the fact that the aircraft manufacturer is able to realize the higher potential the project could yield even though there is an additional initial and recurring cost. This conclusion is independent of the uncertainties in the political state of the world as shown in the hybrid-lattice case, though the political uncertainty undoubtedly still affects the overall profit of such an endeavor. It is entirely possible that variations in the economic model variables could change the profitability of the two designs, and this would be a point to consider if building an aircraft manufacturing facility.

The model outcomes varied due to changes to parameters in the model. The primary parameters that varied between the three models were economic uncertainty, political uncertainty, how costs and profits were modeled, and what type of put option was used. As shown in Table 11, this influenced the overall results as illustrated by a variation in expected value. All other parameters such as discount rate, cost and profit per aircraft, and cost of flexibility I kept constant to those values stated in Tables 1 and 5.

**Table 11: Parameter Changes by Model**

| <b>Model:</b>    | <b>Economic Uncertainty</b> | <b>Political Uncertainty</b> | <b>Costs/Profits</b>      | <b>Put Option</b> | <b>EV Flexible</b> | <b>EV Fixed</b> |
|------------------|-----------------------------|------------------------------|---------------------------|-------------------|--------------------|-----------------|
| <b>Two-Stage</b> | Modeled per 5 year period   | Modeled per 5 year period    | Modeled per 5 year period | European          | \$146              | \$142           |
| <b>Lattice</b>   | Modeled yearly              | Not modeled                  | Modeled yearly            | American          | \$156              | \$148           |
| <b>Hybrid</b>    | Modeled yearly              | Modeled per 5 year period    | Modeled Yearly            | American          | \$192              | \$166           |

From the two-stage decision model to the decision analysis using the lattice, the uncertainty of war was removed, and the uncertainty of the economy was more finely modeled. In addition, costs and profits were modeled on a per year basis giving a much more robust model. The lattice model also allowed the aircraft manufacturer to utilize his flexibility at the end of any year, whereas the two-stage decision only allowed the aircraft manufacturer to utilize flexibility at the end of year 5. I appreciated how the lattice approach allowed me to use the American Put Option, which is likely a more realistic option for an aircraft manufacturer to use. I also thought that modeling the economic uncertainty by year was much more robust than modeling it every 5 years, however I also wanted to model the political uncertainty and be able to compare the two models for the full 10 years.

The hybrid decision analysis allowed me to reintroduce the political uncertainty while keeping the robustness of the lattice decision analysis, along with the American call

option. I felt this model was the most useful model in that it modeled what could occur in the real world much closer than the other two models. It was also the most intellectually stimulating model for me to build as I had to create my own worksheets for it. Many of the other models I had partial worksheets for, or worksheets that were very similar. The hybrid model allowed me to take what I had learned in class and try to apply it from scratch.

It is interesting to note that while the expected value of the designs changed over the three models, the greatest deviation between them is \$46 million, or approximately a fifth of the value found. It is likely that the most influential parameter is the put option due to the nature of the problem. During the first five years, the aircraft manufacturer will lose revenue both from penalties and missed business with the European option if demand exceeds 10 airplanes. With the American put option, he is able to increase supply to 12 aircraft, thus meeting increased demand immediately. Because demand tended to increase rather than decrease a further interesting study to conduct would be to analyze if a 12 aircraft plant design that could expand was a better choice than a fixed 12 aircraft plant design, or one of the plant designs already modeled.

Future work could also reduce the many simplifications in the cost and profit structure, as well as the model of the uncertainties. Most of the cost and profit structures I extrapolated from what information I found in articles on the tanker acquisition, which essentially amounted to guesswork. It would be possible to modify each of the models to reduce these simplifications given time and data on actual cost and profit structures. Additionally, the uncertainties each had a mean and distribution which I simplified to a demand probability distribution as shown in Table 3. With the lattice and hybrid model, I was able to model economic uncertainty to a greater degree than this probability distribution, yet I did not model political uncertainty in such a manner. This simplification was completed on the basis of time, but in the future I could use simulation to not simplify the uncertainties in such a manner.

This application project allowed me to practice utilizing many of the tools and techniques introduced in our class. For the two-stage decision analysis, the decision analysis using a lattice, and in my hybrid lattice analysis I had to take the methodologies introduced in class and tailor them to my particular project forcing me to really consider how the models worked, and how I could make them work for me. This is certainly a skill that I will need in the future and was a very interesting intellectual endeavor.

### ***References:***

- 1) "KC-135 Stratotanker" *Air Force Link* <http://www.af.mil/factsheets/factsheet.asp?fsID=110> 9/28/08
- 2) "Boeing C-135 Stratolifter/KC-135 Stratotanker." *Uswarplanes.net* <http://www.uswarplanes.net/kc135.html> 9/28/08
- 3) "KC-135 Stratotanker" *Wikipedia* [http://en.wikipedia.org/wiki/KC-135\\_Stratotanker](http://en.wikipedia.org/wiki/KC-135_Stratotanker) 9/28/08

- 4) "US Military Spending" *Infoplease*. <http://www.infoplease.com/ipa/A0904490.html> 9/28/08
- 5) "Economic History of the United States" *Wikipedia*.  
[http://en.wikipedia.org/wiki/Economic\\_history\\_of\\_the\\_United\\_States](http://en.wikipedia.org/wiki/Economic_history_of_the_United_States) 9/28/08
- 6) "Dow Jones Industrial Average Index" *Google Finance*  
<http://finance.google.com/finance?q=INDEXDJX:.DJI> 9/28/08
- 7) "World War II Aircraft Production" *Wikipedia*  
[http://en.wikipedia.org/wiki/World\\_War\\_II\\_aircraft\\_production](http://en.wikipedia.org/wiki/World_War_II_aircraft_production) 9/28/08