Intelligent Parking System's Deployment Strategy Evaluation

Abstract

A design team has planned to create a startup company that implements a new citywide intelligent parking system in the Greater Boston area within a 5-year timeframe from year 2008 to 2012. In order to ensure the system's profitability, the startup needs to employ a system deployment strategy that optimizes expected net present value (NPV) of the business project. Two system deployment strategies are identified: fixed and flexible deployment strategies. In order to determine the optimal strategy, two "real-option analyses on the system are conducted: decision tree analysis and lattice analysis. The decision tree analysis predicts the optimal strategy to be the flexible deployment strategy, whereas the lattice analysis predicts the fixed deployment strategy. An extra sensitivity study is conducted to show that the lattice analysis predictions are highly sensitive to the expected values of market uncertainties but relatively insensitive to market volatility.

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Nomenclature

Parameter	Description	Unit/Value
ΔT	Time Increment	1 year
σ	Normalized Standard Deviation	3.70%
a_i	Initial Vehicle Registrations Forecast	1.74%
C1	1st Chance Node of Decision Tree	---
C2	2nd Chance Node of Decision Tree	---
d	Binomial Model Downside Factor	0.964
D1	1st Decision Node of Decision Tree	---
D2	2nd Decision Node of Decision Tree	---
EV	Expected Value	$
HD	High Demand Level	---
HF	High Funding Level	---
i	Period Starting in 2008	Year 'i'
i2	Period Starting in 1987	Year 'i'
I	Inflation Rate	3.4%
LD	Low Demand Level	---
LF	Low Funding Level	---
MD	Medium Demand Level	---
MF	Medium Funding Level	---
NPV	Net Present Value	$
p	Binomial Upside Probability	0.735
PDF	Probability Distribution Function	---
PV	Present Value	$
T	Corporate Tax Rate	34.0%
T_{sensor}	Maximum Number of Sensors to Be Implemented in Greater Boston	60,000
u	Binomial Model Upside Factor	1.038
v	Average Annual Percentage V_{reg} Increase	1.74%
V_{reg}	Number of Vehicles Registered in the City	---
x	Number of Sensors Implemented in Greater Boston in a Any Year	---
Year 0	January 1, 2008	---
Year 'z'	December 31 of the Calendar Year "2008+z" (when z is not 0)	---

1.0 Introduction

1.1 Background

There is often value associated with incorporating flexibility in system design. By being flexible, a system can adapt to market conditions more readily to optimize overall system value. However, there is often a price for adding flexibility to a system. This paper studies the value and cost of flexibility of a conceptual parking system using two different analysis tools: decision tree analysis and lattice analysis.

During the past summer, a multi-discipline team was assembled to form a startup company that solves the on-street parking problems in major cities. The idea was inspired by certain team members' constant struggle to find street parking after work in the Boston and Cambridge areas. The solution is to attach cheap, off-the-shelf sensors onto parking meters. The sensors will detect the presence of vehicles, and relay the information to user devices (i.e. mobile phone and GPS) via server networks. In addition to the sensors, the system uses extra information such as weather, time of year, traffic volume, and special events (e.g. baseball games). By considering both spot availability and environmental factors, a computer algorithm calculates the probability of reaching certain parking spots near the destination, and selects the optimal parking spot for the user. To better understand the system, the system architecture is shown in Figure 1-1.

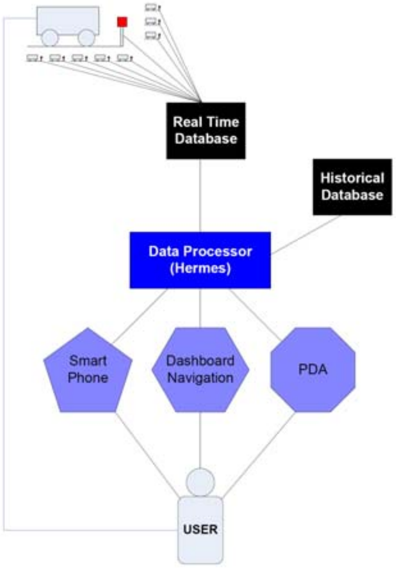

Figure 1-1: System Architecture

The system has important societal benefits, and is potentially profitable. Successful implementation is likely to reduce traffic congestion: research shows that 30% of Boston downtown traffic is caused by people searching for parking.[1] The system is also highly scalable as the startup can always build a small system in a local neighborhood and expand until the system covers crucial locations of the city. In addition, there is a large perceived market, as any large city in the world may benefit significantly from such a system. Finally, the system is financially attractive as it may generate stable long-term cash flow from monthly users' subscription fees.

The system is a large-scale system solution that will be implemented in the Greater Boston area. It will incur a high estimated startup cost within a five-year implementation phase from 2008 to 2012. In addition, the startup team is going to invest significant time in the project. It is crucial to develop a system deployment strategy that optimizes value of the business project in order to ensure system sustainability and strengthen justification for startup involvement.

1.2 Structure of Paper

The paper is divided into seven parts. Section 1 is the introduction above. Section 2 describes the main assumptions of the system deployment model. Section 3 explains and compares the fixed and flexible system deployment options. Section 4 describes a 2-stage decision tree analytic approach to evaluate the optimal deployment strategy based on those options. Section 5 describes an alternative lattice analytic approach to evaluate the appropriate strategy based on the same options. Section 6 compares the two analyses and forms a set of recommendation for the startup. Finally, Section 7 is the concluding section that summarizes the benefits of the analyses and the value of flexibility.

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[1] Web resource from: http://www.wired.com/news/technology/wireless/0,70826-0.html?tw=rss.index

2.0 Defining System Deployment Strategies

2.1 Fixed deployment strategy

Sixty thousand sensors are deployed over the 5-year period in the Greater Boston area. The startup will commit all 60,000 sensors to be manufactured by a manufacturing firm in Year 0 (January 1, 2008) and installed by Boston union workers. The sensors will be deployed at a constant rate throughout the 5-year period. The main advantages of this strategy are the reduced overall manufacturing cost by bulk buying from manufacturers and the reduced labor cost by establishing a long-term 5-year contract. The main disadvantage is the potentially high downside-risk from unfavorable market conditions, such as low demand and funding levels.

2.2 Flexible deployment strategy

Only 24,000 of the 60,000 sensors will be contracted to the same manufacturing firm and Boston union worker in Year 0 (January 1, 2008). The sensors will be deployed at a constant rate during the first 2-year period. At the end of Year 2 (December 31, 2009), the demand level can be evaluated. Based on the demand level, the startup can decide to halt business operation any time from Year 2 onwards, exercising a PUT option. The main advantage of this strategy is the ability to minimize downside-risk from unfavorable market conditions. The main disadvantages are the higher manufacturing and labor costs.

3.0 Initial System Deployment Model

3.1 Model Overview

The output of interest is the net present value (NPV) of the business project over the 5-year period. System cost and revenue over the same period are needed to derive the NPV output.

3.2 Main Model Assumptions

3.2.1 Overall Assumptions

1. There are only two expansion strategies: fixed and flexible deployment strategies.

2. The fixed deployment strategy incurs lower annual costs but requires a commitment to purchase and install 60,000 sensors over 5 years.

3. The flexible deployment strategy incurs higher annual costs but only requires commitment to purchase and install 24,000 sensors over 2 years. There is an option to stop operation at any year-end from Year 2 onwards.

4. The discount rate (DR) is set to be 25% due to the high-risk nature of this startup venture.

5. The inflation rate (I) is set to be 3.4%.[2]

6. The corporate income tax rate (T) is set to be 34.0%.[3]

7. There is no amortization in asset value within the 5-year time period.

3.2.2 System Cost Assumptions

1. The system total costs for fixed and flexible deployment strategies in the next 5 years are deterministic as illustrated in Table 3-1. Refer to Appendix A and B for cost estimate justifications when employing the fixed and flexible deployment strategies respectively.

Table 3-1: System Cost Schedule over the 5-Year Period for Both Fixed and Flexible deployment strategies

Strategy	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	TOTAL
Fixed	$ 490,800	$ 1,230,800	$ 1,160,800	$ 1,160,800	$ 1,160,800	$ 670,000	**$ 5,874,000**
Flexible	$ 541,800	$ 1,317,800	$ 1,247,800	$ 1,247,800	$ 1,247,800	$ 706,000	**$ 6,309,000**

2. If the flexible option is exercised when employing the flexible deployment strategy at any given yearly period, the annual costs for subsequent years become zero.

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[2] Same rate as 2005 U.S. consumer price index inflation rate). Web resource from: http://www.gpec.org/infocenter/Topics/Economy/USInflation.html

[3] Same rate as corporate tax rate within the $0.335M - $10M bracket. Web resource from: http://www.smbiz.com/sbrl001.html

3.2.3 System Revenue Assumptions

1. The following items are the main factors that affect system revenue:
    a) Number of users paying annual fee for service subscription [demand level]
    b) Ability to attract funding from different financial sources [funding level]
    c) The government's permission to implement the system [ignore]
    d) System's competitiveness over other similar companies [ignore]

2. The model assumes that the government will grant permission to implement the system. Thus, item c) is ignored.

3. The model assumes that no other competitors will attempt to provide a dynamic parking solution in the near future. Thus, item d) is ignored.

4. The total demand quantity or number of service subscriptions (D) is linearly correlated with the number of sensors deployed.

$$D = V_{reg} \times 5\% \times \frac{x}{T_{Sensor}} \qquad \text{(Eq. 1-1)}$$

where 'V_{reg}' is total number of vehicle registered in City of Boston; 'x' is the number of sensors implemented; 'T_{Sensor}' is the maximum number of sensors to be implemented at the end of the Year 5 (60,000 sensors in total). 'x' is incremented by 12,000 sensors annually.

5. One important consequence from the above assumptions is that the analyses of this paper focuses on only two uncertainties: funding level and demand level. The funding and demand estimates at different levels over the next five years are illustrated in Table 3-2. Refer to Appendix C and D for the funding and demand estimate justifications respectively.

Table 3-2: System Revenue Schedule over the 5-Year Period for Both Fixed and Flexible deployment strategies

Type	Level	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	TOTAL
Funding	**Low**	$1,387,625	$ -	$1,237,801	$ -	$ -	$ -	**$ 2,625,426**
Funding	**Medium**	$1,387,625	$ -	$5,550,502	$ -	$ -	$ -	**$ 6,938,127**
Funding	**High**	$1,387,625	$ -	$9,840,771	$ -	$ -	$ -	**$11,228,396**
Demand	**Low**	$ -	$251,464	$ 510,974	$ 778,724	$1,054,912	$1,339,738	**$ 3,935,812**
Demand	**Medium**	$ -	$502,927	$1,021,948	$1,557,449	$2,109,824	$2,679,477	**$ 7,871,625**
Demand	**High**	$ -	$754,391	$1,532,922	$2,336,173	$3,164,736	$4,019,215	**$11,807,437**

6. If the PUT option is exercised when employing the flexible deployment strategy at any yearly period, the annual revenues for subsequent years become zero.

4.0 Real Option Analysis I: 2-Stage Decision Tree Analysis

4.1 Overview

A 2-stage decision tree is used to evaluate the optimal strategy for maximizing the business project's NPV given the two mentioned deployment strategies. The 1st stage is defined as the time period from Year 0 to Year 2, whereas the 2nd stage is defined as the time period from Year 2 to Year 5. The beginning of the decision tree starts at Year 0. The startup team must decide whether to employ the fixed or flexible deployment strategy. At Year 2, the demand level will be known. At this point, the flexible deployment strategy provides an option to close down operation. This option is not available when fixed deployment strategy is employed. The decision to exercise the flexible option occurs only in Year 2 in order to simplify the decision tree analysis.

4.2 Treatment of Model Uncertainties

The two main uncertainties for this analysis are the demand level and funding level. The analysis assumes the demand and funding levels have 3 discrete levels: high, medium, and low. It also assumes that the probability for different funding levels depends on the demand levels. The higher the demand level, the greater the funding level, as the investment into the startup becomes more attractive to the funding sources. The analysis assumes that demand level can be accurately determined in Year 2. This allows the conditional probability of the funding level to be determined. The probabilities are summarized in Table 4-1.[4]

Table 4-3: Probabilities of Different Demand and Funding Levels at Year 2

Demand Level	P(D)	P(LF/D)	P(MF/D)	P(HF/D)
High	1/3	1/10	1/10	4/5
Medium	1/3	1/3	1/3	1/3
Low	1/3	4/5	1/10	1/10

4.3 Decision Tree Construction

The decision tree for the two system deployment options is shown in Figure 4-1.

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[4] 'LF', 'MF', 'HF' are low, medium, and high funding level respectively. P('funding level'/D) is the conditional probability of a particular funding level given a demand level.

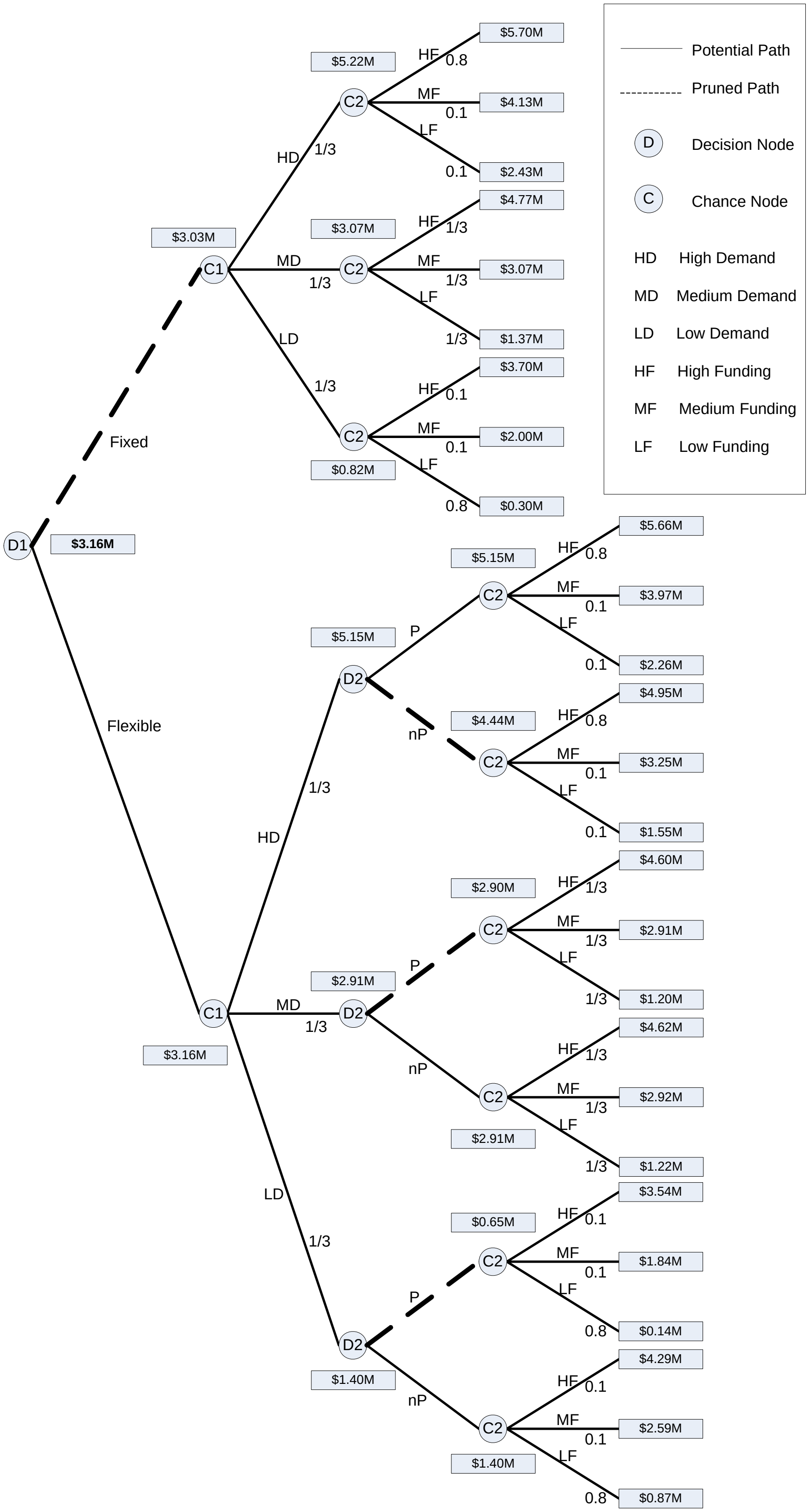

Figure 4-2: Decision Tree

In Figure 4-1, the value of each ending branch is the NPV after Year 5 for a particular deployment plan. There are 27 ending branches/plans in total. Each branch is characterized by four nodes: D1, C1, D2, and C2. D1 is the decision node for adopting a fixed or flexible deployment strategy at Year 0. C1 is the chance node of different demand levels determined at Year 2. D2 is the decision node for choosing to continue or stop operation after Year 2, and this node is only available when flexible deployment strategy is adopted. C2 is the chance node for different funding levels at Year 2.

The first step to construct the decision tree is to calculate the NPV at the end of each branch. The subsequent nodal values can be calculated from those branch values. Refer to Appendix E for the calculations of the NPV of each branch and each node.

4.4 Results of Decision Tree Analysis

4.4.1 Expected Value Outcome and Value of Flexible Option

From the decision tree analysis, the highest NPV of $3.16 million is obtained by employing the flexible deployment strategy. The main benefit of the flexible deployment strategy stems from the ability to stop operation when the demand level is low. The NPV at low demand level of the flexible deployment strategy is 41.4% greater than that of the fixed deployment strategy. As a result, the flexible deployment strategy becomes marginally more beneficial than the fixed deployment strategy.

The value of the exercising the PUT option is the difference between the NPV from employing the flexible deployment strategy and that of the fixed deployment option. The value of the PUT option is only $0.06 million.

4.4.2 Optimal Strategy

The optimal strategy is to employ the flexible deployment strategy to deploy 12,000 sensors per year during the first 2-year period. Then, if the demand level is high, the business operation should be continued. However, if the demand level is medium or low, the business operation should be halted.

4.5 Further Discussions on Optimal Strategy

Both fixed and flexible deployment strategies yield very similar NPVs. The NPV of the flexible deployment option is only 4.1% higher than that of the fixed deployment option. Therefore, the calculated NPV from the decision analysis alone may not give conclusive evidence to choose flexible deployment strategy over the fixed deployment strategy.

Other items of interest include the maximum and minimum outcomes from each of the system deployment strategies. The maximum NPV of the flexible deployment strategy is 5.5% less than that of the fixed deployment strategy, whereas the minimum NPV of flexible deployment strategy is 41.4 % greater than that of the fixed deployment option. Although the fixed deployment strategy allows for slightly greater maximum NPV, the flexible deployment allows for a relatively strong bottom-line NPV. Thus, the flexible deployment option is especially suited for risk-averse business owners.

Finally, this 2-stage decision tree analysis only analyzes the startup team's decision of exercising the PUT option in Year 2. A more thorough decision analysis should involve repeating the decision tree analysis for cases of exercising the option in Year 3, 4, and 5. This allows the startup to fine-tune the optimal strategy. For example, if further analyses predict that exercising the PUT option in Year 4 have the highest NPV, then the optimal strategy is to exercise the PUT option in Year 4 instead of Year 2.

5.0 Real Options Analysis II: Lattice Analysis

5.1 Overview

A binomial lattice model is created to evaluate the NPV of both the fixed and flexible deployment strategies by calculating the annual present value (PV) from Year 0 to 5. The value of exercising the PUT option is also calculated. In addition, a sensitivity study is performed to understand the sensitivity of the PUT option's value to the DR and the expected demand level uncertainty, and the volatility of that uncertainty.

5.2 Treatment of Model Uncertainty

In this lattice analysis, the two market uncertainties (the demand level and funding level) are reduced to one. The demand level is the chosen uncertainty, whereas the funding level is set at the medium level. In Eq. 1-1, the demand level is a function of the number of vehicle registered in Boston (V_{reg}) and the number of sensors installed (x). Since 'x' increments at a regular rate per year, 'V_{reg}' is the main volatile variable that governs the demand level uncertainty. Historical 'V_{reg}' data are used to project future V_{reg} trend, as shown in Figure 5-1.[5]

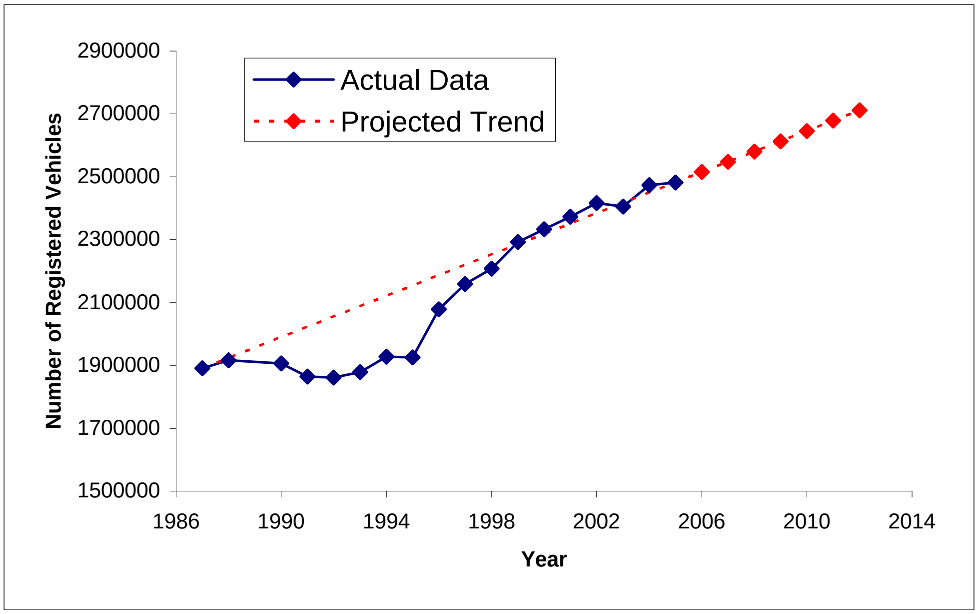

Figure 5-3: Historical Data and Projected Trend of Vehicle Registrations in Greater Boston Area

5.3 Lattice Construction

5.3.1 Development of Probability Distribution Function for Demand Uncertainty

Six pieces of information are required from V_{reg} to initialize the binomial lattice model: the initial forecast (a_i), the time increment in a given period (Δt), the normalized standard deviation of V_{reg} (σ), percentage increase of V_{reg} per period (v), the binomial model upside probability (p), the binomial model upside factor (u), and the binomial

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[5] Web resource from: http://www.tbf.org/indicators2004/transportation/indicators.asp?id=2633&crosscutID=322&crosscutName=Sustainable%20Development

* The V_{reg} data from 2006-2012 are generated from linear extrapolation.

model downside factor (d). 'a_i', 'Δt', 'σ', and 'v' can be derived from the data in Table 5-1, whereas 'p', 'u', and 'd' can be derived from 'a_i', 'Δt', 'σ', using the following equations:

$$p = 0.5 + 0.5 \cdot \left(\frac{v}{\sigma} \right) \cdot (\Delta t)^{0.5} \qquad \text{(Eq. 5-1)}$$

$$u = e^{\left[\sigma \cdot (\Delta t)^{0.5} \right]} \qquad \text{(Eq. 5-2)}$$

$$d = \frac{1}{u} \qquad \text{(Eq. 5-3)}$$

The values of the input variables for the binomial lattice model are summarized in Table 5-1. Refer to Appendix F1 for details of estimating and calculating those stated variables.

Table 5-4: Summary of Main Input Variables of Binomial Lattice Model:

Parameter	Variable	Value
Initial Vehicle Registrations Forecast	a_i	2,547,459
Time Increment	ΔT	1 year
Normalized Standard Deviation	σ	3.70%
Average Annual Increase	v	1.74%
Binomial Upside Probability	p	0.735
Binomial Model Upside Factor	u	1.038
Binomial Model Downside Factor	d	0.964

Upon inserting the value of 'p', 'u', and 'd' into the binomial lattice model, a probability lattice and a demand revenue lattice can be created. The information from the lattices can combine to form the probability distribution functions (PDF) of the annual demand revenues, as shown in Figure 5-2. Refer to Appendix F2 for more details on the binomial lattice model calculations.

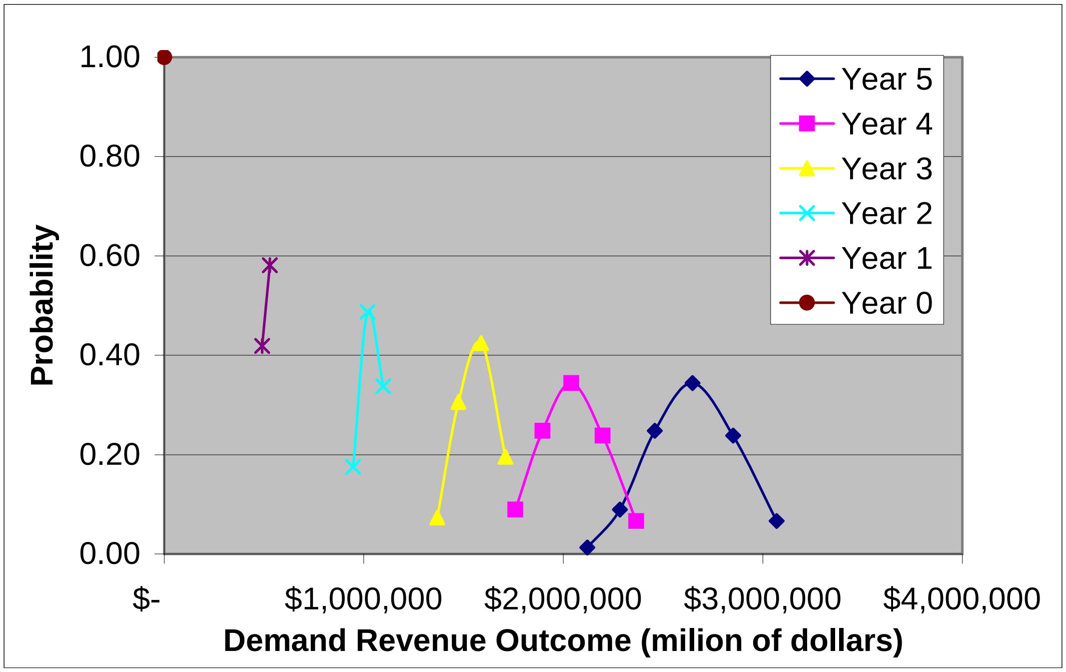

Figure 5-4: Resulting PDFs of Annual Demand Revenues Based on Binomial Lattice Model

5.3.2 Development of Nominal Profit Lattice to Calculate NPV for Fixed Deployment Strategy

In order to determine the NPV for the fixed deployment strategy, a nominal profit lattice is constructed from the previously developed demand revenue lattice, funding revenue schedule, and cost schedule, as shown in Table 5-2.

Table 5-5: Outcome Lattice for Nominal Profit (Fixed Deployment Strategy)

Year	0	1	2	3	4	5
Nominal Profit:	$ 896,825	$ (701,948) $ (739,650)	$5,487,599 $5,409,329 $5,336,639	$ 548,627 $ 426,760 $ 313,582 $ 208,471	$ 1,205,047 $ 1,036,383 $ 879,744 $ 734,272 $ 599,170	$ 2,399,686 $ 2,180,845 $ 1,977,606 $ 1,788,856 $ 1,613,561 $ 1,450,764

The annual expected values (EV) are then calculated from the nominal profit lattice. The next step is to calculate the annual present values (PV) of each year by applying T, I, and DR to the EV of each year. Finally NPV is calculated by summing the annual PVs. Refer to Appendix F3 for more details on the nominal profit lattice construction and the NPV calculations from the nominal profit lattice.

5.3.3 Development of Present Value Lattice to Calculate NPV for Flexible Deployment Strategy

In order to determine the NPV for the flexible deployment strategy, a different profit lattice is constructed, as shown in Table 5-3.

Table 5-6: Outcome Lattice for Nominal Profit (Flexible Deployment Strategy)

Year	0	1	2	3	4	5
Nominal Profit:	$ 845,825	$ (788,948)	$5,400,599	$ 461,627	$ 1,118,047	$ 2,363,686
		$ (826,650)	$5,322,329	$ 339,760	$ 949,383	$ 2,144,845
			$5,249,639	$ 226,582	$ 792,744	$ 1,941,606
				$ 121,471	$ 647,272	$ 1,752,856
					$ 512,170	$ 1,577,561
						$ 1,414,764

Instead of calculating expected value of each year, each value in the nominal profit lattice is converted into a corresponding PV. In addition, since the startup has a PUT option of shutting down operation during and after Year 2, this feature is incorporated in the PV lattice from Year 2 onwards. The resulting PV lattice is shown in Table 5-4.

Table 5-7: Outcome Lattice for Present Value (Flexible Deployment Strategy)

Year	0	1	2	3	4	5
Present Value:	**$2,915,665**	$3,117,868	$4,801,664	$ 1,701,974	$ 1,898,077	$ 1,560,033
		$2,948,649	$4,565,845	$ 1,456,618	$ 1,678,349	$ 1,415,598
			$4,447,605	$ 1,228,753	$ 1,474,285	$ 1,281,460
				$ 1,327,971	$ 1,284,769	$ 1,156,885
					$ 1,108,764	$ 1,041,191
						$ 933,744

The highlighted value is the final NPV for employing the flexible deployment strategy.

5.4 Results of Lattice Analysis

From the lattice analysis, the highest NPV of $3.07 million is obtained by employing the fixed deployment strategy. Since the flexible deployment strategy generates a lower NPV than the fixed deployment strategy do, the value of the PUT option is a negative value of -$0.16 million.

One reason behind the negative value is that the PUT option is never exercised within the 5-year timeframe, as shown in Table 5-5.

Table 5-8: Lattice Showing Whether Option Is Exercised (Flexible Deployment Strategy)

Year	1	2	3	4	5
PUT Option Exercised?			FALSE FALSE FALSE	FALSE FALSE FALSE FALSE	FALSE FALSE FALSE FALSE FALSE

The highlighted cells in Table 5-5 are the only situations, where the PUT option can be exercised. A value of "FALSE" is denoted to any cell, where the PUT option is not exercised. The lattice analysis suggests that shutting down operation will always lower NPV given the demand level uncertainty assumption. Since the PUT option is not exercised, the flexibility advantage of the flexible deployment strategy is never realized. In this scenario, the higher manufacturing and labor costs of the flexible deployment strategy makes the strategy less favorable when compared to the fixed deployment strategy. Thus, the optimal strategy is to employ the fixed deployment strategy. This involves installing 12,000 sensors per year over the 5-year period.

5.5 Further Discussions: Sensitivity Analysis

5.5.1 Study 1: PUT Option Values at Different Demand Levels and Discount Rates

In order to understand the sensitivity of the results, the DR and demand level are scaled in order to observe the relative change in the PUT option value. In the sensitivity study, the DR ranges from 0 to 75%[6], whereas the demand level is scaled form 50% to 150% of the original demand level. The results are generated using a 2-way Excel data table, as shown in Table 5-6.

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[6] A reasonable discount rate for risky business can be as high as 75% due to high systemic risk and illiquidity of investments. Web resource: http://www.pacesettercapital.com/old-05-05-2003/files/Valuation_Article_Website.pdf#search=%22business%20discount%20rate%20approximation%22

Table 5-9: PUT Option Values at Different DR and Demand Levels (σ=3.7%)

Demand Scalar	Discount Rate			
	0%	25%	50%	75%
0.5	$ 954,210	$ 658,755	$ 456,621	$ 325,418
0.6	$ 592,742	$ 513,222	$ 385,715	$ 286,157
0.7	$ 231,274	$ 367,690	$ 314,810	$ 246,896
0.8	$ (106,198)	$ 173,237	$ 186,411	$ 153,052
0.9	$ (255,973)	$ (5,258)	$ 64,069	$ 15,366
1.0	$ (265,172)	$ (166,213)	$ (74,948)	$ 102,752
1.1	$ (265,172)	$ (166,213)	$ (84,084)	$ 35,150
1.2	$ (265,172)	$ (166,213)	$ (122,510)	$ (65,992)
1.3	$ (265,172)	$ (166,213)	$ (122,510)	$ (99,201)
1.4	$ (265,172)	$ (166,213)	$ (122,510)	$ (99,201)
1.5	$ (265,172)	$ (166,213)	$ (122,510)	$ (99,201)

One important realization from the PUT option value shown in Table 5-6 is that the option values are highly sensitive to the assumptions of the demand level and DR. When the demand level is high and the DR is low, the PUT option values tend to be negative. This implies fixed deployment strategy is preferred. The opposite is true for low demand level and high DR, where the flexible deployment strategy is preferred.

One explanation to those results is the likelihood of PUT option being exercised when employing the flexible deployment strategy. When DR is high, the future profits are highly discounted. As the profit prospect of continuing operation becomes worse when DR becomes higher, the likelihood of exercising the PUT option increases. When the demand level is low, the future profits from the demand revenues become smaller. Since the business project is more likely to run into loss, the likelihood of exercising the PUT option also increases. When the tendency of exercising the PUT option increases, the flexible deployment strategy also becomes more favorable.

5.5.2 Study 2: PUT Option Values and Uncertainty Volatility

Another intuitive exercise is to understand how volatility of the demand level uncertainty affects the PUT option values. A sensitivity study is performed again with the same ranges of demand level and DR. However, there is one difference between this study and the previous one: volatility is increased by changing 'σ' from 3.7% to 18.5% (5 times the original value). The resulting option values are shown in Table 5-7.

Table 5-10: PUT Option Values at Different DR and Demand Levels (σ=18.5%)

Demand Scalar	Discount Rate			
	0%	25%	50%	75%
0.5	$ 639,751	$ 534,709	$ 398,128	$ 293,915
0.6	$ 356,271	$ 333,555	$ 315,524	$ 248,353
0.7	$ 88,070	$ 221,804	$ 196,633	$ 152,337
0.8	$ (31,486)	$ 106,597	$ 129,512	$ 109,991
0.9	$ (125,904)	$ (13,814)	$ (1,485)	$ 84,724
1.0	$ (204,463)	$ (49,884)	$ (18,234)	$ (15,452)
1.1	$ (229,927)	$ (100,893)	$ (59,104)	$ (24,332)
1.2	$ (241,725)	$ (144,002)	$ (68,241)	$ (50,668)
1.3	$ (253,523)	$ (148,295)	$ (108,276)	$ (55,707)
1.4	$ (265,172)	$ (152,587)	$ (110,172)	$ (89,686)
1.5	$ (265,172)	$ (166,213)	$ (112,069)	$ (90,643)

If the PUT option values in Table 5-6 are compared to those in Table 5-7, only one option value switches sign (the one at DR=50% and Demand Scalar = 0.9). With five-fold increase in 'σ', there is only one case where the fixed deployment strategy should be switched to the flexible deployment strategy. Thus, this study concludes that the optimal strategy is relatively insensitive to the volatility of the chosen demand level uncertainty.

6.0 Recommendations

6.1 Project Feasibility and Optimal Deployment Strategy

There are two important conclusions from the decision tree and lattice analyses. First of all, both analyses conclude that the NPV from implementing the project is approximately $3 million. Thus, the business project is a feasible one.

Secondly, the analyses have conflicting conclusions on the optimal deployment strategy. The decision tree analysis supports the flexible deployment strategy, whereas the lattice analysis supports the fixed deployment strategy. On closer inspection, both analyses have similar results on NPV values for both strategies, as shown in Table 6-1.

Table 6-11: Summary of NPV Results from Both Decision Tree and Lattice Analyses

Analysis	Strategy	NPV			Mean	% EV Deviation from Mean
		EV	Max	Min		
Decision Tree	Fixed	$ 3,034,552	$ 5,704,217	$ 304,828	$ 3,044,465	-0.3%
Decision Tree	Flexible	$ 3,156,356	$ 5,663,327	$ 886,837	$ 3,044,465	3.7%
Lattice	Fixed	$ 3,071,288	---	---	$ 3,044,465	0.9%
Lattice	Flexible	$ 2,915,665	---	---	$ 3,044,465	-4.2%

The four NPVs are all within ±5% from the mean, which may imply that there is no strong, conclusive evidence to prefer one strategy to the other. One additional piece of information from the decision tree is that the flexible deployment option has significantly lower downside-risk. Thus, the recommendation is to implement flexible deployment strategy given the current model assumptions. However, the demand level or DR assumptions may change. In such a scenario, the recommendation is to apply the strategy according to the results of the lattice analysis's sensitivity study.

6.2 Preferred Analysis Technique

Both the decision tree and lattice analyses have their strengths and weaknesses. The decision tree analysis allows the startup to identify 27 distinct outcomes clearly, while determining the optimal NPV outcome. However, discrete levels need to be set for each main market uncertainty. Only 3 levels are set as the number of possible branches and outcomes increases exponentially with the number of discrete levels of each uncertainty. A sensitivity study on the uncertainties can also be challenging.

The lattice analysis's main advantage for this project is its ease to analyze a wide range of demand levels and DR from a sensitivity study. As a result, multiple strategies from a large number of demand levels and DRs can quickly be determined. However, the analysis is less intuitive due to the obscure nature of the binomial lattice model. Another weakness in the analysis is that the analysis tool seems only comfortable at accommodating for one uncertainty (demand level) to be analyzed at a time. If the other main model uncertainty (funding level) is also analyzed using the binomial lattice model, the resulting lattice analysis will immediately become very complicated.

Due to the pros and cons of each analysis, it is difficult to choose the preference on a particular analysis for this project. One possible deciding factor is that the funding level uncertainty has little historical information. In order to fully benefit from the lattice analysis, the main uncertainties should have good trends of historical data. Since the decision analysis is more forgiving with lack of data, decision analysis is recommended to be the primary analysis tool for the startup team. However, if more data for the funding level arrive or if market uncertainty assumptions change, the lattice analysis may be a stronger analysis tool.

6.3 Suggestions for Further Analyses

A recommendation for the decision tree analysis is to repeat the analysis for cases such that the decision nodes for exercising the PUT option occur at Year 3, Year 4, and Year 5 respectively. This can better reflect reality by relaxing the startup's requirement to make the decision of exercising the option in Year 2 (similar to the European option) and allowing the startup to make the decision anytime between Year 2 and Year 5 (similar to the American option).

In order to generate a more thorough lattice analysis, the analysis should be repeated for high and low funding levels. This helps to address the lattice analysis's inherent weakness of analyzing only one uncertainty (demand level) by attempting to expand the single-uncertainty analysis into a 2-uncertainty analysis.

7.0 Conclusions

The analyses attempt to understand the profitability of the parking system and its optimal deployment strategy. Both analyses for both system deployment strategies predicts that the startup can expect to create a business project with an estimated $3 million NPV within 5 years. Therefore, implementing the system certainly seems profitable and sustainable. However, as successful technology ventures often create value in the dollar range of more than tens of millions, the NPV figure is not particularly attractive for the startup team to pursue the project.

As for the optimal deployment strategy, if the startup team receives no further information on its system financial assumptions, the flexible deployment strategy seems more appropriate due to its lower downside-risk based on decision tree analysis alone. However, if the startup team receives more information on the demand level, funding level, or DR, a further lattice analysis combined with a sensitivity study may help to define a more optimal strategy. In addition, information on demand volatility does not seem to be as pertinent as the actual expected demand level and estimated DR. Thus, the startup team should not expect the optimal strategy to change significantly due to the demand level variation over the next 5 years.

The analyses have not necessarily increased the value of the business project significantly since the analyses give similar, non-conclusive NPV results. However, the analyses have helped the startup team to understand the value and risk of the system implementation more clearly. More specifically, if more market information is available, the team can react to more readily to market conditions by performing further analysis to optimize the system deployment strategy.

Appendix

Appendix A – Cost Estimation for Fixed deployment strategy

A1. Number and Cost of Vehicle Detection Sensors

The number of parking meters in Cambridge is approximately 2,700[7]; the number in Boston is approximately 7,300.[8] The number of unrestricted on-street parking spaces is estimated to be 5 times the number of metered parking spaces (~50,000 unrestricted on-street parking spaces).[9] This brings the total number of sensors needed to 60,000.

The chosen vehicle detection sensor on each parking meter is Honeywell's Anisotropic Magneto-Resistive (AMR) sensor.[10] The sensor costs $20 each.[11]

A2. Number and Cost of Network Devices and Main Processor

The design team has chosen Zigbee wireless mesh network as the network node system. The estimated cost is $0.50 per node.[12] One node is needed per sensor, and therefore 60,000 nodes are required.

The wireless nodes will be connected to a server. This server is going to distribute computing loads to the computer processors that are connected to the server. The startup team estimates that one new computer process and two new servers are required per 10,000 wireless nodes. Each server is estimated to cost $1,000 each, whereas each processor is estimated to cost $2,000 each.

A3. Cost of Manufacturing Sensor Devices

Manufacturing cost for one sensor device is assumed to be the same as the cost of purchasing the sensor at $20 per sensor.

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[7] Web resource from: http://www.wpi.edu/Academics/Depts/IGSD/Projects/Boston/Center/Projects/IQP_public/D03/Cambridge%20Parking%20Regulation/D03_Presentation-Cambridge_Parking_Reg-Apr25.pdf#search=%22estimated%20number%20of%20parking%20meters%20harvard%20square%22

[8] Web resource from: http://www.cityofboston.gov/transportation/parking.asp

[9] Web resource from: http://www.bostonmpo.org/bostonmpo/resources/parking/ParkingReport1.pdf#search=%22number%20of%20parking%20meters%20kendall%20square%22

[10] Web resource from: http://shop.ssec.honeywell.com/shopdisplayproducts.asp?id=6&subcat=10&cat=HMC1xxx+%2F+HMC6xxx+%2D+Products+and+Accessories

[11] Web resource from: http://www.ssec.honeywell.com/magnetic/datasheets/an218.pdf

[12] Web resource from: http://www.itarchitect.com/shared/printableArticle.jhtml?articleID=18900058

A4. Legal Cost

Legal counseling will be required to ensure that legal and financial records for incorporation and initial round of investments from different funding sources are correct and compliant. Brown Rudick's Emerging Companies Program provides such a counseling startup package at $20,000. There may be an annual cost of extra legal counseling, lawsuits, and insurance payments. The total annual legal cost is estimated to be the same as the initial legal counseling cost at $20,000.

A5. Fixed Labor Cost

During the 5-year implementation phase, a team of full-time and part-time employees needs to be hired and paid. Here is the estimate for the minimum set of positions required, summing up to $220,000 salary payout per year: [13]

- One administrative / accounting clerk ($32,000/year)
- One network planning manager ($62,000/year)
- One production engineer ($60,000/year)
- One marketing coordinator ($51,000/year)
- Two part-time research students ($15,000/year/person)*

A6. Variable Labor Cost

As the parking system expands in the 5-year period, additional wages must be paid install and maintain the system:

- A union worker is assumed to take one hour to install each sensor, and the wage is assumed to be $30/hr ($30/sensor)*
- One trained technician is needed to fix and maintain sensors for every 5,000 sensors ($35,000/year)*

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[13] Web resource (MonsterTrak's Salary Center) from:
http://salary.monster.com/salarywizard/layoutscripts/swzl_newsearch.asp
All salaries, except for ones with *, are estimated using MonsterTrak's Salary Center. The salary represents the median salary level for Boston. * denotes to salaries that are ballpark estimations.

A7. Cost Summary

The information from the research is summarized in Table A-1, and the total system cost is estimated to be $5,874,000 over the 5-year period for the fixed deployment strategy.

Table A-12: System Cost Estimation over the 5-Year Period (Fixed deployment strategy)

Parameter	Quantity	Cost per unit	Cost
Vehicle Sensors	60000	$ 20	$ 1,200,000
Network Nodes	60000	$ 0.50	$ 30,000
Main Processor	6	$ 2,000	$ 12,000
Server	12	$ 1,000	$ 12,000
Manufacturing Cost	60000	$ 20	$ 1,200,000
Annual Legal Cost	5	$ 20,000	$ 100,000
Annual Employee Salary	5	$ 220,000	$ 1,100,000
Installation Cost	60000	$ 30	$ 1,800,000
Technician Cost	12	$ 35,000	$ 420,000
TOTAL	**---**	**---**	**$ 5,874,000**

The annual cost schedule for the five-year period is also estimated. Sensor purchasing and manufacturing costs will be incurred and distributed evenly from Year 0 to Year 4. Salary and other labor costs will be incurred and distributed evenly from Year 1 to Year 5. The cost schedule is illustrated in Table A-2.

Table A-13: System Cost Schedule Estimate over the 5-Year Period (Fixed deployment strategy)

Cost Items	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	TOTAL
Vehicle Sensors	$ 240,000	$ 240,000	$ 240,000	$ 240,000	$ 240,000	$ -	$ 1,200,000
Network Nodes	$ 6,000	$ 6,000	$ 6,000	$ 6,000	$ 6,000	$ -	$ 30,000
Main Processor	$ 2,400	$ 2,400	$ 2,400	$ 2,400	$ 2,400	$ -	$ 12,000
Server	$ 2,400	$ 2,400	$ 2,400	$ 2,400	$ 2,400	$ -	$ 12,000
Manufacturing Cost	$ 240,000	$ 240,000	$ 240,000	$ 240,000	$ 240,000	$ -	$ 1,200,000
Annual Legal Cost	$ -	$ 20,000	$ 20,000	$ 20,000	$ 20,000	$ 20,000	$ 100,000
Annual Employee Salary	$ -	$ 220,000	$ 220,000	$ 220,000	$ 220,000	$ 220,000	$ 1,100,000
Installation Cost	$ -	$ 360,000	$ 360,000	$ 360,000	$ 360,000	$ 360,000	$ 1,800,000
Technician Cost	$ -	$ 140,000	$ 70,000	$ 70,000	$ 70,000	$ 70,000	$ 420,000
Total	**$ 490,800**	**$ 1,230,800**	**$ 1,160,800**	**$ 1,160,800**	**$ 1,160,800**	**$ 670,000**	**$ 5,874,000**

Appendix B – Cost Estimation for Flexible deployment strategy

By employing flexible deployment strategy, the overall cost of the system is increased due to less favorable terms of shorter-term contracts for the suppliers and installation workers. The additional costs are assumed to be:

1. 10% increase in vehicle sensors cost
2. 50% increase in network node cost
3. 10% increase in manufacturing cost
4. 10% increase in installation cost

Incorporating the additional assumptions, the results are summarized in Table B-1, and the total system cost is estimated to be $6,309,000 over the 5-year period for the flexible deployment strategy. Note that this cost is only true when the PUT option is not exercised over the 5-year period.

Table B-14: System Cost Estimation over the 5-Year Period (Flexible deployment strategy without Exercising the PUT Option)

Parameter	Quantity	Cost per unit	Cost
Vehicle Sensors	60000	22	$ 1,320,000
Network Nodes	60000	0.75	$ 45,000
Main Processor	6	2000	$ 12,000
Server	12	1000	$ 12,000
Manufacturing Cost	60000	22	$ 1,320,000
Annual Legal Cost	5	20000	$ 100,000
Annual Employee Salary	5	220000	$ 1,100,000
Installation Cost	60000	33	$ 1,980,000
Technician Cost	12	35000	$ 420,000
TOTAL	---	---	**$ 6,309,000**

The five-year annual cost schedule for employing the flexible deployment strategy is also estimated by the same methodology as the cost schedule in Appendix A. Sensor purchasing and manufacturing costs will be incurred and distributed evenly from Year 0 to Year 4. Salary and other labor costs will be incurred and distributed evenly from Year 1 to Year 5. The cost schedule is illustrated in Table B-2.

Table B-15: System Cost Schedule Estimate over the 5-Year Period (Flexible Case without Exercising the PUT Option)

Cost Items	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	TOTAL
Vehicle Sensors	$ 264,000	$ 264,000	$ 264,000	$ 264,000	$ 264,000	$ -	$ 1,320,000
Network Nodes	$ 9,000	$ 9,000	$ 9,000	$ 9,000	$ 9,000	$ -	$ 45,000
Main Processor	$ 2,400	$ 2,400	$ 2,400	$ 2,400	$ 2,400	$ -	$ 12,000
Server	$ 2,400	$ 2,400	$ 2,400	$ 2,400	$ 2,400	$ -	$ 12,000
Manufacturing Cost	$ 264,000	$ 264,000	$ 264,000	$ 264,000	$ 264,000	$ -	$ 1,320,000
Annual Legal Cost	$ -	$ 20,000	$ 20,000	$ 20,000	$ 20,000	$ 20,000	$ 100,000
Annual Employee Salary	$ -	$ 220,000	$ 220,000	$ 220,000	$ 220,000	$ 220,000	$ 1,100,000
Installation Cost	$ -	$ 396,000	$ 396,000	$ 396,000	$ 396,000	$ 396,000	$ 1,980,000
Technician Cost	$ -	$ 140,000	$ 70,000	$ 70,000	$ 70,000	$ 70,000	$ 420,000
Total	**$ 541,800**	**$ 1,317,800**	**$ 1,247,800**	**$ 1,247,800**	**$ 1,247,800**	**$ 706,000**	**$ 6,309,000**

Appendix C – Funding Estimation

C1. Potential Funding Sources

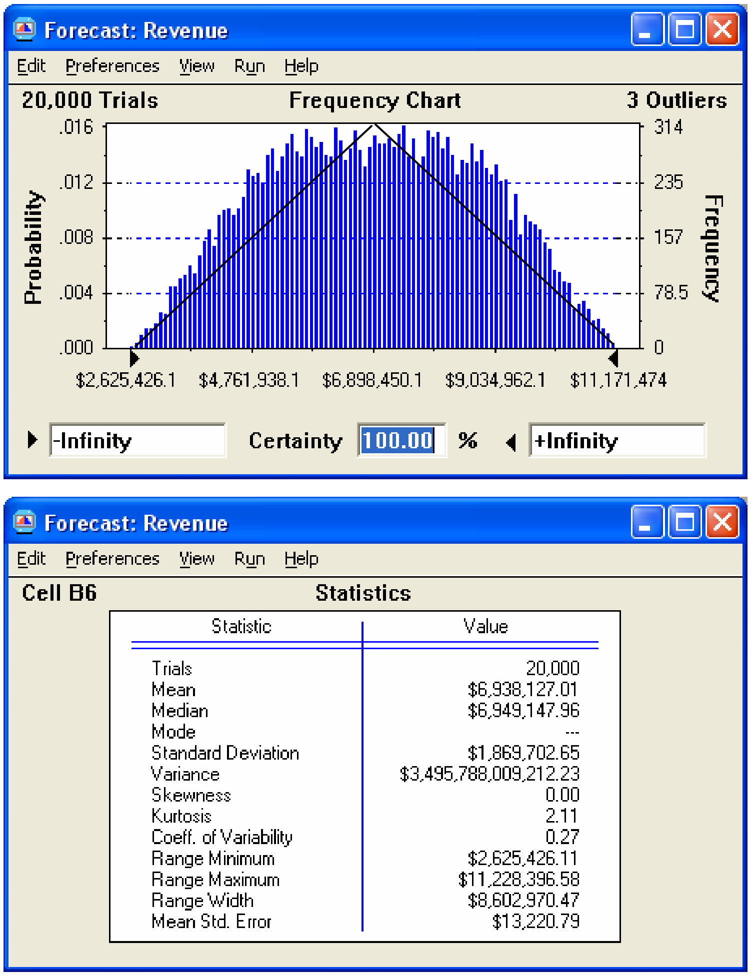
Four groups of funding sources and their potential contributions are identified:

- Government contract ($5 million +/- $3 million)
- Venture capitalists ($500,000 to $3 million)
- Angel investors ($0 to $200,000)
- Small business funds ($0 to $200,000)

The above estimates are all ballpark figures based on team members' entrepreneurship experiences.

C2. Funding Estimates at Different Funding Levels

Since there is no information on the accuracy of the funding figures, each of above bulleted funding revenue sources is assumed uniformly distributed. An extra step is taken to sum up the different distributions above to simulate a total distribution of potential funding. A Monte Carlos simulation is created, as shown in Figure C-1.



Cell B6

Statistics

Statistic	Value
Trials	20,000
Mean	$6,938,127.01
Median	$6,949,147.96
Mode	---
Standard Deviation	$1,869,702.65
Variance	$3,495,788,009,212.23
Skewness	0.00
Kurtosis	2.11
Coeff. of Variability	0.27
Range Minimum	$2,625,426.11
Range Maximum	$11,228,396.58
Range Width	$8,602,970.47
Mean Std. Error	$13,220.79

Figure C-5: Resulting Funding Distribution Based on Monte Carlos Simulation (20,000 trials)

In Figure C-1, a triangular distribution is used to approximate the funding revenue PDF with its lower limit at $2.63 million, its likeliest value at $6.94 million, and it upper limit at $11.17 million respectively. Those three values form the low, medium, and high funding levels respectively.

The model assumes two rounds of funding: one at Year 0 and one at Year 2. The funding amounts for the different funding levels at Year 0 are the same at $1,387,625 (1/5 of the likeliest value). However, the funding amounts for the different funding levels at Year 2 are dependent on the demand level. It is assumed that the demand level can be determined in Year 2. The resulting funding schedule is illustrated in Table 2-2.

Appendix D – Demand Estimation

D1. Demand Quantities and Revenue Estimation

The demand quantity can be approximated by Eq. 3-1. The main contributing factor to the uncertainty is 'V_{reg}'. The data for 'V_{reg}' are shown in Table D-1.[14]

Table D-16: Vehicle Registration Data for Greater Boston

Year	V_{reg}	Year	V_{reg}
1987	1891001	1997	2158444
1988	1916435	1998	2207747
1999	---	1999	2292119
1990	1906248	2000	2332632
1991	1864022	2001	2372546
1992	1861607	2002	2416851
1993	1878500	2003	2405524
1994	1927985	2004	2472926
1995	1925259	2005	2481813
1996	2077820		

The subscription cost per device is $20 per year. By linearly extrapolating the number of vehicle registered, utilizing Eq. A-1, and multiplying the quantity by the subscription cost, the demand revenue for over the five-year period can be determined as shown in Table D-2.

Table D-17: Demand Quantity and Revenue Estimates over the 5-Year Period

Paremeter	V_{reg}	Demand Quantity	Benefit per unit	Demand Revenue
Demand in Year 1	2514636	25146	$ 20	$ 502,927
Demand in Year 2	2554870	51097	$ 20	$ 1,021,948
Demand in Year 3	2595748	77872	$ 20	$ 1,557,449
Demand in Year 4	2637280	105491	$ 20	$ 2,109,824
Demand in Year 5	2679477	133974	$ 20	$ 2,679,477
Total Demand Revenue	**---**	**---**	**---**	**$ 7,871,625**

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[14] Web resource from: http://www.tbf.org/indicators2004/transportation/indicators.asp?id=2633&crosscutID=322&crosscutName=Sustainable%20Development

D2. Demand Revenue Estimations at Different Demand Levels

The three levels of demand (low, medium, and high) can be determined in Year 2. Each level of demand is assumed to occur with equal probability of 1/3. In addition, the demand revenue at the medium demand level is the same as the total demand revenue shown in Table D-2. The demand revenues at low and high demand levels are 50% decrease and 50% increase in the demand revenue from the medium level respectively. This is summarized in Table D-3.

Table D-18: Demand Probability and Revenue at Different Demand Levels

Demand Level	Probability	Demand Revenue
Low	1/3	$ 3,935,812
Medium	1/3	$ 7,871,625
High	1/3	$ 11,807,437

Appendix E – Decision Tree Calculations

E1. Calculating the NPV of Each Branch

As explained in Section 4.3, the values for each ending branch (NPV over 5 years) must be calculated in order to construct the decision tree. There are 27 branches in total and each branch has a unique nodal ID from D1, C1, D2, and C2. The first step is to create a table of demand revenue schedules for each of these branches using data from Table 2-2, as illustrated in Table E-1.

Table E-19: Demand Revenue Schedules for All Branches in the Decision Tree

Node				Time Period					
D1	**C1**	**D2**	**C2**	**Year 0**	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**
Fixed	HD	---	HF	$ -	$ 754,391	$ 1,532,922	$ 2,336,173	$ 3,164,736	$ 4,019,215
Fixed	HD	---	MF	$ -	$ 754,391	$ 1,532,922	$ 2,336,173	$ 3,164,736	$ 4,019,215
Fixed	HD	---	LF	$ -	$ 754,391	$ 1,532,922	$ 2,336,173	$ 3,164,736	$ 4,019,215
Fixed	MD	---	HF	$ -	$ 502,927	$ 1,021,948	$ 1,557,449	$ 2,109,824	$ 2,679,477
Fixed	MD	---	MF	$ -	$ 502,927	$ 1,021,948	$ 1,557,449	$ 2,109,824	$ 2,679,477
Fixed	MD	---	LF	$ -	$ 502,927	$ 1,021,948	$ 1,557,449	$ 2,109,824	$ 2,679,477
Fixed	LD	---	HF	$ -	$ 251,464	$ 510,974	$ 778,724	$ 1,054,912	$ 1,339,738
Fixed	LD	---	MF	$ -	$ 251,464	$ 510,974	$ 778,724	$ 1,054,912	$ 1,339,738
Fixed	LD	---	LF	$ -	$ 251,464	$ 510,974	$ 778,724	$ 1,054,912	$ 1,339,738
Flexible	HD	P	HF	$ -	$ 754,391	$ 1,532,922	$ 2,336,173	$ 3,164,736	$ 4,019,215
Flexible	HD	P	MF	$ -	$ 754,391	$ 1,532,922	$ 2,336,173	$ 3,164,736	$ 4,019,215
Flexible	HD	P	LF	$ -	$ 754,391	$ 1,532,922	$ 2,336,173	$ 3,164,736	$ 4,019,215
Flexible	HD	nP	HF	$ -	$ 754,391	$ 1,532,922	$ -	$ -	$ -
Flexible	HD	nP	MF	$ -	$ 754,391	$ 1,532,922	$ -	$ -	$ -
Flexible	HD	nP	LF	$ -	$ 754,391	$ 1,532,922	$ -	$ -	$ -
Flexible	MD	P	HF	$ -	$ 502,927	$ 1,021,948	$ 1,557,449	$ 2,109,824	$ 2,679,477
Flexible	MD	P	MF	$ -	$ 502,927	$ 1,021,948	$ 1,557,449	$ 2,109,824	$ 2,679,477
Flexible	MD	P	LF	$ -	$ 502,927	$ 1,021,948	$ 1,557,449	$ 2,109,824	$ 2,679,477
Flexible	MD	nP	HF	$ -	$ 502,927	$ 1,021,948	$ -	$ -	$ -
Flexible	MD	nP	MF	$ -	$ 502,927	$ 1,021,948	$ -	$ -	$ -
Flexible	MD	nP	LF	$ -	$ 502,927	$ 1,021,948	$ -	$ -	$ -
Flexible	LD	P	HF	$ -	$ 251,464	$ 510,974	$ 778,724	$ 1,054,912	$ 1,339,738
Flexible	LD	P	MF	$ -	$ 251,464	$ 510,974	$ 778,724	$ 1,054,912	$ 1,339,738
Flexible	LD	P	LF	$ -	$ 251,464	$ 510,974	$ 778,724	$ 1,054,912	$ 1,339,738
Flexible	LD	nP	HF	$ -	$ 251,464	$ 510,974	$ -	$ -	$ -
Flexible	LD	nP	MF	$ -	$ 251,464	$ 510,974	$ -	$ -	$ -
Flexible	LD	nP	LF	$ -	$ 251,464	$ 510,974	$ -	$ -	$ -

The second step is to create a table of funding revenue schedules for each of these branches using data from Table 2-2, as illustrated in Table E-2

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Table E-20: Funding Revenue Schedules for All Branches in the Decision Tree

Node				Time Period					
D1	**C1**	**D2**	**C2**	**Year 0**	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**
Fixed	HD	---	HF	$ 1,387,625	$ -	$ 9,840,771	$ -	$ -	$ -
Fixed	HD	---	MF	$ 1,387,625	$ -	$ 5,550,502	$ -	$ -	$ -
Fixed	HD	---	LF	$ 1,387,625	$ -	$ 1,237,801	$ -	$ -	$ -
Fixed	MD	---	HF	$ 1,387,625	$ -	$ 9,840,771	$ -	$ -	$ -
Fixed	MD	---	MF	$ 1,387,625	$ -	$ 5,550,502	$ -	$ -	$ -
Fixed	MD	---	LF	$ 1,387,625	$ -	$ 1,237,801	$ -	$ -	$ -
Fixed	LD	---	HF	$ 1,387,625	$ -	$ 9,840,771	$ -	$ -	$ -
Fixed	LD	---	MF	$ 1,387,625	$ -	$ 5,550,502	$ -	$ -	$ -
Fixed	LD	---	LF	$ 1,387,625	$ -	$ 1,237,801	$ -	$ -	$ -
Flexible	HD	P	HF	$ 1,387,625	$ -	$ 9,840,771	$ -	$ -	$ -
Flexible	HD	P	MF	$ 1,387,625	$ -	$ 5,550,502	$ -	$ -	$ -
Flexible	HD	P	LF	$ 1,387,625	$ -	$ 1,237,801	$ -	$ -	$ -
Flexible	HD	nP	HF	$ 1,387,625	$ -	$ 9,840,771	$ -	$ -	$ -
Flexible	HD	nP	MF	$ 1,387,625	$ -	$ 5,550,502	$ -	$ -	$ -
Flexible	HD	nP	LF	$ 1,387,625	$ -	$ 1,237,801	$ -	$ -	$ -
Flexible	MD	P	HF	$ 1,387,625	$ -	$ 9,840,771	$ -	$ -	$ -
Flexible	MD	P	MF	$ 1,387,625	$ -	$ 5,550,502	$ -	$ -	$ -
Flexible	MD	P	LF	$ 1,387,625	$ -	$ 1,237,801	$ -	$ -	$ -
Flexible	MD	nP	HF	$ 1,387,625	$ -	$ 9,840,771	$ -	$ -	$ -
Flexible	MD	nP	MF	$ 1,387,625	$ -	$ 5,550,502	$ -	$ -	$ -
Flexible	MD	nP	LF	$ 1,387,625	$ -	$ 1,237,801	$ -	$ -	$ -
Flexible	LD	P	HF	$ 1,387,625	$ -	$ 9,840,771	$ -	$ -	$ -
Flexible	LD	P	MF	$ 1,387,625	$ -	$ 5,550,502	$ -	$ -	$ -
Flexible	LD	P	LF	$ 1,387,625	$ -	$ 1,237,801	$ -	$ -	$ -
Flexible	LD	nP	HF	$ 1,387,625	$ -	$ 9,840,771	$ -	$ -	$ -
Flexible	LD	nP	MF	$ 1,387,625	$ -	$ 5,550,502	$ -	$ -	$ -
Flexible	LD	nP	LF	$ 1,387,625	$ -	$ 1,237,801	$ -	$ -	$ -

The third step is to create a table of cost schedules for each of these branches using data from Table 2-1, as illustrated in Table E-3.

Table E-21: Cost Schedules for All Branches in the Decision Tree

Node				Time Period					
D1	C1	D2	C2	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5
Fixed	HD	---	HF	$ (490,800)	$ (1,230,800)	$ (1,160,800)	$ (1,160,800)	$ (1,160,800)	$ (670,000)
Fixed	HD	---	MF	$ (490,800)	$ (1,230,800)	$ (1,160,800)	$ (1,160,800)	$ (1,160,800)	$ (670,000)
Fixed	HD	---	LF	$ (490,800)	$ (1,230,800)	$ (1,160,800)	$ (1,160,800)	$ (1,160,800)	$ (670,000)
Fixed	MD	---	HF	$ (490,800)	$ (1,230,800)	$ (1,160,800)	$ (1,160,800)	$ (1,160,800)	$ (670,000)
Fixed	MD	---	MF	$ (490,800)	$ (1,230,800)	$ (1,160,800)	$ (1,160,800)	$ (1,160,800)	$ (670,000)
Fixed	MD	---	LF	$ (490,800)	$ (1,230,800)	$ (1,160,800)	$ (1,160,800)	$ (1,160,800)	$ (670,000)
Fixed	LD	---	HF	$ (490,800)	$ (1,230,800)	$ (1,160,800)	$ (1,160,800)	$ (1,160,800)	$ (670,000)
Fixed	LD	---	MF	$ (490,800)	$ (1,230,800)	$ (1,160,800)	$ (1,160,800)	$ (1,160,800)	$ (670,000)
Fixed	LD	---	LF	$ (490,800)	$ (1,230,800)	$ (1,160,800)	$ (1,160,800)	$ (1,160,800)	$ (670,000)
Flexible	HD	P	HF	$ (541,800)	$ (1,317,800)	$ (1,247,800)	$ (1,247,800)	$ (1,247,800)	$ (706,000)
Flexible	HD	P	MF	$ (541,800)	$ (1,317,800)	$ (1,247,800)	$ (1,247,800)	$ (1,247,800)	$ (706,000)
Flexible	HD	P	LF	$ (541,800)	$ (1,317,800)	$ (1,247,800)	$ (1,247,800)	$ (1,247,800)	$ (706,000)
Flexible	HD	nP	HF	$ (541,800)	$ (541,800)	$ (541,800)	$ -	$ -	$ -
Flexible	HD	nP	MF	$ (541,800)	$ (541,800)	$ (541,800)	$ -	$ -	$ -
Flexible	HD	nP	LF	$ (541,800)	$ (541,800)	$ (541,800)	$ -	$ -	$ -
Flexible	MD	P	HF	$ (541,800)	$ (1,317,800)	$ (1,247,800)	$ (1,247,800)	$ (1,247,800)	$ (706,000)
Flexible	MD	P	MF	$ (541,800)	$ (1,317,800)	$ (1,247,800)	$ (1,247,800)	$ (1,247,800)	$ (706,000)
Flexible	MD	P	LF	$ (541,800)	$ (1,317,800)	$ (1,247,800)	$ (1,247,800)	$ (1,247,800)	$ (706,000)
Flexible	MD	nP	HF	$ (541,800)	$ (541,800)	$ (541,800)	$ -	$ -	$ -
Flexible	MD	nP	MF	$ (541,800)	$ (541,800)	$ (541,800)	$ -	$ -	$ -
Flexible	MD	nP	LF	$ (541,800)	$ (541,800)	$ (541,800)	$ -	$ -	$ -
Flexible	LD	P	HF	$ (541,800)	$ (1,317,800)	$ (1,247,800)	$ (1,247,800)	$ (1,247,800)	$ (706,000)
Flexible	LD	P	MF	$ (541,800)	$ (1,317,800)	$ (1,247,800)	$ (1,247,800)	$ (1,247,800)	$ (706,000)
Flexible	LD	P	LF	$ (541,800)	$ (1,317,800)	$ (1,247,800)	$ (1,247,800)	$ (1,247,800)	$ (706,000)
Flexible	LD	nP	HF	$ (541,800)	$ (541,800)	$ (541,800)	$ -	$ -	$ -
Flexible	LD	nP	MF	$ (541,800)	$ (541,800)	$ (541,800)	$ -	$ -	$ -
Flexible	LD	nP	LF	$ (541,800)	$ (541,800)	$ (541,800)	$ -	$ -	$ -

The forth step is to create a table of nominal profit schedules for each of these branches by subtracting cost in Table E-3 from the revenues from Table E-1 and E-2, as illustrated in Table E-4.

Table E-22: Nominal Profit (Before Tax) Schedules for All Branches in the Decision Tree

Node				Time Period					
D1	**C1**	**D2**	**C2**	**Year 0**	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**
Fixed	HD	---	HF	$ 896,825	$ (476,409)	$ 10,212,893	$ 1,175,373	$ 2,003,936	$ 3,349,215
Fixed	HD	---	MF	$ 896,825	$ (476,409)	$ 5,922,624	$ 1,175,373	$ 2,003,936	$ 3,349,215
Fixed	HD	---	LF	$ 896,825	$ (476,409)	$ 1,609,923	$ 1,175,373	$ 2,003,936	$ 3,349,215
Fixed	MD	---	HF	$ 896,825	$ (727,873)	$ 9,701,919	$ 396,649	$ 949,024	$ 2,009,477
Fixed	MD	---	MF	$ 896,825	$ (727,873)	$ 5,411,650	$ 396,649	$ 949,024	$ 2,009,477
Fixed	MD	---	LF	$ 896,825	$ (727,873)	$ 1,098,949	$ 396,649	$ 949,024	$ 2,009,477
Fixed	LD	---	HF	$ 896,825	$ (979,336)	$ 9,190,945	$ (382,076)	$ (105,888)	$ 669,738
Fixed	LD	---	MF	$ 896,825	$ (979,336)	$ 4,900,676	$ (382,076)	$ (105,888)	$ 669,738
Fixed	LD	---	LF	$ 896,825	$ (979,336)	$ 587,975	$ (382,076)	$ (105,888)	$ 669,738
Flexible	HD	P	HF	$ 845,825	$ (563,409)	$ 10,125,893	$ 1,088,373	$ 1,916,936	$ 3,313,215
Flexible	HD	P	MF	$ 845,825	$ (563,409)	$ 5,835,624	$ 1,088,373	$ 1,916,936	$ 3,313,215
Flexible	HD	P	LF	$ 845,825	$ (563,409)	$ 1,522,923	$ 1,088,373	$ 1,916,936	$ 3,313,215
Flexible	HD	nP	HF	$ 845,825	$ 212,591	$ 10,831,893	$ -	$ -	$ -
Flexible	HD	nP	MF	$ 845,825	$ 212,591	$ 6,541,624	$ -	$ -	$ -
Flexible	HD	nP	LF	$ 845,825	$ 212,591	$ 2,228,923	$ -	$ -	$ -
Flexible	MD	P	HF	$ 845,825	$ (814,873)	$ 9,614,919	$ 309,649	$ 862,024	$ 1,973,477
Flexible	MD	P	MF	$ 845,825	$ (814,873)	$ 5,324,650	$ 309,649	$ 862,024	$ 1,973,477
Flexible	MD	P	LF	$ 845,825	$ (814,873)	$ 1,011,949	$ 309,649	$ 862,024	$ 1,973,477
Flexible	MD	nP	HF	$ 845,825	$ (38,873)	$ 10,320,919	$ -	$ -	$ -
Flexible	MD	nP	MF	$ 845,825	$ (38,873)	$ 6,030,650	$ -	$ -	$ -
Flexible	MD	nP	LF	$ 845,825	$ (38,873)	$ 1,717,949	$ -	$ -	$ -
Flexible	LD	P	HF	$ 845,825	$ (1,066,336)	$ 9,103,945	$ (469,076)	$ (192,888)	$ 633,738
Flexible	LD	P	MF	$ 845,825	$ (1,066,336)	$ 4,813,676	$ (469,076)	$ (192,888)	$ 633,738
Flexible	LD	P	LF	$ 845,825	$ (1,066,336)	$ 500,975	$ (469,076)	$ (192,888)	$ 633,738
Flexible	LD	nP	HF	$ 845,825	$ (290,336)	$ 9,809,945	$ -	$ -	$ -
Flexible	LD	nP	MF	$ 845,825	$ (290,336)	$ 5,519,676	$ -	$ -	$ -
Flexible	LD	nP	LF	$ 845,825	$ (290,336)	$ 1,206,975	$ -	$ -	$ -

The final step is to apply the corporate tax rate (T) of 34.0% (defined in Section 1.2), incorporate inflation rate (I) of 3.4%, and apply a DR of 25%. The following equation can be used to calculate the PV at each period (i):

$$PV = P_{Nominal} \times \frac{(1-T)}{[(1+DR)\times(1+I)]^{i}} \qquad \text{(Eq. E-1)}$$

NPV is determined by summing the PV value in each period, as shown in Table E-5.

Table E-23: Present Value and NPV (After Tax) Schedules for All Branches in the Decision Tree

Node				Time Period						NPV
D1	C1	D2	C2	Year 0	Year 1	Year 2	Year 3	Year 4	Year 5	
Fixed	HD	---	HF	$ 591,905	$ (368,595)	$ 4,034,889	$ 359,276	$ 473,921	$ 612,822	**$ 5,704,217**
Fixed	HD	---	MF	$ 591,905	$ (243,273)	$ 2,339,898	$ 359,276	$ 473,921	$ 612,822	**$ 4,134,549**
Fixed	HD	---	LF	$ 591,905	$ (243,273)	$ 636,045	$ 359,276	$ 473,921	$ 612,822	**$ 2,430,695**
Fixed	MD	---	HF	$ 591,905	$ (371,680)	$ 3,833,014	$ 121,244	$ 224,439	$ 367,683	**$ 4,766,606**
Fixed	MD	---	MF	$ 591,905	$ (371,680)	$ 2,138,024	$ 121,244	$ 224,439	$ 367,683	**$ 3,071,615**
Fixed	MD	---	LF	$ 591,905	$ (371,680)	$ 434,170	$ 121,244	$ 224,439	$ 367,683	**$ 1,367,762**
Fixed	LD	---	HF	$ 591,905	$ (500,087)	$ 3,631,140	$ (116,789)	$ (25,042)	$ 122,545	**$ 3,703,672**
Fixed	LD	---	MF	$ 591,905	$ (500,087)	$ 1,936,149	$ (116,789)	$ (25,042)	$ 122,545	**$ 2,008,681**
Fixed	LD	---	LF	$ 591,905	$ (500,087)	$ 232,296	$ (116,789)	$ (25,042)	$ 122,545	**$ 304,828**
Flexible	HD	P	HF	$ 558,245	$ (287,698)	$ 4,000,517	$ 332,683	$ 453,346	$ 606,235	**$ 5,663,327**
Flexible	HD	P	MF	$ 558,245	$ (287,698)	$ 2,305,526	$ 332,683	$ 453,346	$ 606,235	**$ 3,968,336**
Flexible	HD	P	LF	$ 558,245	$ (287,698)	$ 601,673	$ 332,683	$ 453,346	$ 606,235	**$ 2,264,483**
Flexible	HD	nP	HF	$ 558,245	$ 108,557	$ 4,279,442	$ -	$ -	$ -	**$ 4,946,244**
Flexible	HD	nP	MF	$ 558,245	$ 108,557	$ 2,584,452	$ -	$ -	$ -	**$ 3,251,253**
Flexible	HD	nP	LF	$ 558,245	$ 108,557	$ 880,598	$ -	$ -	$ -	**$ 1,547,400**
Flexible	MD	P	HF	$ 558,245	$ (416,105)	$ 3,798,643	$ 94,650	$ 203,864	$ 361,096	**$ 4,600,393**
Flexible	MD	P	MF	$ 558,245	$ (416,105)	$ 2,103,652	$ 94,650	$ 203,864	$ 361,096	**$ 2,905,402**
Flexible	MD	P	LF	$ 558,245	$ (416,105)	$ 399,799	$ 94,650	$ 203,864	$ 361,096	**$ 1,201,549**
Flexible	MD	nP	HF	$ 558,245	$ (19,850)	$ 4,077,568	$ -	$ -	$ -	**$ 4,615,963**
Flexible	MD	nP	MF	$ 558,245	$ (19,850)	$ 2,382,577	$ -	$ -	$ -	**$ 2,920,972**
Flexible	MD	nP	LF	$ 558,245	$ (19,850)	$ 678,724	$ -	$ -	$ -	**$ 1,217,119**
Flexible	LD	P	HF	$ 558,245	$ (544,512)	$ 3,596,768	$ (143,382)	$ (45,617)	$ 115,958	**$ 3,537,460**
Flexible	LD	P	MF	$ 558,245	$ (544,512)	$ 1,901,777	$ (143,382)	$ (45,617)	$ 115,958	**$ 1,842,469**
Flexible	LD	P	LF	$ 558,245	$ (544,512)	$ 197,924	$ (143,382)	$ (45,617)	$ 115,958	**$ 138,616**
Flexible	LD	nP	HF	$ 558,245	$ (148,257)	$ 3,875,693	$ -	$ -	$ -	**$ 4,285,681**
Flexible	LD	nP	MF	$ 558,245	$ (148,257)	$ 2,180,702	$ -	$ -	$ -	**$ 2,590,690**
Flexible	LD	nP	LF	$ 558,245	$ (148,257)	$ 476,849	$ -	$ -	$ -	**$ 886,837**

E2. Calculating the NPV at Each Node

At each chance node, the NPV of the node is the expected value of the outcomes of that node. Each branch of a chance node is associated with an NPV and a probability of its branches. By summing the product of NPV and probability of each branch, the nodal expected value of the chance node is determined.

At each decision node, the value of the node is the highest NPV among the decision node's branches. Note that the branches that do not possess the highest NPV need to be pruned. The value of each node is calculated in Table E-6.

Table E-24: Expected Value of Each Node in the Decision Tree

C2 Node Expected Value

D1	C1	D2	HF	MF	LF	P(HF/D)	P(MF/D)	P(LF/D)	EV
Fixed	HD	---	$ 5,704,217	$ 4,134,549	$ 2,430,695	0.80	0.10	0.10	$ 5,219,898
Fixed	MD	---	$ 4,766,606	$ 3,071,615	$ 1,367,762	0.33	0.33	0.33	$ 3,068,661
Fixed	LD	---	$ 3,703,672	$ 2,008,681	$ 304,828	0.10	0.10	0.80	$ 815,098
Flexible	HD	P	$ 5,663,327	$ 3,968,336	$ 2,264,483	0.80	0.10	0.10	$ 5,153,943
Flexible	HD	nP	$ 4,946,244	$ 3,251,253	$ 1,547,400	0.80	0.10	0.10	$ 4,436,861
Flexible	MD	P	$ 4,600,393	$ 2,905,402	$ 1,201,549	0.33	0.33	0.33	$ 2,902,448
Flexible	MD	nP	$ 4,615,963	$ 2,920,972	$ 1,217,119	0.33	0.33	0.33	$ 2,918,018
Flexible	LD	P	$ 3,537,460	$ 1,842,469	$ 138,616	0.10	0.10	0.80	$ 648,885
Flexible	LD	nP	$ 4,285,681	$ 2,590,690	$ 886,837	0.10	0.10	0.80	$ 1,397,107

D2 Node Expected Value

D1	C1	D2	D2 Decision
Flexible	HD	$ 5,153,943	Continue
Flexible	MD	$ 2,918,018	Stop
Flexible	LD	$ 1,397,107	Stop

C1 Node Expected Value

D1	HD	MD	LD	Probability	EV	
Fixed	$ 5,219,898	$ 3,068,661	$ 815,098	0.33	$ 3,034,552	
Flexible	$ 5,153,943	$ 2,918,018	$ 1,397,107	0.33	$ 3,156,356	<-- D1 Decision

Appendix F – Lattice Calculations

F1. Setting Up the Lattice Input Variables

The historical data of Figure 5-1 are shown in Table F-1.

Table F-25: Vehicle Registration Data for Greater Boston

Year	V_reg	Year	V_reg
1987	1891001	1997	2158444
1988	1916435	1998	2207747
1999	---	1999	2292119
1990	1906248	2000	2332632
1991	1864022	2001	2372546
1992	1861607	2002	2416851
1993	1878500	2003	2405524
1994	1927985	2004	2472926
1995	1925259	2005	2481813
1996	2077820		

In order to project future vehicle registration from year 2008 to 2012, the number of vehicle registration is assumed to increase at a constant rate (v) from 1987 to 2005. 'v' is used to linear extrapolate vehicle registration from year 2008 to 2012, as shown in Figure F-2. It is calculated to be 1.74%.

Table F-26: Projected Vehicle Registration for Greater Boston

Project Year	Calendar Year	V_{reg}
0	2008 Beginning	2,547,459
1	2008 End	2,580,282
2	2009 End	2,613,105
3	2010 End	2,645,927
4	2011 End	2,678,750
5	2012 End	2,711,573

'a_i' is the forecast in Year 0, which is 2,547,459 vehicles. Since an effective lattice often involves a five or more-period analysis and the startup is interested in NPV maximization within 5 years, 'Δt' is defined as 1 year. In order to determine 'σ', the data needs to be normalized by the following equation:

$$\text{Normalized } V_{reg\_i2} = V_{reg\_i2} / (1 + v)^{i2} \qquad \text{(Eq. G-1)}$$

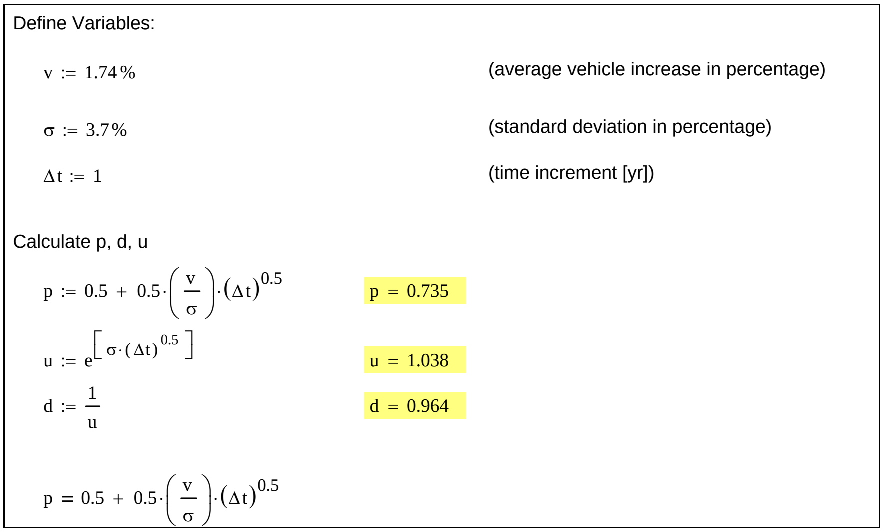
where i2 is the number of years since 1987. The resulting normalized data are shown in Table F-3.

Table F-27: Normalized Vehicle Registration Data for Greater Boston

Year	Year Counter (i2)	V_{reg}	Normalized V_{reg}
1987	0	1,891,001	1,891,001
1988	1	1,916,435	1,883,738
1990	3	1,906,248	1,810,334
1991	4	1,864,022	1,740,030
1992	5	1,861,607	1,708,127
1993	6	1,878,500	1,694,220
1994	7	1,927,985	1,709,184
1995	8	1,925,259	1,677,648
1996	9	2,077,820	1,779,696
1997	10	2,158,444	1,817,211
1998	11	2,207,747	1,827,007
1999	12	2,292,119	1,864,466
2000	13	2,332,632	1,865,048
2001	14	2,372,546	1,864,597
2002	15	2,416,851	1,867,010
2003	16	2,405,524	1,826,555
2004	17	2,472,926	1,845,698
2005	18	2,481,813	1,820,728

'σ' can then be determined by finding the standard deviation of the normalized V_{reg}, and then dividing by the standard deviation with the starting value of 1,891,001 vehicles in 1987. This yields 'σ' of 3.70%.

Finally, u, d, and p are calculated using the MathCad code in Figure E-1 using equations from Eq. 5-1, 5-2, and 5-3. The values are all summarized in Table 5-1.



Define Variables:

$v := 1.74\%$ (average vehicle increase in percentage)

$\sigma := 3.7\%$ (standard deviation in percentage)

$\Delta t := 1$ (time increment [yr])

Calculate p, d, u

$p := 0.5 + 0.5 \cdot \left(\frac{v}{\sigma}\right) \cdot (\Delta t)^{0.5}$ $p = 0.735$

$u := e^{\left[\sigma \cdot (\Delta t)^{0.5}\right]}$ $u = 1.038$

$d := \frac{1}{u}$ $d = 0.964$

$p = 0.5 + 0.5 \cdot \left(\frac{v}{\sigma}\right) \cdot (\Delta t)^{0.5}$

Figure F-6: MathCad Code for Calculating 'p', 'u', and 'd'

F2. Generating Probability Distribution Function for Demand Level

The vehicle registration probability and outcome lattice can be generated by inputting ‘a_i’, ‘p’, ‘d’, and ‘u’ into an online binomial lattice model spreadsheet, as shown in Table F-4 and Table F-5. [15]

Table F-28: Probability Lattice for Vehicle Registered

Year	0	1	2	3	4	5
Probabilities:	1.00	0.58	0.34	0.20	0.11	0.07
		0.42	0.49	0.42	0.33	0.24
			0.18	0.31	0.36	0.34
				0.07	0.17	0.25
					0.03	0.09
						0.01

Table F-29: Outcome Lattice for Vehicle Registered (no. of vehicle registered)

Year	0	1	2	3	4	5
Vehicle Registered:	2547459	2644262	2744744	2849045	2957308	3069686
		2455750	2549069	2645934	2746479	2850845
			2367343	2457303	2550680	2647606
				2282119	2368840	2458856
					2199963	2283561
						2120764

Using the demand equation from Eq. 1-1 and multiplying each figure by the price of subscription ($20/subscription), the outcome lattice for vehicle registered can be converted into the outcome lattice for the demand level, as shown in Figure F-6.

Table F-30: Outcome Lattice for Demand Revenue

Year	0	1	2	3	4	5
Demand Revenue:	$ -	$ 528,852	$1,097,898	$ 1,709,427	$ 2,365,847	$ 3,069,686
		$ 491,150	$1,019,628	$ 1,587,560	$ 2,197,183	$ 2,850,845
			$ 946,937	$ 1,474,382	$ 2,040,544	$ 2,647,606
				$ 1,369,271	$ 1,895,072	$ 2,458,856
					$ 1,759,970	$ 2,283,561
						$ 2,120,764

Each figure of the demand level is plotted against its associated probability. This results in a number of annual demand revenue PDF curves, as shown in Figure 5-2.

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[15] Web resource from: http://ardent.mit.edu/real_options/Real_opts_data/spreadsheets/Excel/binomial%20tree%20model.xls

F3. Developing Nominal Profit Lattice to Calculate NPV for Fixed Deployment Strategy

A lattice of nominal profit for the fixed deployment strategy is created by summing the demand revenue lattice (Table F-6) with funding revenue schedule (medium level, Table 5-3) and deducting the cost schedule (fixed schedule, Table 3-1). The resulting nominal profit lattice is shown in Figure F-7.

Table F-31: Outcome Lattice for Nominal Profit (Fixed Deployment Strategy)

Year	0	1	2	3	4	5
Nominal Profit:	$ 896,825	$ (701,948)	$5,487,599	$ 548,627	$ 1,205,047	$ 2,399,686
		$ (739,650)	$5,409,329	$ 426,760	$ 1,036,383	$ 2,180,845
			$5,336,639	$ 313,582	$ 879,744	$ 1,977,606
				$ 208,471	$ 734,272	$ 1,788,856
					$ 599,170	$ 1,613,561
						$ 1,450,764

The expected value (EV) for each year is calculated by summing the product of each figure in Table F-7 with its associated probability in Table F-1. PV for each period can then be calculated using Eq. E-1. The sum of the PV will then form the NPV of $3.07M, as shown in Table F-32.

Table F-32: Final NPV Calculation (Fixed Deployment Strategy)

Year	0	1	2	3	4	5
EV	$ 896,825	$ (717,745)	$5,422,989	$ 399,971	$ 934,782	$ 1,967,799
EV+Tax	$ 591,905	$ (473,712)	$3,579,172	$ 263,981	$ 616,956	$ 1,298,747
PV	$ 591,905	$ (366,508)	$2,142,503	$ 122,259	$ 221,071	$ 360,057
NPV	**$3,071,288**					

F4: Developing PV Lattice to Calculate NPV for Flexible Deployment Strategy

Using the same methodology as Appendix F3, a nominal profit lattice can be created, as shown in Table F-9.

Table F-33: Outcome Lattice for Nominal Profit (Flexible Deployment Strategy)

Year	0	1	2	3	4	5
Nominal Profit:	$ 845,825	$ (788,948)	$5,400,599	$ 461,627	$ 1,118,047	$ 2,363,686
		$ (826,650)	$5,322,329	$ 339,760	$ 949,383	$ 2,144,845
			$5,249,639	$ 226,582	$ 792,744	$ 1,941,606
				$ 121,471	$ 647,272	$ 1,752,856
					$ 512,170	$ 1,577,561
						$ 1,414,764

In order to incorporate PUT option into the lattice, a PV lattice must be created. The first step is to construct the PV lattice at Year 5. This is simply the nominal profit lattice at Year 5 with tax deduction. This forms the first column of the PV lattice, as shown in Table F-10

Table F-34: Outcome Lattice for PV (Flexible Deployment Strategy) – Only Populated Year 5

Year	0	1	2	3	4	5
					(C1)	$ 1,560,033 (uS)
						$ 1,415,598 (dS)
						$ 1,281,460
						$ 1,156,885
						$ 1,041,191
						$ 933,744

In Table F-10, 'C1' denotes the current cell of interest; 'uS' denotes the upside value of 'C1'; dS denotes the downside value of 'C1'. In order to calculate the PV at 'C1', the cell must contain a formula that relates the DR, I, T, and EV of the next period. In addition, option should be exercised when the expected NPV for next year is less than the NPV of the current year. The following formula is used for the lattice calculation:

$$C1 = \begin{cases} \left[PV\_i + \dfrac{uS \cdot P + dS \cdot (1 - P)}{(1 + DR) \cdot (1 + I)} \right] \cdot (1 - T) & EV(NPV)_{i+1} > NPV_i \\ PV\_i \cdot (1 - T) & \text{Otherwise} \end{cases} \qquad \text{(Eq. G-1)}$$

Reusing the same formula for Year 3 and 4, the semi-populated lattice is shown in Table F-11.

Table F-35: Outcome Lattice for PV (Flexible Deployment Strategy) – Only Populated Year 2-5

Year	0	1	2	3	4	5
		(C2)	$4,801,664 (uS)	$ 1,701,974	$ 1,898,077	$ 1,560,033
			$4,565,845 (dS)	$ 1,456,618	$ 1,678,349	$ 1,415,598
			$4,447,605	$ 1,228,753	$ 1,474,285	$ 1,281,460
				$ 1,327,971	$ 1,284,769	$ 1,156,885
					$ 1,108,764	$ 1,041,191
						$ 933,744

Since the option can only be exercised from Year 2 onwards, the calculations for the PV lattice in Year 0 and 1 can simply use the following equation:

$$C2 = \left[PV\_i + \frac{uS \cdot P + dS \cdot (1 - P)}{(1 + DR) \cdot (1 + I)} \right] \cdot (1 - T) \qquad \text{(Eq. G-2)}$$

The resulting PV lattice is shown in Table 5-4.