Real-Options Analysis: A Luxury-Condo Building in Old-Montreal

Abstract: In this paper, we apply concepts from real-options analysis to the design of a luxury-condo building in Old-Montreal, Canada. We present the conditions that apply to this specific case by simulating, in a realistic manner, the legal, commercial, and environmental constraints that apply to such project. We then simulate the number of condo condos sold as contracts signed during a 12-months period using a uniformly distributed probability function and apply a two-stage decision analysis to determine whether a fixed or flexible approach to design is preferable. We also repeat this decision analysis using lattices to simulate the growth in the number of contracts signed. Results from random simulations show that the flexible design is much more profitable than its fixed counterpart. Results from lattice decision analysis show that both designs are not profitable, but that a European call option "in-project" that takes advantage of a higher than expected number of contracts considerably reduces losses.

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Introduction

The object of this study is to apply concepts in real-options analysis (ROA) to the evaluation of a luxury condo project in Old-Montreal, Canada. It was inspired from a current project that has been going on for the last two and a half years, and where various problems were encountered related to the number of condos sold and construction efficiency.

Therefore, this paper applies concepts in ROA to model some of the features and uncertainties that may be faced during such venture. It also aims at evaluating how flexibility can be integrated into a real estate project, and how useful it might be. The major constraint in this area is that for security issues, construction has to stop once new owners begin entering the building. If the “real option” provided by the flexible design is to be exercised, it has to be done during a certain construction timeframe. Also, most of the figures and metrics stated below are in realistic accordance with the current building legislations in the province of Quebec, and with the numbers provided from the current project.

Basic Data

When a future owner decides to buy a condo, he or she signs a contract with the project entrepreneur to receive delivery at the end of construction, which lasts for 12 months. In this project, the major system concept we are studying is therefore flexibility with regards to the number of contracts that are signed within the 12-months sales period, which corresponds to the duration of construction. The value of the flexibility is calculated by comparing two building designs, one that incorporates the ability to adjust to an uncertain number of contracts, and the other that does not.

Fixed Design Project

The project goal is to build a 24 luxury condo building in Old-Montreal over a 12-months period. The building is defined as a standalone “tower”, with underground parking garage, and only one construction site. At the beginning of construction, 3 contracts have already been signed and analysts project this number to increase by 3 every month for the 12 following months. With this information in hand, the entrepreneur decides to construct a 24 condos building. Should the number of contracts become higher than 24 condos during the construction period, he will not be able to take advantage of it.

Average revenue per condo (because they are not all sold at the same price) is set to 350 000$. The buyer pays an initial deposit at the time where he buys the condo, which corresponds to 10% of the full condo price. The difference is paid at month 12.

Six months after the beginning of construction, heating and electricity costs occur to preserve the structure and maintain the building. On average, these cost are in the order of 5000$ per month until the end of the project. The construction costs is on average 150

000$ per condo. It is also spread evenly over the 12-months period. Buying the construction site costs 1 000 000$, while decontamination and urban "landscape" preservation rules for new buildings cost 100 000$[1].

The discount rate is taken to be 10% for all 12 months of the project. This rate corresponds to the possible rate of return of similarly risky projects, as risk-free interest rates in Quebec are currently fairly low (~5%). Once all condos are sold, the entrepreneur does not own the building any longer and is not responsible for any operating cost.

Flexible Design Project

In this design, flexibility in-project is incorporated to be able to adapt in light of uncertain number of contracts signed. To take this uncertainty into account, the features presented below are added to the model. The entrepreneur has 12 months to sell as many condos as possible. The selling office displays graphical arts of the future condos, as well as floor layouts and architects' plans. It will be possible for potential buyers to visit a "demo" condo after month 6 of construction.

The entrepreneur decides to incorporate flexibility by building a minimum of 18 condos, and a maximum of 30 condos depending the number of contracts that are signed at the end of the sales period. This flexibility incurs additional costs to get a stronger structure (e.g. larger columns, stronger floors, walls, and ceilings), provide the possibility for adding more plumbing and electrical systems, and for building more parking lots if necessary. Another cost is incurred for compensating the entrepreneur for other construction projects he might have to refuse because of the uncertain duration of the project. The cost of the flexibility in the structure is 300 000$, and the compensation to the entrepreneur is 300 000$, for a total cost of flexibility of 600 000$.

If more than 30 condos are built, payment for the first 18 condos is made at month 12, while all extra condos are paid for at month 18. If more than 18 condos are required, the contractor will build one or two series of 6 additional condos (e.g. if number of contracts signed is 19, the contractor will build 6 more condos. If it is 27, it will build 12 extra condos). The "additional owners" pay for the condos at the end of month 18.

The construction duration of 12 months corresponds to the sales period. It is possible to continue construction up to a maximum of 18 months if additional condos have to be built. After this, new owners will start moving into the building and no more construction will be permitted due to safety regulations. Any extra month of construction incurs 3000$ in damage and interest paid to the current owners for every 6 months delay periods, per condo bought (due to Quebec's legislation). For simplicity, the 3000$ will be distributed over the extra 6-months period. Six additional condos cost 10% per condo per series of 6 (165 000$ for the first 6 extra condos and 176 000$ for 12 extra condos). If the total number of contracts signed is increased over 18, the compensation and maintenance

[1] In this case, the City of Montreal demands that entrepreneurs build new buildings in accordance to specific design rules, materials, etc.

costs are automatically added to the total costs at the end of the 12th month up to month 18. The simulation of number of contracts signed is capped to a maximum of 30.

Uncertainties and Metrics

In this design project, the main uncertainty is obviously the number of signed contracts. Especially in the current economical context prevailing in Montreal, construction has stopped to increase and has been stagnating during the last year. For the part of the analysis presented below where random simulation of the number of contracts is used, it will be simulated with a uniformly distributed random number varying between 0 and 3 every month for the 12-months duration of the initial sales/construction period. The numbers 0 and 3 are arbitrary and reflect a certain level of effectiveness in signing new contracts each month. They are also based on current numbers obtained from the project that motivates this paper. Each 12-months sales period gives a final random number of signed contracts that is obtained by adding all the contracts signed in each month. The outcome of one 12-months sales period represents 1 possible scenario. We randomly simulate 1000 such scenarios.

To simplify the simulation, we assume that once a contract is signed, it cannot be revoked. Note that this assumption will not hold when growth in the number of contracts signed will be simulated using a lattice. This will result in tangible difference in the profitability of each project, depending on whether we use a two-stage decision analysis with random simulation or lattice to evaluate the value of the flexibility.

Structure of the Paper

The paper is structured as follows. In the subsequent section, a two-stages decision analysis using random number of signed contracts is presented. The performance of both designs is compared based on profits generated. In the next section, we model the number of contracts signed using a slightly different approach, which involves lattice development. In the following section, we apply a decision analysis framework on our lattice development to see the range of profits that are generated when the number of signed contracts is simulated through a lattice. Finally, we conclude this paper with a few remarks on the results obtained through the various approaches.

Two-Stages Decision Analysis

In this section, we use a two-stage decision analysis approach to evaluate the performance of both fixed and flexible designs based on profit. For both designs, we assume the same random number of signed contracts described above. On one side, the fixed design cannot adapt to changes in the number of signed contracts, while the flexible one provides the possibility to build additional condos if necessary (either 6 or 12 more on top of the original 18 condos). This number of signed contracts is modeled as described in the previous section. The fixed design builds 24 condos, while the flexible design builds fewer condos at the beginning (18), invests in flexibility to be able to build more condos, and eventually decided whether to expand or not depending on the final number of contracts signed at the end of the 12-months sales period.

Therefore, based on 1000 simulations with the above designs, Figure 1 shows the decision-tree associated with our two-stages decision analysis:

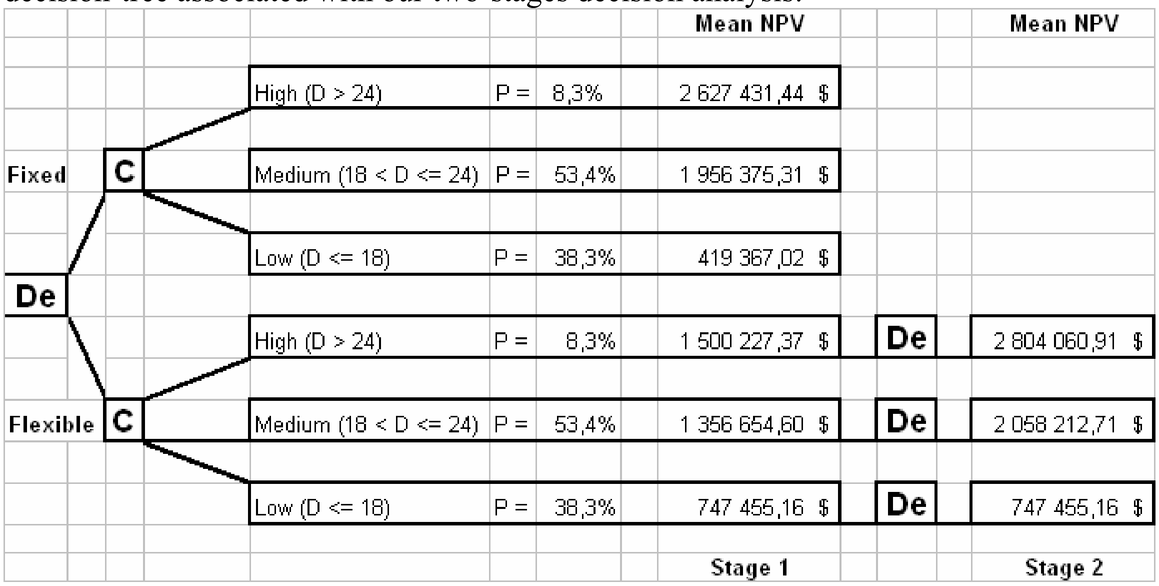


Figure 1: Two-stages decision-tree for decision analysis of fixed vs. flexible systems design. "De" stands for a decision-node, "C" represents a chance node, "D" represents the number of signed contracts, while P is the probability associated with a given interval of signed contracts.

To exemplify our calculations, we walk through a possible path. Suppose at the first decision node "De", decision is made to take a flexible design. Than 1000 scenarios (or 12-months sales period) are simulated, which corresponds to the chance node "C". We then sample the number "D" of contracts signed according to three intervals: high (D > 24), medium (18 < D ≤ 24), and low (D ≤ 18). Each interval has a probability "P" associated with it, which corresponds to the number of scenarios that created the number of signed contracts corresponding to the interval, divided by 1000. For example, in case a high number of contracts is signed, P = 8,3% means that in 8,3% of the 1000 scenarios, more than 24 contracts were signed. After Stage 1, which we just described, the decision to expand or not is made, which corresponds to the second decision node "De".

On Figure 1, we note that after the second decision node of the flexible design there is no subsequent chance node. The reason for this is inherent to the design of our project, which is similar to a European call option. The decision to expand or not is done at month 12 exactly after the sales period is over. The decision to expand is done only once at that specific time, and depends on how many contracts were signed during the sales period. If the number of contracts signed is 18 or below, no expansion occurs. If it is between 19 and 24, we expand and build 6 more condos, irrespective of how many contracts were signed. If it is between 25 and 30, we expand to 12 more condos. This constraint is specific to this project, where construction cannot be continued once new owners have moved definitely into the building. The expected net present values (NPVs) are calculated for the two periods in Table 1 and Table 2.

Table 1: Probability and mean Net Present Value (NPV) for various levels of contracts signed (low, medium, high) for the fixed (a) and flexible (b) designs. The expected value at the end of Stage 1 (see Figure 1) is also shown.

Fixed Case (1st period)		
Number of contracts signed	Probability	Mean NPV
Low (D <= 18)	38,3%	419 367,02 $
Medium (18 < D <= 24)	53,4%	1 956 375,31 $
High (D > 24)	8,3%	2 627 431,44 $
Expected NPV	1 423 398,79 $	

(a)

Flexible Case (1st period)		
Number of contracts signed	Probability	Mean NPV
Low (D <= 18)	38,3%	747 455,16 $
Medium (18 < D <= 24)	53,4%	1 356 654,60 $
High (D > 24)	8,3%	1 500 227,37 $
Expected NPV	1 135 247,75 $	

(b)

Table 2: Probability and mean NPV for various levels of contracts signed. The expected value at the end of Stage 2 (see Figure 1) is also shown.

Flexible Case (2nd period)		
Number of contracts signed	Probability	Mean NPV
Low (D <= 18)	38,3%	747 455,16 $
Medium (18 < D <= 24)	53,4%	2 058 212,71 $
High (D > 24	8,3%	2 804 060,91 $
Expected NPV	1 618 097,97 $	

From Table 1, we see that after a period of 12 months (Stage 1), the fixed design has generated a higher expected NPV (1 423 398,79 $ vs.1 135 247,75 $). That is, the price paid to add flexibility to the flexible design is counterbalanced by the lower amount of initial capital investment (i.e. 18 condos were built instead of 24 for the fixed case) but a fixed 24 condos building is still more profitable. On the other hand, once decision has been made to build additional condos in the 2nd stage of the flexible design, the expected NPV increases from 1 135 247,75 $ to 1 618 097,97 $. This represents an increase of 482 850,21 $ with respect to the expected value for the 1st stage of the fixed design. The value of the European call option is therefore the expected NPV for the flexible case at the end of the 2nd period and the expected NPV for the fixed case at the end of the 1st period: 1 618 097,97 $ - 1 423 398,79 $ = 194 699,17 $.

Lattice Development of Number of Signed Contracts

The major uncertainty in this project is the number of contracts signed within the 12-months sales period. Table 3 summarizes the values that were assigned in this particular project. Note that the volatility (σ) associated with the number of signed contracts was assigned a high value of 50% per month because it is highly volatile, while the projected growth of 3 contracts signed per month was assigned to remain coherent with the assumptions presented earlier. We have divided the 12-months sales period into the 6 required periods that cover exactly 2 months each.

Table 3: Summary of assigned values for the lattice modeling of contracts signed.

Assigned Values	
Initial number of contracts signed	3
Projected number of contract growth (per month)	3
Duration of sales period (months)	12
Total number of periods	6
Number of periods between 2 periods (Δt)	1
Initial probability	100%
Volatility of number of contracts signed (σ) per month	50,0%

The growth rate per period of 2 months (v) as well as the values u, d, and p are presented in Table 4. The growth rate (v) was calculated as the projected number of contracts growth per period divided by the total projected number of contracts for the 12 months sales period. The values u, d, and p were calculated using the following equations:

$$u = e^{\sigma\sqrt{\text{Vt}}}$$

$$d = e^{-\sigma\sqrt{\text{Vt}}}$$

$$p = 0.5 + 0.5(v/\sigma)\sqrt{\text{Vt}}$$

Table 4: Values calculated from the assigned values in Table 3 used to create the outcome and probability lattices.

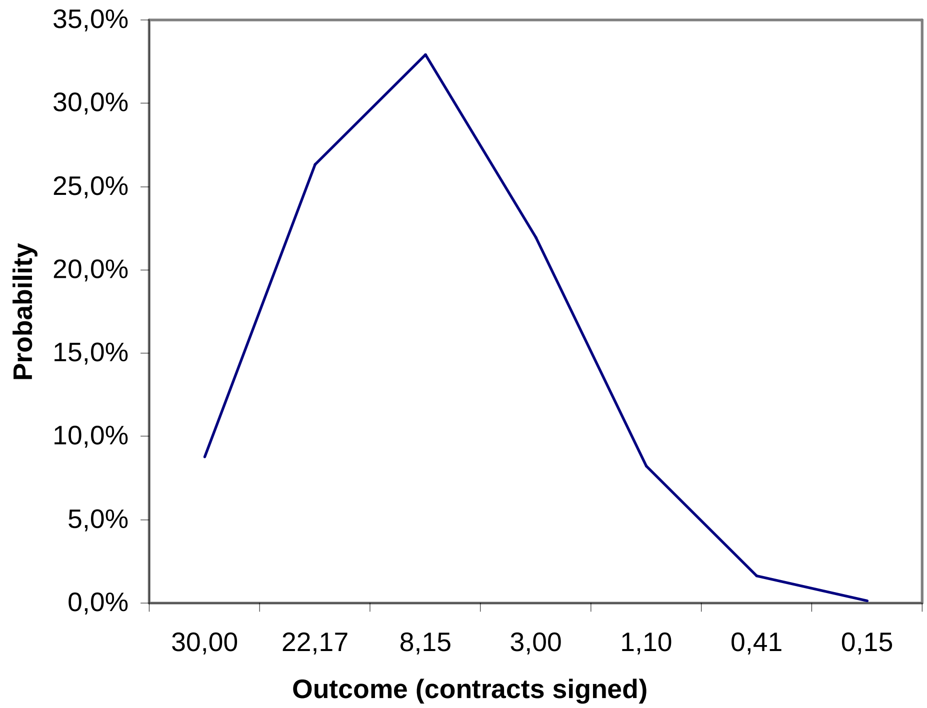
Calculated Values	
Number of months per period (months)	2
Projected number of contracts growth per period	6
Total projected number of contracts	36
Growth rate per period (v)	16,7%
Volatility of number of contracts signed (σ) per period	70,7%
u	1,649
d	0,607
p	66,7%

The outcome lattice in signed contracts is shown in Table 5 with the corresponding probability lattice. Figure 2 also displays the probability density function associated with the outcome lattice and probabilities in Period 6. Note that the number of contracts signed is capped to 30 condos in Table 5 as it is the maximum number of condos that will be built in the flexible design project in any case.

Table 5: Lattice development of number of contracts signed for the condo project using the values presented in Table 4. The corresponding probability lattice is also shown. (mo = months)

	P0 (0 mo)	P1 (2 mo)	P2 (4 mo)	P3 (6 mo)	P4 (8 mo)	P5 (10 mo)	P6 (12 mo)
Contracts signed:	3,00	4,95	8,15	13,45	22,17	30,00	30,00
		1,82	3,00	4,95	8,15	13,45	22,17
			1,10	1,82	3,00	4,95	8,15
				0,67	1,10	1,82	3,00
					0,41	0,67	1,10
						0,25	0,41
							0,15
Probabilities:	100,0%	66,7%	44,4%	29,6%	19,8%	13,2%	8,8%
		33,3%	44,4%	44,4%	39,5%	32,9%	26,3%
			11,1%	22,2%	29,6%	32,9%	32,9%
				3,7%	9,9%	16,5%	21,9%
					1,2%	4,1%	8,2%
						0,4%	1,6%
							0,1%

Figure 2: Probability density function associated with the above lattice for Period 6.



Decision Analysis using Lattice

In this section, we investigate the exercise of a European call option "in-project". That is, we explore the capability of taking advantage of upside potential for profit provided by a higher number of signed contracts than expected. This flexibility is provided by the stronger structure built into the original design of our condo building, and by payment of compensating fees to the contractor for the uncertainty in duration of the project.

The parameters to the decision lattice, the modeled number of signed contracts as well as the associated probabilities are presented in Table 3 and Table 4. The net revenues in the case where no flexibility is built into the system is shown in Table 6a, while the flexible case is shown in Table 6b. Note that most revenues are not generated until month twelve in both cases when owners pay the remaining cost of the condo (315 000$) after an initial 10% deposit of 35 000$. Additional revenues at month 18 are generated when additional owners (those that bought their condo after the initial 18 were sold) also make the final payment on their respective condo.

Table 6: Net revenues for both fixed (a) and flexible (b) designs

	P0 (0 mo)	P1 (2 mo)	P2 (4 mo)	P3 (6 mo)	P4 (8 mo)	P5 (10 mo)	P6 (12 mo)
Net Revenue:	-1 205 000,00 $	-531 884,27 $	-487 696,14 $	-424 842,24 $	-304 726,46 $	-335 850,89 $	8 840 000,00 $
(fixed case)		-536 314,28 $	-558 685,72 $	-541 884,27 $	-497 696,14 $	-424 842,24 $	6 677 931,55 $
			-561 372,66 $	-584 941,62 $	-568 685,72 $	-541 884,27 $	2 071 080,19 $
				-586 571,33 $	-594 801,33 $	-584 941,62 $	376 314,28 $
					-595 789,80 $	-600 781,54 $	-247 155,25 $
						-601 381,08 $	-476 516,88 $
							-557 723,58 $

(a)

	P0 (0 mo)	P1 (2 mo)	P2 (4 mo)	P3 (6 mo)	P4 (8 mo)	P5 (10 mo)	P6 (12 mo)
Net Revenue:	-1 730 000,00 $	-381 884,27 $	-337 696,14 $	-274 842,24 $	-154 726,46 $	-185 850,89 $	10 323 204,34 $
(flexible case)		-386 314,28 $	-408 685,72 $	-391 884,27 $	-347 696,14 $	-274 842,24 $	6 744 820,29 $
			-411 372,66 $	-434 941,62 $	-418 685,72 $	-391 884,27 $	2 221 080,19 $
				-436 571,33 $	-444 801,33 $	-434 941,62 $	526 314,28 $
					-445 789,80 $	-450 781,54 $	-97 155,25 $
						-451 381,08 $	-326 516,88 $
							-407 723,58 $

(b)

Deciding whether to exercise the European call option is also important, and it happens only once at the end of month 12. In the flexible case, the option to expand is exercised at month 12 if and only if the number of signed contracts is above the original number of 18 condos that have been built. Hence, the criterion for exercising is not based on revenues but rather on how much contracts were signed at the end of month 12. Table 7 shows the period corresponding to the various possible states of number of signed contracts where the option can be exercised.

Table 7: Decision lattice showing when the call option should be exercised to expand and build additional condos. Notice that exercise can only happen once at the end of month 12, similar as a European option.

	P0 (0 mo)	P1 (2 mo)	P2 (4 mo)	P3 (6 mo)	P4 (8 mo)	P5 (10 mo)	P6 (12 mo)
Exercise Call Option:	NO	NO	NO	NO	NO	NO	YES
(based on demand,		NO	NO	NO	NO	NO	YES
not revenues)			NO	NO	NO	NO	NO
				NO	NO	NO	NO
					NO	NO	NO
						NO	NO
							NO

In order to calculate the NPV for the fixed and flexible cases, we weigh the net revenues presented in Table 6 by their corresponding probability and find an expected value for each of the six periods. We then discount the expected net revenues for the 12 months period of the project to find the NPV using the discount rate of 10% distributed over the 6 periods. Table 8 shows those results for both cases and demonstrates that both projects are losing money. However, the flexible design project reduces losses.

Table 8: Expected, present value and net present value obtained by weighting the various states in each period by the associated probability, and discounting for each 2 months period at r = 10%. The results from the non-flexible case are shown in a), while those for the non-flexible case are shown in b).

Month	0	2	4	6	8	10	12
E[Net Revenues]	-1 205 000,00 $	-533 360,94 $	-527 433,34 $	-518 428,53 $	-491 414,29 $	-485 976,17 $	3 270 352,74 $
PV[Net Revenues]	-1 205 000,00 $	-524 581,48 $	-510 212,49 $	-493 246,65 $	-459 848,51 $	-447 274,08 $	2 960 363,95 $
NPV (fixed case)	-679 799,27 $						

(a)

Month	0	2	4	6	8	10	12
E[Net Revenues]	-1 730 000,00 $	-383 360,94 $	-377 433,34 $	-368 428,53 $	-341 414,29 $	-335 976,17 $	3 515 507,36 $
PV[Net Revenues]	-1 730 000,00 $	-377 050,58 $	-365 110,03 $	-350 532,67 $	-319 483,70 $	-309 219,76 $	3 182 280,96 $
NPV (flexible case)	-269 115,78 $						

(b)

From these Tables, we calculate the value of the call option as:

NPV(flexible) – NPV(non-flexible) = -269 115,78 $ -(-679 799,27 $) = 410 683,49 $

We see that adding flexibility to the project reduces losses by an expected amount of 410 683,49 $. However, this difference may not be worth enough for an entrepreneur to invest in the flexibility as both projects themselves show negative NPVs and most likely will not be deemed worth undertaking.

Concluding Remarks

This project shows that a flexible approach to design can be beneficiary to the domain of real estates. The specific constraints imposed to this model were chosen to resemble those of a project currently being completed in Old-Montreal. It is possible and realistic that these constraints lead to net losses and negative net present values, showing the difficulty of making profit in that sector of economic activity. However, both two-stages and lattice decision analyses showed that flexibility can certainly increase prospects for higher profits by taking advantage of higher than expected number of signed contracts. Hence, different sets of constraints could very well lead to profitable projects where flexibility in design adds to the value of the project. This "in-project" flexibility is different from the "on-project" flexibility that is often exploited in the domain of real estate and building construction. It exploits flexibility in the design of the building itself instead of taking advantage of multiple sites when the number of contracts increases, which is already done quite frequently by entrepreneurs.

As introduced earlier, the discrepancy in profitability results between the two-stages and lattice decision analyses comes from the fact that in the first case our simulation was using a uniform probability distribution, and because we did not allow for contracts to be revoked. In the second case where lattice development was used to simulate the number of contracts signed, we allowed for contract revocations and simulated the contract growth using an exponential function, which led to a lognormal probability distribution of number of contracts signed (see Figure 2). Future work should be devoted to allow for contract revocation in the two-stages decision analysis, with similar probability distribution, volatility and mean as used for the lattice development case to see if similar profitability results can be obtained.

This application gave me good hands-on experience to search flexibility in a specific project and evaluate whether it is worth investing in it or not. It paved the way to new approaches in the practice of engineering and outlined an exciting new paradigm in systems research that I will continue to explore during my master's degree.