

**1.146 – Engineering Systems Analysis for Design  
Application Portfolio:**

**Construction of a New Rapid Transit Corridor**

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## Abstract

This application portfolio evaluates design options for a planned rapid transit corridor. The case is hypothetical and involves the proposed construction of a light rail line in a transit corridor which currently has a ridership of about 6000 trips per day. Significant growth is projected in the area, which would be reflected in increased ridership over the next 24 years; ridership growth would be intensified by improved levels of service provided by a rapid transit system such as the one which is proposed. Two design options are considered: A fixed design in which the light rail line would be constructed immediately and a flexible design, in which a bus rapid transit (BRT) system, designed to be easily upgradable to a light rail line, would first be constructed. After the end of the lifespan of the first set of buses (12 years), the decision could be made to either maintain the BRT system or to upgrade it to a light rail. The BRT system involves lower initial capital expenditure, but has higher operating costs and lower capacity than the light rail system.

This analysis explores the differences between the fixed and the flexible design in view of the top uncertainty, ridership. Two methods are used to represent the future development of ridership: A decision tree based on the city planning staff's predictions and a lattice analysis. The analysis is carried out from the perspective of the city council and the transit agency, i.e. it involves all operating costs but only 20% of the capital costs since the remaining 80% are taken over by the Federal Transit Administration under its "New Starts" program.

The results show that the flexible design results not only in a higher Expected Net Present Value (ENPV), but also helps limit potential downside risks if ridership does not grow as anticipated. However, it is found that the final decision should not only be based on the financial aspects, since the flexible approach delays the investments to a time when levels of political support and Federal funding available for the project are not known. In this respect, the fixed approach fares better.

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# 1 Introduction to the system and basic characteristics

Our mission is to recommend an investment strategy for an urban transit corridor. We are dealing with a long corridor which is part of a larger transit network. The corridor is approximately 12km (7.5 mi) long and leads from a city's suburbs to the central business district. The city and the transit authority are planning an upgrade of the corridor to a high-frequency rapid transit service, and there are currently two design options under consideration:

1. Build a bus rapid transit corridor which is designed to be easily upgradable to a light rail line, operating on a separate right of way. Operate it for the lifespan of the buses and then re-evaluate the situation with new demand (and revenue) figures. Either the bus rapid transit corridor can be kept in operation or it can be converted to a light rail line.
2. Build the light rail line immediately.

These options will be described in more detail in part 3.

Given that major transit investment projects tend to be very unique in terms of their costs and characteristics, most of this case is hypothetical, using realistic assumptions. Many characteristics of the system, such as ridership figures and cost estimates, are based on the Miami streetcar project, which is currently in its final planning phase. However, our hypothetical case simplifies the situation to make it tractable for our purposes.

The corridor currently has a demand of about 6000 passenger trips per day, 50% of which are commuting trips in the AM and PM peak hours (0700-1000 and 1600-1900). There is a regular bus service operating in this corridor, but due to automobile congestion it often experiences severe delays. Therefore, the city council would like to upgrade it to a rapid transit corridor and is suggesting the construction of a light rail line as it is expecting a large increase in demand within the next 20 years. Since peak hour demand drives capacity, the system design will need to be able to accommodate the expected levels of ridership during peak hours. For simplicity, we will assume that the demand has the following pattern:

- AM peak ridership is 25% of total daily ridership on the inbound route, evenly spread out over the 3 peak hours. For simplicity, we assume that all jobs are located in the central business district, i.e. the peak load point is reached right before the vehicles enter downtown.
- PM peak ridership is again 25% of total daily ridership on the inbound route, evenly spread out over the 3 peak hours. The same assumption holds as above.
- The remaining 50% of daily trips are during off-peak hours and peak hours in the non-peak direction.

For analyzing this system, we use two models<sup>1</sup>:

- A model for determining the vehicle requirements as a function of demand and vehicle characteristics. With a set of basic assumptions regarding running times and turnaround times, this can be accomplished fairly easily.
- A cost model, taking into account both fixed and variable costs. The latter are caused by the direct operation of the vehicles and by its maintenance.

The investment costs for vehicle procurement and the variable costs of vehicle operation have been obtained from statistics compiled by the American Public Transportation Association

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<sup>1</sup> This report will not focus further on the models and only present their results. Both models are based on material from lecture 1.258J, Public Transportation Systems, taught at MIT in the spring semester of 2008.

(APTA). The estimates for the construction costs are based on the Miami Streetcar project, assuming that the costs for traffic control & construction mobilization, roadway construction, station construction and utility relocation are shared by the bus and streetcar alternative. The only difference is the construction of tracks and catenary, which is a major cost factor. Additionally, if the busway is constructed first, then the stations can be built to a lower standard and upgraded when the light rail line is put in place. In addition, if the construction of the light rail line is delayed to a later point, then the mobilization and traffic control costs will need to be paid again.

All analyses will be based on a real discount rate of 5%. This assumes that costs and revenues adjust to inflation.

## **2 Uncertainties**

### **2.1 Description of uncertainties**

First, we need to look at the uncertainties related to this project. Except for the top uncertainty, ridership, these will not be considered further in the analysis, but in a real case, they would have an important influence on the design of the project. As a matter of fact, ridership is connected with several of the underlying “secondary” uncertainties, as it is driven by fuel price, policy, automobile ownership, etc.

- Customer demand (top uncertainty): While ridership forecasts can give a bandwidth within which ridership is expected to be in the future, precise predictions cannot be made. To a certain extent the development of ridership depends on economic activity in the city and needs to be considered exogenous. On the other hand, the city council has influence over ridership through parking policies and restrictions on automobile access to the central business district.
- Cost of energy: The variation of fuel and electricity prices has an impact on the operating costs of rapid transit buses and light rail vehicles. More importantly, it drives the operating cost of public transit’s main “competitor”, the automobile, and is thus connected to ridership.
- Direct policy implications: It is unclear to what degree future political leaders of the city will back the public transit system and, more importantly, what federal funding schemes will be available in the future. This is particularly important in the two-stage approach which defers a large capital investment to a point in time where the political and economic circumstances are unknown. While there would be federal support available now for the construction of the light rail line, this may not be the case anymore in the future.
- Indirect policy implications: Environmental and land-use legislation may have an influence on automobile use in the future, which in return has an impact on ridership.
- Land use development: Land use patterns can develop differently than foreseen. The alignment of a major infrastructure project like a rapid transit line is very difficult to alter once it is constructed, even if it does not respond to prevailing travel patterns anymore.
- Automobile ownership: This is dependent on many factors, including demographical shifts, the costs of ownership, the quality of the public transportation system (!), residential density and land use patterns etc.
- Public transportation technologies: Delaying the construction of the streetcar system may let us profit from possible cost savings offered by better technology at the point of construction.
- Industry structure: The majority of public transportation systems in the United States is owned and operated by public-sector agencies. This might change in the future if the

market is liberalized and more private-sector operators enter it. This means that in the future upgrade project, the city might face a completely different decision environment.

## 2.2 Analysis of the top uncertainty

Ridership is the key characteristic of our rapid transit line's performance. In its case studies, the city planning staff used a mode choice model and a large-scale regional model to predict ridership. Based on the model predictions, we will assume that in the first 12 years, the ridership can be an average of 6000, 9000 or 12000 per day. The planning staff thinks that there is significant potential for development in the area, and that the chance of ridership being 12000 by year 12 is 40%. The other two outcomes have a 30% likelihood associated with them. In the second term (year 12-24), ridership is predicted to increase, such that it will be between an average of 10000 and 18000, with a most likely outcome of 15000 passengers per day.

- If ridership is 12k in year 12, then there is a 50% chance it will be 18k in year 24, a 30% chance it will be 14k and a 20% chance that it will drop to 10k.
- If ridership is 9k in year 12, the probabilities are: 18k – 30%, 14k – 40% and 10k – 30%.
- If ridership is 6k in year 12, the probabilities are: 18k – 20%, 14k – 30% and 10k – 50%.

The distribution of trips throughout the day is assumed to remain as described above.

With respect to system capacity, a bus system would be able to handle a demand of about 16000 per day, although this is close to such a system's technical capacity limit. The "comfort zone" for passengers is limited at about 100 passengers in an articulated bus – in our ridership model, that would be approximately 14000 trips per day.

We will assume that the development of ridership is exogenous, i.e. that it does not depend on the choice of transit technology. This assumption helps us maintain path independence and simplifies the analysis. However, under "real" circumstances, this assumption would not hold, as fixed-guideway systems (like light rail lines) generally attract more ridership than traditional bus services.

## 2.3 System performance metrics

The primary objective of the city is not to make money, but to provide a transportation service to the community, the benefits of which are generally external to the system (reduced congestion, environmental benefits, higher-density development). This system requires very high capital investments, and the city and the Federal Transit Administration do not expect to re-coup all those costs, so it is highly likely that the total NPV will be negative in any case.

Since we have assumed that ridership is independent from the choice of technology, the objective is to provide an adequate transportation service for the given demand in the most cost-effective way. Revenue is a function of ridership and goes both towards financing the day-to-day operational costs and towards paying off debt from construction and vehicle purchase; for the "bottom line" NPV calculation, we don't distinguish between fixed and variable costs.

## 3 Details on the design options

Some major system parameters have already been identified in the planning process, namely the route alignment and the fact that it is to be a rapid transit corridor (i.e. high-frequency service) with a dedicated right of way. The city has also stated its preference for a light rail system, but is open to other design suggestions. This represents the **inflexible base design**: If it were to

immediately construct the complete light rail line, the city would need to pay for construction mobilization and traffic control costs only once and would benefit from lower operating costs from the beginning away. However, a light rail line is a high-capacity system, and ridership levels – especially during the first years of operation – may not justify the high investments required.

As an alternative, we will examine a **flexible design option**, where the corridor is first designed as a busway, but is constructed for light rail standards, so that a possible upgrade only involves minimal additional investments, i.e. laying tracks, installing the overhead power system and upgrading stations. We define one decision point along the way, after the lifespan of the first fleet of buses has passed (year 12), where we make a decision as to whether we upgrade the busway to a light rail line or keep the bus line in place. This decision will be based on the ridership levels observed in year 12 and the possible future development of ridership.

Of course there would be a number of other flexibilities in the system, but this one was chosen because it ties together capacity (the lower bound for which is a function of the major uncertainty, ridership) with capital investments, and it is a more efficient way of responding to increases in ridership than to build large (“over-designed”) infrastructure and then vary the number of vehicles. While the latter approach has often been practiced in the past, the flexible design suggested here is a more innovative approach which some major transit agencies have been experimenting with in recent years (such as RATP in Paris).

The fixed decision point at year 12 was partly introduced in order to avoid complications: The buses have a pre-determined life span, and if they were not to be used in this particular corridor anymore before the 12 years are over, it would be no problem for a large transit agency to absorb them into other parts of its operations. However, this would hide some of the costs of the flexible solution, since the transit agency would still need to shoulder the costs of the buses in the first place, but the operating costs would occur somewhere else in the network. Selling the buses was not considered as an option either because that would make the decision dependent on another factor which is typically highly uncertain – the salvage value of the buses.

The following table gives details on the construction costs. The numbers are based on the Miami streetcar project and have been adjusted to fit the hypothetical case under consideration.

Item	Cost estimate (US\$)	New busway	New light rail line	Light rail upgrade
Traffic control and mobilization	8650000	Yes	Yes	Yes
Segregated right of way	22670000	Yes	Yes	No
Stations	4885000	Yes: Half	Yes	Yes: Half
Utilities (does not include overhead power system)	11273000	Yes	Yes	No
Tracks	27195000	No	Yes	Yes
Overhead power system	20400000	No	Yes	Yes

**Vehicle requirements:** We assume an average operating speed (including layovers) of 15 mph for buses and 18 mph for light rail vehicles. The fleet size is driven by peak hour vehicle

requirements. Using the above assumptions for operating speed, it would be 12 buses or 9 light rail vehicles. The costs are assumed to be the following<sup>2</sup>: \$812000 for a bus and \$3950000 for a light rail vehicle, with a lifespan of 12 years for a bus and 35 years for a light rail vehicle. These costs have to be added to the initial capital investments. However, since our time horizon is not the full lifespan of a light rail vehicle, we have to scale down the price for light rail vehicles to exclude the portion of their lifespan not covered by the planning horizon. This can be thought of (in a simplified way) as their remaining value at the end of the 24 years. For example, for a vehicle bought at year 12, we consider only 1/3 of its original purchase price.

Under the Federal Transit Administration's "New Starts" program, the US government will take over 80% of the capital investment costs (including vehicles). In its investment decision, the city only considers its own cost. It has to pay for 100% of the operating costs.

The bus system in its planned layout is not able to carry more than 14000 passengers per day, and any demand above that will result in high crowding and passenger complaints. Thus, the basic decision rule is that for any level of ridership above 14k, we should have a light rail system in place. If we do not construct the light rail and ridership goes beyond 14k, we will need to rent 3 additional buses, at a cost of \$100000 per year and vehicle and without federal assistance

**Cost of flexibility:** Given the project assumptions, the only additional costs which are incurred if the flexible design is chosen are the additional construction costs in year 12 if the system is upgraded. We will only have to pay these costs, which amount to 8.65 Million Dollars, if we actually exercise the option. Since they are incurred in year 12, they are discounted, which makes the value of incorporating flexibility into the system dependent on the discount rate. Note that the 8.65 Million\$ are costs in addition to the actual construction costs of the upgrade, which are 50 Million\$.

The reason why there are no upfront costs is because our base case is defined as the complete light rail system, and the flexible case includes only a partial construction of that system. If the base case were a bus system, then the flexibility to construct a light rail would add additional upfront construction costs to the project.

## 4 Valuation of alternatives

The following section presents various methodologies of assessing the value of the two alternatives, which would ultimately lead to a decision on which of the two alternatives to choose.

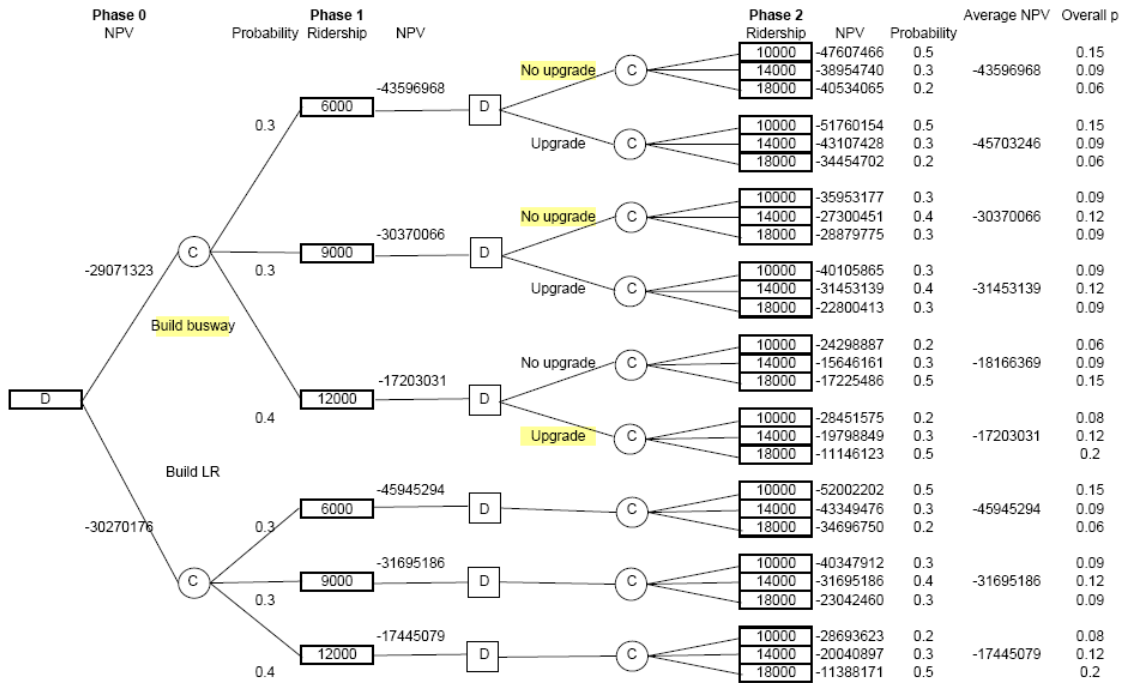
### 4.1 Decision Tree

The following decision tree shows the NPVs and optimal decisions (marked in yellow) based on the assumptions of ridership development described in part 2:

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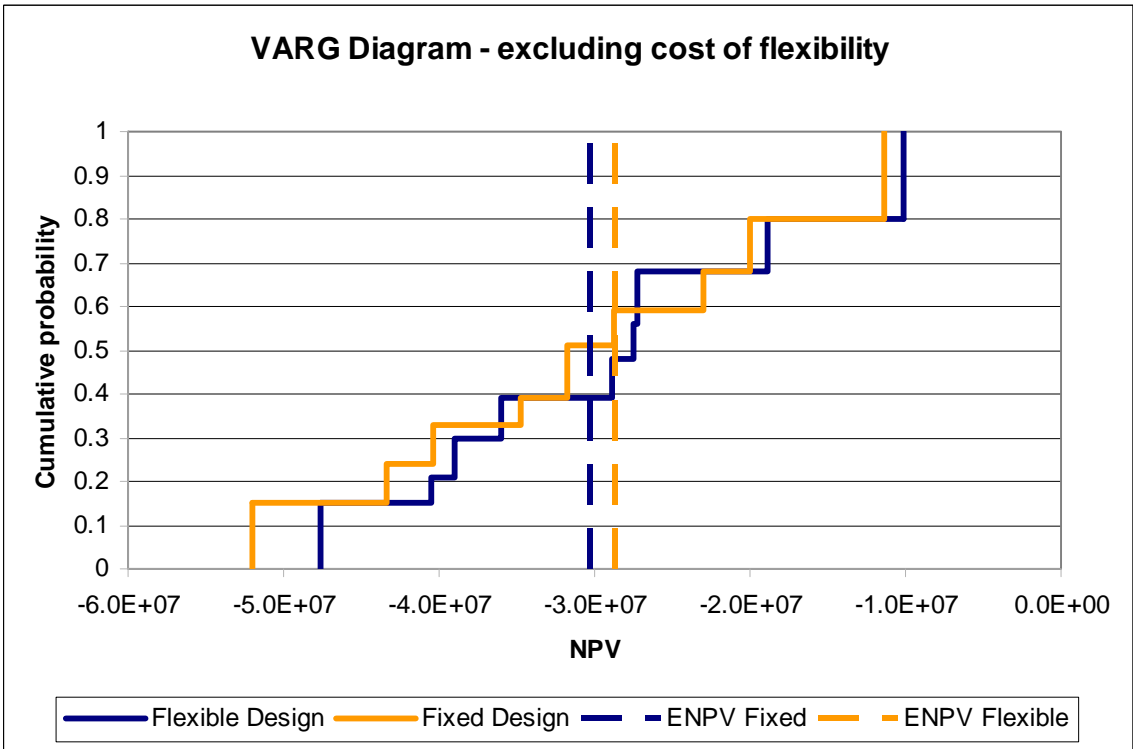
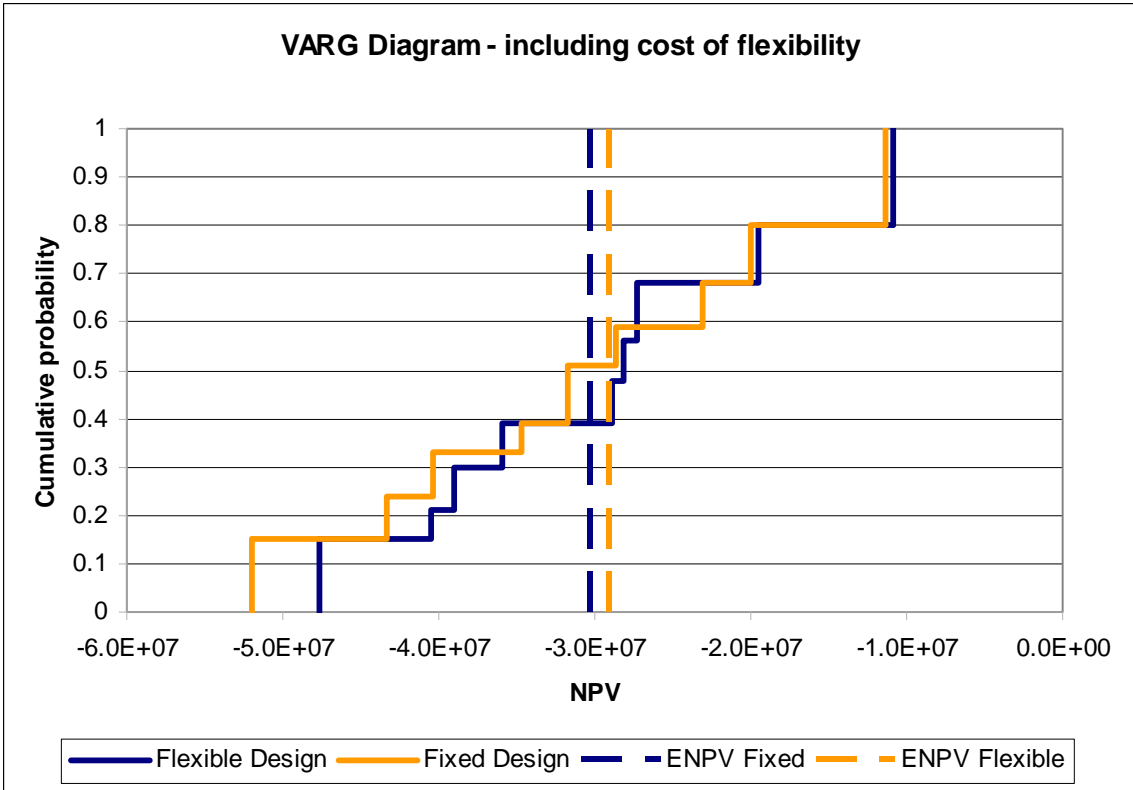
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We can see that the higher average NPV can be achieved by opting for the flexible design. We also see how the flexibility can be exercised: If ridership remains below the expected level in the first phase, then it is more advisable not to upgrade to the light rail, no matter what levels of ridership are to be expected in the second phase.

The following diagrams show the VARG curves for two different cases. One is the VARG diagram including the cost of flexibility (as described above), the other one excludes it. We can see the difference between the VARG curve including the additional costs and the one excluding it; this is the cost of the flexibility.



The VARG diagrams show the advantage of the flexible approach: Its VARG curve is shifted to the right with respect to the fixed design, thus limiting downside risks and potentially adding

upside opportunities. In this context, in which we are not actually trying to make money but to provide a service in the most cost-effective way, the limitation of risks is more important than the potential upside gain in NPV. We can see that only on two occasions the fixed design is better than the flexible design (blue curve above orange curve); these are the cases in which we exercise our flexibility and upgrade to the light rail in year 12, and then ridership drops.

## 4.2 Key figures based on decision tree

Next, we will look at a few single measures characterizing the two options:

	Flexible Design	Fixed Design	Which is preferable?
Max NPV	-11146123	-11388171	Flexible
Min NPV	-47607466	-52002202	Flexible
ENPV	-29071323	-30270176	Flexible
Initial CapEx	11444400	23754600	Flexible
ENPV/CapEx	-2.540222536	-1.274286897	Flexible

This table confirms the findings from above. Initially, the B/C ratio and the IRR were also intended to be included in the analysis, but they were dropped because the B/C ratio changes with each of the chance outcomes, and the IRR cannot be computed because the average NPV of the projects is negative. However, an interesting metric is the break-even point, i.e. the discount rate at which the two projects have the same ENPV. It was calculated to be 4.1%.

## 4.3 Lattice Analysis

For the lattice analysis, we have to slightly alter the assumptions on ridership development, since the assumptions stated above and used in the decision tree contain a discontinuity at year 12 and are not path-independent. Hence we will develop the lattice based only on the projected ridership in year 24. Since a lattice with one-year time increments is simply too large to be presented in this report, a 4-year lattice was constructed, which is divided into 6 four-year periods from year 0 to year 24. A one-year lattice with 24 periods was also constructed, for validation purposes only, and is not shown here as it was not used in the analysis.

The ridership projections based on the city's development plans state that "the" most likely ridership will be 15000 by year 24. There is no available data on yearly volatility of ridership numbers, but the forecasters have given an upper and lower bound for their projection, so the numbers can be summed up as follows:

Current ridership of the bus system before construction of the light rail: 6000 pax/day  
 Projected average ridership by year 24: 15000 pax/day  
 Projected maximum ridership by year 24: 18000 pax/day  
 Projected minimum ridership by year 24: 10000 pax/day

Thus we have growth in all cases. To find  $u$  and  $d$ , we make the following assumption: If we have a "u" event every period, we will have 18000 riders by year 24, if we have a "d" event every period, we will have 10000 riders per day. So with 6 periods, we can determine  $u$  and  $d$  by simply calculating the 6th root of the total growth in these two extreme cases. Then we can use the Excel solver to "optimize"  $p$  such that the average of all outcomes is 15000:

$S = 6000$  pax/day (year 0)  
 $MAX = 18000$  pax/day (year 24/period 6)  
 $MIN = 10000$  pax/day (year 24/period 6)  
 $AVG = 15000$  pax/day (year 24/period 6)

$$u = \sqrt[6]{\frac{MAX}{S}} \approx 1.201, \quad d = \sqrt[6]{\frac{MIN}{S}} \approx 1.089. \quad \text{The average growth rate is: } v = \sqrt[6]{\frac{AVG}{S}} \approx 1.152$$

Both  $u$  and  $d$  are greater than 1 because we expect an increase of ridership in any case. The result for  $p$  found with the Excel solver is 0.68, making  $(1-p) = 0.32$ . The resulting lattices are shown below.

### OUTCOME LATTICE

Year	0	4	8	12	16	20	24
	6000.00	7206.00	8654.41	10393.94	12483.12	14992.23	18005.67
		6534.00	7847.33	9424.65	11319.00	13594.12	16326.54
			7115.53	8545.75	10263.44	12326.39	14804.00
				7748.81	9306.32	11176.89	13423.44
					8438.45	10134.58	12171.63
						9189.47	11036.56
							10007.34

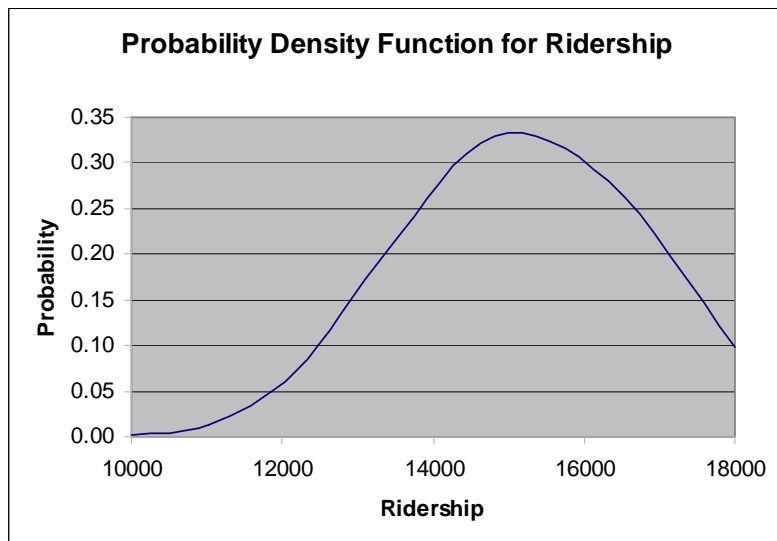
### PROBABILITY LATTICE

Year	0	4	8	12	16	20	24
	1.00	0.68	0.46	0.31	0.21	0.15	0.10
		0.32	0.44	0.44	0.40	0.34	0.28
			0.10	0.21	0.28	0.32	0.33
				0.03	0.09	0.15	0.21
					0.01	0.04	0.07
						0.00	0.01
							0.00
<b>Sum of p</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>	<b>1.00</b>

The following table shows the probabilities associated with the ridership by year 24, and the average projected ridership for year 24 (the basis for the probability calculation). The 4-year steps make the outcome a little more coarse than with 1-year steps, but they are a very good approximation.

Ridership in yr 24	p
18006	0.10
16327	0.28
14804	0.33
13423	0.21
12172	0.07
11037	0.01
10007	0.00

**Average 15012.90**



Furthermore, we can calculate the average ridership for the six periods by multiplying the probability lattice with the ridership lattice:

Year	Daily Ridership						
	0	4	8	12	16	20	24
0	4900	4002	3268	2669	2180	1780	
	2091	3415	4184	4556	4651	4558	
		729	1785	2916	3969	4862	
			254	829	1694	2766	
				88	361	885	
					31	151	
						11	
<b>Average per period</b>	<b>6991</b>	<b>8146</b>	<b>9491</b>	<b>11058</b>	<b>12885</b>	<b>15013</b>	

Based on the above ridership figures and probabilities, we can calculate the revenues arising from that ridership and cash flows for various project configurations. These are presented in the lattices below. Since we are working with four-year steps, the figures are discounted to the value of money in the next lower step in order to facilitate the dynamic programming approach presented below. The discounting was done in the following way (example for year 16 – discounted to the value of money in year 12):

$$CF_{16} = \frac{Rv_{16} - C_{16}}{(1 + DR)^4} + \frac{Rv_{16} - C_{16}}{(1 + DR)^3} + \frac{Rv_{16} - C_{16}}{(1 + DR)^2} + \frac{Rv_{16} - C_{16}}{(1 + DR)^1}$$

- CF - Cash Flow
- Rv - Revenue
- C - Costs
- DR - Discount Rate

For the flexible design case, two lattices are constructed, which have the same cash flow numbers from year 0 to year 8. After that, one shows the cash flows if the upgrade is performed, and the other one shows the cash flows without an upgrade. It therefore accounts for the additional costs incurred if ridership on the bus system exceeds 14k passengers and we have to rent additional vehicles. The year 12 column includes the investments into the upgrade (if applicable) or into a new fleet of buses. The following years have different operating costs. For the upgraded case, the cash flows in years 16 to 24 are the same as for the fixed case.

**Revenue Lattice in USD (in four-year increments, discounted to the next lower step)**

Year	0	4	8	12	16	20	24
	0.00	1.12E+07	1.35E+07	1.62E+07	1.94E+07	2.33E+07	2.80E+07
		1.02E+07	1.22E+07	1.46E+07	1.76E+07	2.11E+07	2.54E+07
			1.11E+07	1.33E+07	1.60E+07	1.92E+07	2.30E+07
				1.20E+07	1.45E+07	1.74E+07	2.09E+07
					1.31E+07	1.58E+07	1.89E+07
						1.43E+07	1.72E+07
							1.56E+07

**Cash Flow Lattice - Fixed version**

Year	0	4	8	12	16	20	24
	-2.38E+07	-7.61E+06	-5.36E+06	-2.65E+06	5.93E+05	4.49E+06	9.18E+06
		-8.65E+06	-6.61E+06	-4.16E+06	-1.22E+06	2.32E+06	6.57E+06
			-7.75E+06	-5.53E+06	-2.86E+06	3.50E+05	4.20E+06
				-6.76E+06	-4.34E+06	-1.44E+06	2.05E+06
					-5.69E+06	-3.06E+06	1.09E+05
						-4.53E+06	-1.65E+06
							-3.25E+06

**Cash Flow - Flexible version with no upgrade**

Year	0	4	8	12	16	20	24
	-1.10E+07	-9.49E+06	-7.24E+06	-6.14E+06	-1.29E+06	-4.74E+06	-5.57E+04
		-1.05E+07	-8.49E+06	-7.64E+06	-3.10E+06	4.39E+05	-2.67E+06
			-9.63E+06	-9.01E+06	-4.74E+06	-1.53E+06	-5.03E+06
				-1.02E+07	-6.22E+06	-3.32E+06	1.74E+05
					-7.57E+06	-4.94E+06	-1.77E+06
						-6.41E+06	-3.54E+06
							-5.14E+06

**Cash Flow - Flexible version with upgrade**

Year	0	4	8	12	16	20	24
	-1.10E+07	-9.49E+06	-7.24E+06	-1.61E+07	5.93E+05	4.49E+06	9.18E+06
		-1.05E+07	-8.49E+06	-1.76E+07	-1.22E+06	2.32E+06	6.57E+06
			-9.63E+06	-1.90E+07	-2.86E+06	3.50E+05	4.20E+06
				-2.03E+07	-4.34E+06	-1.44E+06	2.05E+06
					-5.69E+06	-3.06E+06	1.09E+05
						-4.53E+06	-1.65E+06
							-3.25E+06

#### 4.4 Lattice Valuation

We can now analyze the above lattices with a dynamic programming approach. The final columns are the same as in the cash flow matrices above, i.e. the ENPV in year 24 for the fixed version is equal to the cash flow in year 24 for the fixed version, etc.

From there on backwards, the columns include the cash flows for the period under consideration plus the discounted cash flows from the following period, multiplied with the probability of attaining those states:

$$ENPV_{20} = CF_{20} + \frac{(p_{up} \cdot CF_{24,up} + p_{down} \cdot CF_{24,down})}{(1 + DR)^4}$$

CF - Cash Flow  
DR - Discount Rate

By construction of the case study, we can only perform the upgrade of the flexible system in year 12. Again, two separate lattices are constructed for the flexible case, one with and one without the upgrade.

**NPV - Fixed version (Dynamic programming procedure)**

Year	0	4	8	12	16	20	24
	-3.81E+07	-1.26E+07	-4.19E+06	3.23E+06	8.82E+06	1.14E+07	9.18E+06
		-1.82E+07	-9.97E+06	-2.43E+06	3.61E+06	7.10E+06	6.57E+06
			-1.52E+07	-7.57E+06	-1.11E+06	3.24E+06	4.20E+06
				-1.22E+07	-5.39E+06	-2.58E+05	2.05E+06
					-9.27E+06	-3.43E+06	1.09E+05
						-6.31E+06	-1.65E+06
							-3.25E+06

**NPV - Flexible version with no upgrade (Dynamic programming procedure)**

Year	0	4	8	12	16	20	24
	-3.62E+07	-2.39E+07	-1.64E+07	-1.04E+07	-4.97E+06	-5.47E+06	-5.57E+04
		-2.80E+07	-2.00E+07	<b>-1.29E+07</b>	-5.56E+06	-2.38E+06	-2.67E+06
			-2.38E+07	<b>-1.62E+07</b>	-8.11E+06	-4.30E+06	-5.03E+06
				<b>-1.96E+07</b>	-1.01E+07	-3.69E+06	1.74E+05
					-1.40E+07	-6.86E+06	-1.77E+06
						-9.74E+06	-3.54E+06
							-5.14E+06

**NPV - Flexible version with upgrade (Dynamic programming procedure)**

Year	0	4	8	12	16	20	24
	-3.79E+07	-2.51E+07	-1.72E+07	<b>-1.03E+07</b>	8.82E+06	1.14E+07	9.18E+06
		-3.08E+07	-2.29E+07	-1.59E+07	3.61E+06	7.10E+06	6.57E+06
			-2.82E+07	-2.11E+07	-1.11E+06	3.24E+06	4.20E+06
				-2.57E+07	-5.39E+06	-2.58E+05	2.05E+06
					-9.27E+06	-3.43E+06	1.09E+05
						-6.31E+06	-1.65E+06
							-3.25E+06

If we look at the two lattices for the flexible case, we can compare the NPV figures for year 12; if the “no upgrade” lattice has a higher NPV for a particular ridership outcome, then it is more advisable not to upgrade. If the version “with upgrade” has a higher NPV, we should upgrade the system. The better decision is marked in bold type and framed.

**Numerical example:**

If, in year 12, we have an average ridership of approx. 9400 passengers (according to the outcome lattice), then we can achieve an NPV of -12.9 million \$ until year 24 if we don't construct the light rail. If we do construct it, we can only achieve an NPV of -15.9 million \$. We therefore choose not to upgrade to the light rail system.

Using the above information, we can determine the decision rule for the flexible option:

**NPV - Flexible version using decision rule (Dynamic programming procedure)**

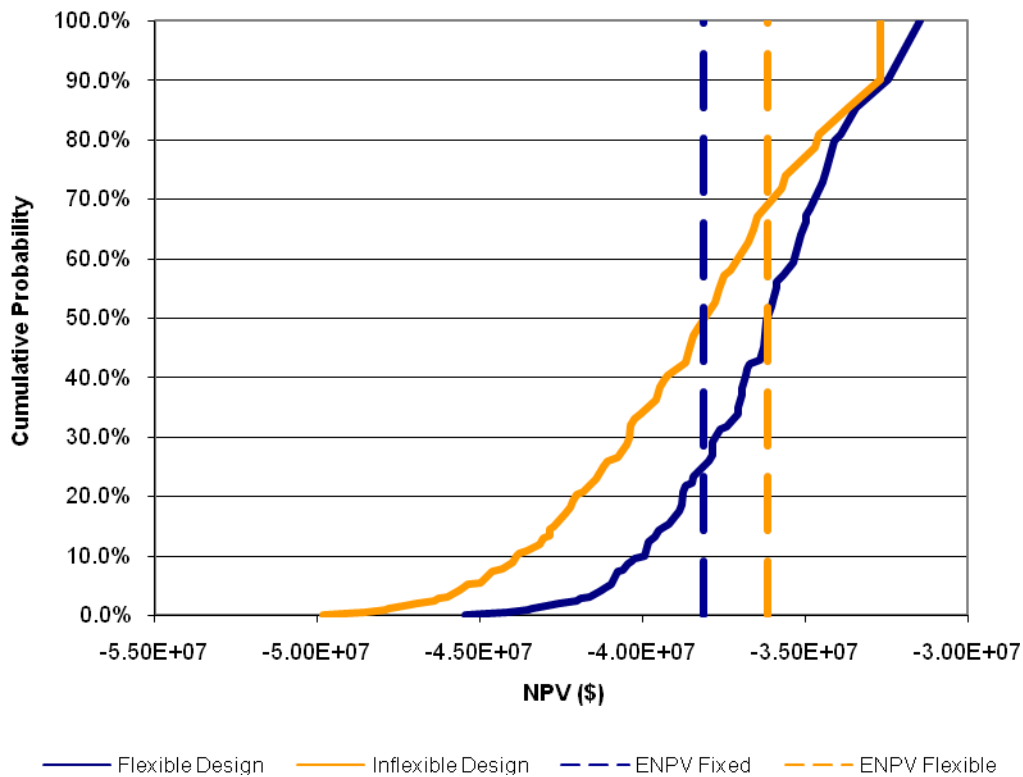
Year	0	4	8	12	Upgrade?	16	20	24
	-3.62E+07	-2.39E+07	-1.64E+07	-1.03E+07	YES	...	...	...
		-2.80E+07	-2.00E+07	-1.29E+07	NO	...	...	...
			-2.38E+07	-1.62E+07	NO	...	...	...
				-1.96E+07	NO	...	...	...
						...	...	...
							...	...
								...

**4.5 VARG Curve based on lattice valuation**

The following figure shows the VARG diagram which was calculated using the cash-flow lattices for the fixed and flexible version. It includes the cost of flexibility and incorporates the decision rule found above: For all possible paths where ridership is approximately 10300 passengers per day in year 12, the NPV is calculated from the cash flow matrix of the upgraded system, and for all other paths it is calculated from the cash flow matrix of the non-upgraded system.



### VARG curves derived from lattice calculation



The above VARG curve confirms the findings from the VARG curves constructed from the decision tree: The flexible design limits downside risks and opens opportunities for upside gains which could not be realized with the inflexible design. As a matter of fact, the difference between the two curves is even more striking here: On no occasion do the two lines intersect, the flexible design is persistently better than the fixed design. Again, since the objective is mostly to provide a service in the most cost-effective way, the limitation of downside risks is very important.

#### 4.6 Key figures based on lattice analysis

The following table reports the same key figures as the table in section 4.2, with the difference that this one is based on the lattice analysis.

	Flexible Design	Fixed Design	Which is preferable?
Max NPV	-31476843	-31718891	Flexible
Min NPV	-45435745	-49830480	Flexible
ENPV	-36165769	-38140085	Flexible
Initial CapEx	11444400	23754600	Flexible
ENPV/CapEx	-3.160128	-1.605587	Flexible

These figures again unambiguously speak for the flexible approach. They are very similar to those presented in section 4.2, which were derived from the decision tree, with the exception of the maximum NPV values. Those are significantly lower here than in the decision tree analysis. A look at how the NPV in the decision tree was calculated explains this: In the decision tree, there is

the assumption that there are only two levels of ridership – one before year 12 and one afterwards. Thus, for the most positive scenarios, the decision tree projects a ridership level of 18000 passengers per day as of year 12. This is reflected in the cash flows (revenue) and results in a significantly higher NPV than in the lattice analysis, where ridership develops incrementally and in the best case reaches 18000 by year 24. This assumption is more realistic than that of the decision tree, and we can conclude that the maximum NPV numbers of the lattice analysis should be used rather than those from the decision tree analysis.

## **5 Conclusions**

### **5.1 Advantages and disadvantages of the flexible design**

The results in section 4 of this assignment give a clear indication that, from a financial point of view, the flexible approach should be chosen. This is validated by the fact that two different ways of modeling the major uncertainty, ridership, lead to the same conclusions.

The flexible approach offers savings through deferring large parts of the capital investment to a later point in time, where knowledge of the way the system operates will be greater and the investments only have to be made if there is a need for them. This is particularly important in the case of a transit corridor, since each project is unique and there is often very limited experience from other projects to learn from. Although large efforts are usually put into ridership projections (one of the key design inputs), they are difficult to verify, and flaws in the assumptions often do not become apparent until the system has been operating for several years. An interesting point is that the two VARG diagrams presented in section 4.1, which use the original probabilities for development of ridership identified by city planning staff, show that the cost of the flexibility is not even a major factor – the shift it introduces into the VARG curve is minimal.

Another point which must be borne in mind is that, given the intense competition between projects seeking Federal funding, proponents of projects may deliberately overstate expected ridership in order to obtain funding. Choosing a flexible approach has advantages for both sides, the project planners and the funding agency. The initial capital expenditure for the funding agency (especially of the Federal side, since they have to carry the 80% of capital costs which were not considered in this analysis) are much lower, and the flexible approach buys time for evaluating the actual performance of the project. The project proponents, on the other hand, have the possibility of providing a large portion of the planned transportation capacity at a fraction of the costs of the full project, thus improving its ENPV/CapEx ratio and its relative standing with respect to competing projects. At the same time, the possibility for constructing a full-fledged light rail system is not lost. Especially in the hypothetical case which was considered in this analysis, where we started with fairly low ridership numbers and predicted large growth rates, the large capacity provided by the fixed approach would not be needed until several years into its operational life.

On the other hand, the fixed approach has a large advantage with respect to the flexible one: Aside from the fact that constructing the entire light rail system at once causes overall smaller capital investments (this can be thought of as economies of scale), it is unclear how the regulatory or policy environment may change in the future, and the funds which are available for construction today (especially Federal grants) may no longer be available at the point in time when the busway would need to be upgraded to a light rail. It is also unclear what level of political support public transportation projects may enjoy from future city councils.

### **5.2 Recommendation for design choice**

Given the above results, the recommendation is to opt for the flexible design option, since it involves less capital expenditure up front and allows an evaluation of the system's performance and of the validity of ridership forecasts; furthermore, the price of this flexibility is very small in

comparison to overall project costs and potential downside risks can be avoided, as was shown in the VARG curves.

However, from the point of view of the city and transit authority, close consideration should be given to the question whether the downsides of the fixed approach are outweighed by the fact that it avoids uncertainties about funding structure and political support for the project in 12 years, when the upgrade would need to be performed. This is especially the case if the two options are only compared in terms of ENPV (using both a decision tree and a lattice analysis), since that difference is not very large relative to the overall project costs.

### **5.3 Lessons learned**

The most important lesson from the project analysis performed above is that predictions and forecasts for project performance can – and should – be scrutinized; in many projects such as this one, there is more flexibility than catches the eye. We have only considered one particular type of flexibility, assuming many design parameters to be given, but in reality there would be many more possibilities: For instance, route alignment could have been considered or the constraint that the system can only be upgraded after 12 years could have been relaxed. However, just as a real project would have a multitude of flexible parameters, there would also be much more uncertainty in it than we assumed in this analysis and some of the decisions to be taken are more political than technical.

The process of analyzing this problem from different sides has shown that the decision tree and lattice methods are adequate for modeling an exogenous uncertainty such as ridership. However, the results are sensitive to the assumptions on how the uncertainty will develop; in this case, we based ourselves on the forecasts made by the city planners, which in turn were derived from models. It is important to recognize that there are also assumptions, simplifications and uncertainties which go into those models, and which propagate into the analyses conducted here. A limitation which was encountered was that the lattice model is unable to represent discontinuities and shifts in probabilities and average growth rate of the uncertain parameter during the life of the project. This, however, is fairly likely in a public transportation project with a long lifespan. On the other hand, the lattice has the advantage of being able to model passenger growth more realistically than the decision tree. This demonstrates the advantage of using both evaluation tools together.

Aside from this, the economic evaluation of projects, in which the ENPV and other key figures are compared to other projects, is also highly dependent on design assumptions. Examples of such parameters are the discount rate, the chosen fare policy or the assumptions on construction costs (in the transportation sector, time and budget overruns on such large projects are quite common). Therefore, even if the above tools are used correctly to evaluate a project, it is possible (and quite easy) to manipulate assumptions in order to achieve a desired outcome.