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Flexible Design of Space Shuttle External Tank
Application Portfolio
Engineering Systems Analysis for Design
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Abstract

Many of the topics presented in the Engineering Systems Analysis for Design class are applied to the flexible design of the external tank of the NASA Space Shuttle. The design variables are the physical dimensions of the tank. Both the price of aluminum and the required payload are uncertain. This analysis concentrates on the variation in payload. Payload has varied significantly historically, and depends on an uncertain political and scientific climate. A Microsoft Excel spreadsheet calculates the profit per launch as a function of the design variables, the aluminum costs, the payload, and other relevant parameters. Three possible designs are presented: a large fixed tank capable of lifting payloads up to 30000 kg, a small fixed tank capable of lifting payloads up to 15000 kg, and a flexible segmented design which can be adapted to be used as either a small or a large tank. A two-stage decision analysis is performed, and shows that the best initial strategy is to use the flexible tank. A lattice model is created and exercised, and values the flexibility at \$0.9 million. A brief evaluation of the pedagogical value of this project concludes that the application was challenging but a good introduction to the practical applications of the techniques.

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1. System definition



Figure 1: Space shuttle external fuel tank. Public domain image from www.nasa.gov

The system under consideration is a redesigned space shuttle external fuel tank, shown in Figure 1. This is a tank attached to the NASA space shuttle, containing both hydrogen and oxygen used by the main space shuttle engines. It is jettisoned after the main engines shut down. It must contain adequate fuel to lift itself, the space shuttle, and the contents of the space shuttle (the payload) to the orbital altitude while overcoming its own drag plus the drag of the shuttle. The tank is modeled as a cylinder with a cone on one end and a hemisphere on the other. Figure 2 shows the nominal design.

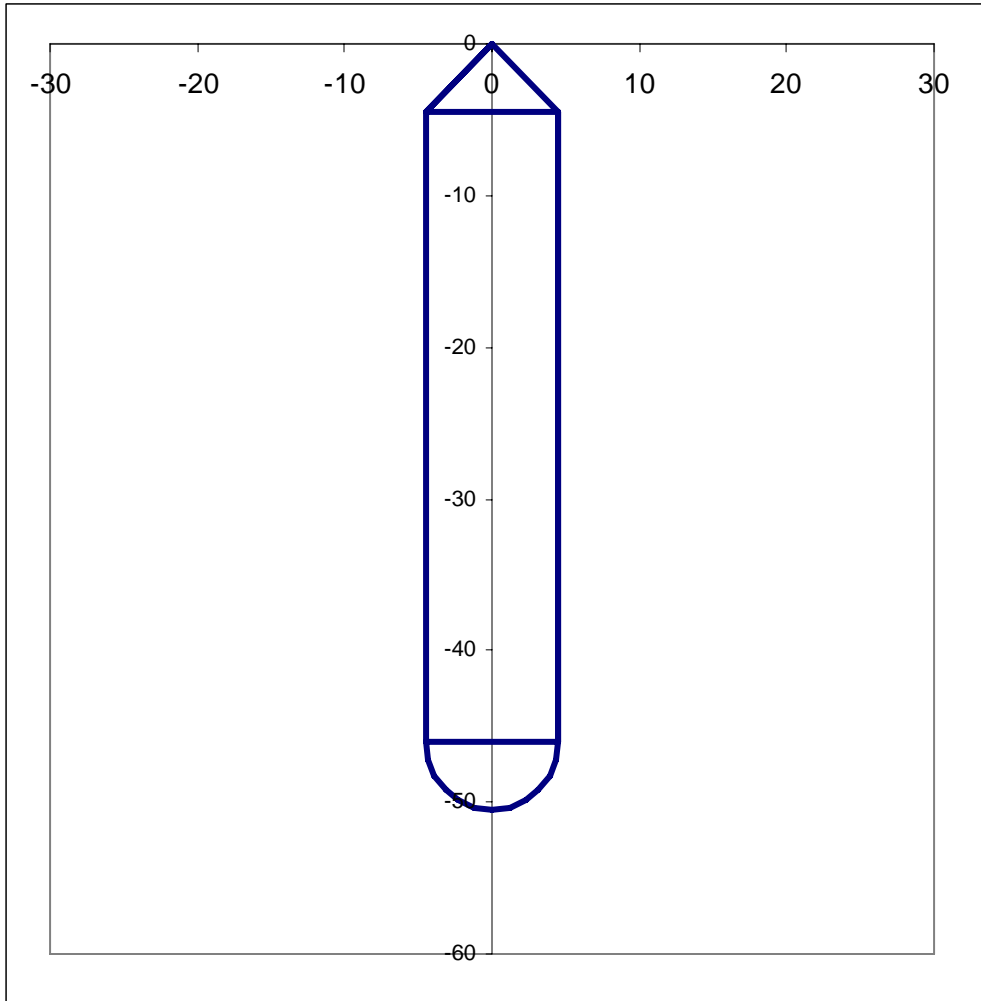


Figure 2: Nominal tank design

The design parameters are the physical dimensions of the system. Table 1 shows the design variables, other parameters, and the output values generated by the model.

DESIGN VARIABLES

Description	Nominal value	Unit
Length of cylindrical portion		41.5m
Tank radius		4.5m
Wall thickness of cylindrical portion		0.7cm
Wall thickness of hemispherical portion		0.8cm
Wall thickness of conical portion		0.75cm
Cone height		4.5m

OTHER PARAMETERS

Description	Nominal value	Unit
Aluminum alloy density		0.0028kg/cm ³
Cost of aluminum alloy		6 \$/kg
Cost of seam manufacture		12\$/m
Allowable stress in walls		40000N/cm ²
Tank pressure		70N/cm ²
Design payload		30000kg
Orbit height		250km
Orbital speed		8km/s
Fixed payload launch costs (incl. nom. tank)		20000\$/kg
Charge to customer to launch payload		21000\$/kg

OUTPUT VALUES

Description	Nominal value	Unit
Tank volume		2926.4m ³
Tank weight		27737.8kg
Tank cost		511424.2\$
Delta payload (from nominal) due to cone drag		0kg
Payload launched		30000kg
Total charge to customer		630000000\$

Table 1: Design variables, parameters, and outputs of the tank model

The designer aims to maximize the profit per flight: the revenue from lifting the payload minus the costs. The design is constrained by the total allowable stresses and by a vibration constraint on the first bending mode of the tank. Of course, it is also constrained by the physical laws of gravity and the rocket equation.

The uncertainties are reflected in the “other parameters” listed in the table. For example, the cost of the aluminum alloy may be uncertain due to fluctuating aluminum prices, or the labor costs may change, affecting the cost of seam manufacture. The customer (NASA) may change the required payload, orbital parameters, or amount paid per kilogram launched. These uncertainties are discussed in the next section.

The model was developed by O. De Weck and K. Willcox for their multidisciplinary system design optimization class. It is implemented in Microsoft Excel.

2. Salient uncertainties

2.1. Payload

The space shuttle program operates in an inherently uncertain environment. One of the major uncertainties affecting the design of the tank is the uncertainty of the mission. As political leaders change, the goals of NASA's space mission change. Currently, NASA is planning to build a new manned vehicle to replace the shuttle for many of the longer-range and scientific missions. Thus, the space shuttle may be relegated to service missions for the International Space Station (ISS). In fact, NASA's current policy is to only allow shuttle missions to the ISS, where it can be inspected externally.

As an indicator for the variability of the mission, we can use mission payload. The payload has a direct effect on the design of the external tank: the higher the payload, the larger the required volume of the tank.

The payload of the space shuttle missions have varied significantly over the life of the shuttle, since the first launch in 1981. The minimum payload was 1801 kg, a space-station service mission in 2000, and the maximum was STS-93's launch of the 22781 kg Chandra X-ray observatory in 1999. A plot of the known payloads over time since the inception of the program is shown in Figure 3.

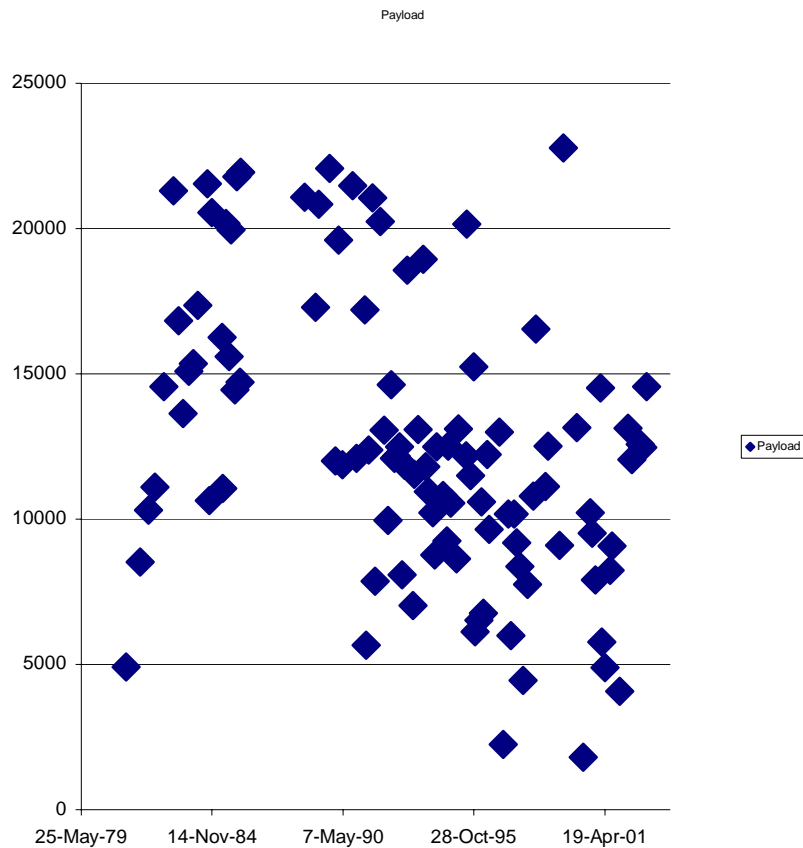


Figure 3: Payloads of space shuttle missions (Data source: Wikipedia, the free encyclopedia, en.wikipedia.org)

Note that the payload of the nominal tank is 30000 kg, greater than that of any mission to date. The tank is thus overdesigned: it is large enough to carry any foreseeable payload, but it often carries much less.

The plot does not show a clear time trend to the data. A linear trend could be fit, but most of the data would lie far from the line. In addition, it cannot be assumed that the changes are due to randomness: they depend on the uncertain political and scientific climate.

I have divided the payload into two categories: light, defined as less than 15000 kg, and heavy, which is between 15000 and 30000 kg. It is assumed that the payload is never greater than 30000 kg. The metric I will be using is the fraction of the flights in a given year that fall into the “heavy” category. The reason for this division is that different tank designs could be appropriate for different classes of payload weight.

Table 2 shows approximate 5-year periods (with the exception of the last, which is shorter) and the number of flights during that period, along with what fraction of those flights was in the “heavy” category.

From	To	Flights	Heavy	
12-Apr-81	28-Jan-86	25	56.5%	
29-Sep-88	5-Apr-91	14	70.0%	
28-Apr-91	19-May-96	28	14.3%	
20-Jun-96	8-Mar-01	16	7.1%	
19-Apr-01	16-Jan-03	11	0%	

Table 2: Percentage of heavy flights over 5-year periods

Over the entire space shuttle program, 27% of the flights have been heavy. While it seems that the percentage of heavy flights is decreasing, the small amount of data does not allow for predictions with any level of confidence. The uncertainty will be characterized in different ways throughout this document appropriate to the method of analysis used.

2.2. Aluminum costs

The cost of the raw aluminum in the nominal tank is 32% of the total cost to manufacture the tank; therefore, variations in this cost have a significant effect on the final cost. Aluminum prices have varied significantly over the last 150 years (I have data dating to 1850, when aluminum was \$17/lb, equivalent to \$295 in 2004 dollars) but I will only use the last 50 years to predict the future price. These last 50 years are shown in Figure 4, with both the actual cost and the costs inflation-adjusted to 2004 dollars.

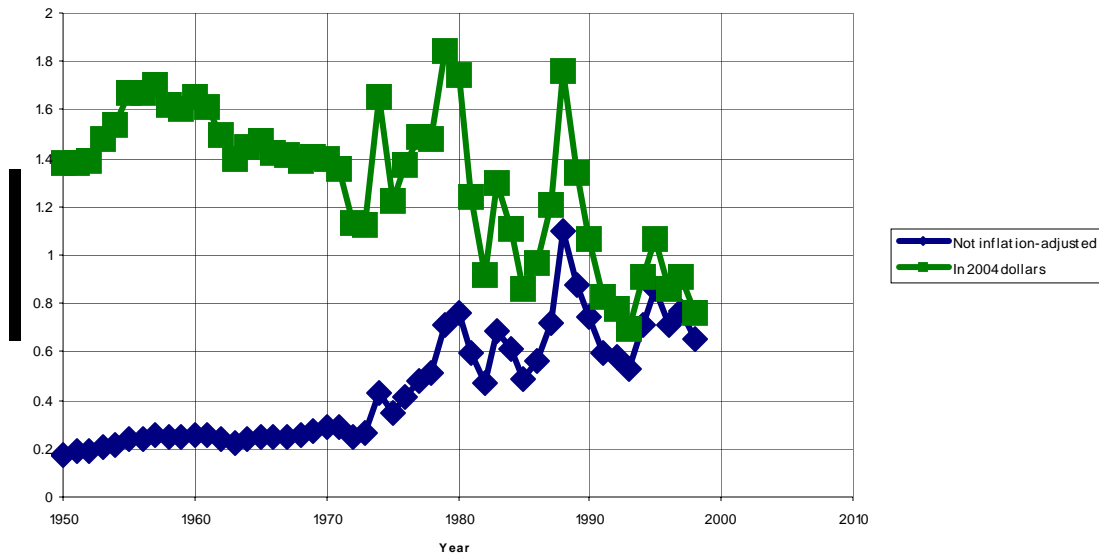


Figure 4: Historical Aluminum Prices. Source: "Metal Prices in the U.S. through 1998", U.S. Geological Survey, 2004, available http://minerals.usgs.gov/minerals/pubs/metal_prices/metal_prices1998.pdf

The price of aluminum depends on a large number of factors, including the state of the world economy and the cost of energy (aluminum smelting requires large amounts of energy). Figure 4 shows the results of price controls in the early 1970s, wars, recessions, and oil crises. These factors are inherently uncertain. Therefore, since very little is known about the future, I will use past data as a guide for the predictions of the future price. I will use the non-inflation adjusted numbers for prediction, since the income to the tank manufacturers is set in a non-inflation-pegged contract price, and inflation is also uncertain.

First, I found an exponential fit to the data, shown on the red line in Figure 5. This curve represents a growth rate of 3.39% annually. The volatility (the standard deviation of the yearly growth rates) over this period is 18.8%.

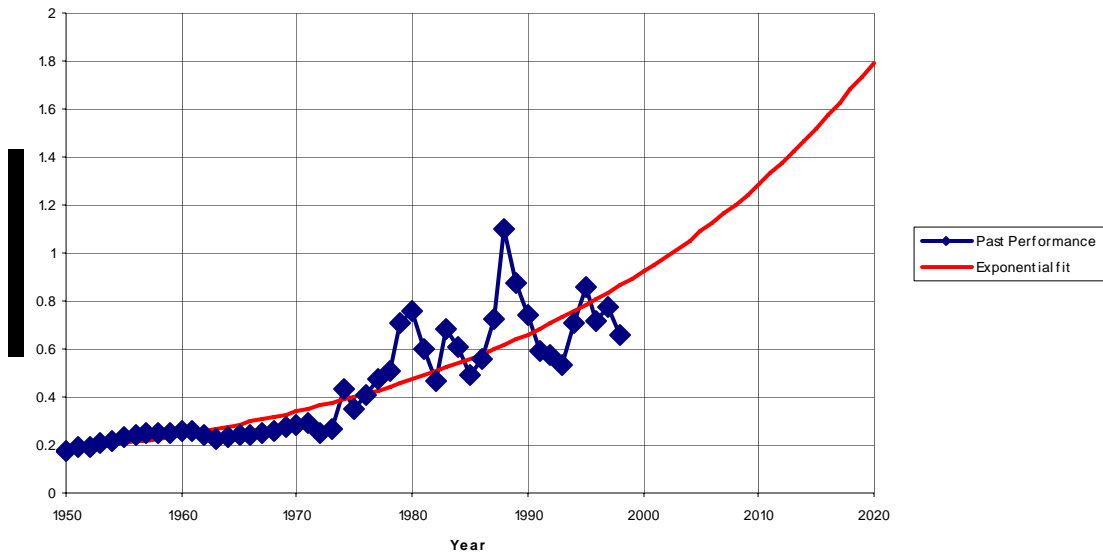


Figure 5: Aluminum price prediction

3. Designs under consideration

Three possible systems are considered. Two are fixed, and optimized for light and heavy payloads respectively. The third is a flexible design. The designs are described in Table 3 and shown in Figure 6. In Table 3, L is the length of the cylindrical section, R is the radius of the tank, t_1 , t_2 , and t_3 are the wall thicknesses of the cylindrical, hemispherical, and conical portions respectively, and h is the height of the cone.

	L cm	R cm	t ₁ cm	t ₂ cm	t ₃ cm	h/R	Cost	Profit per light launch	Profit per heavy launch
Large fixed	9223	344	0.175	0.2	0.1875	3.524	\$752,626	\$514,508,880	\$ 534,508,880
Large flexible	11409	311	0.175	0.2	0.1875	4.755	\$1,070,684	\$405,311,295	\$ 425,311,295
Small fixed	7468	278	0.175	0.2	0.1875	3.778	\$602,047	\$710,784,964	\$ 0
Small flexible	5704	311	0.175	0.2	0.1875	4.755	\$658,718	\$604,777,356	\$ 0

Table 3: Description of Possible Designs

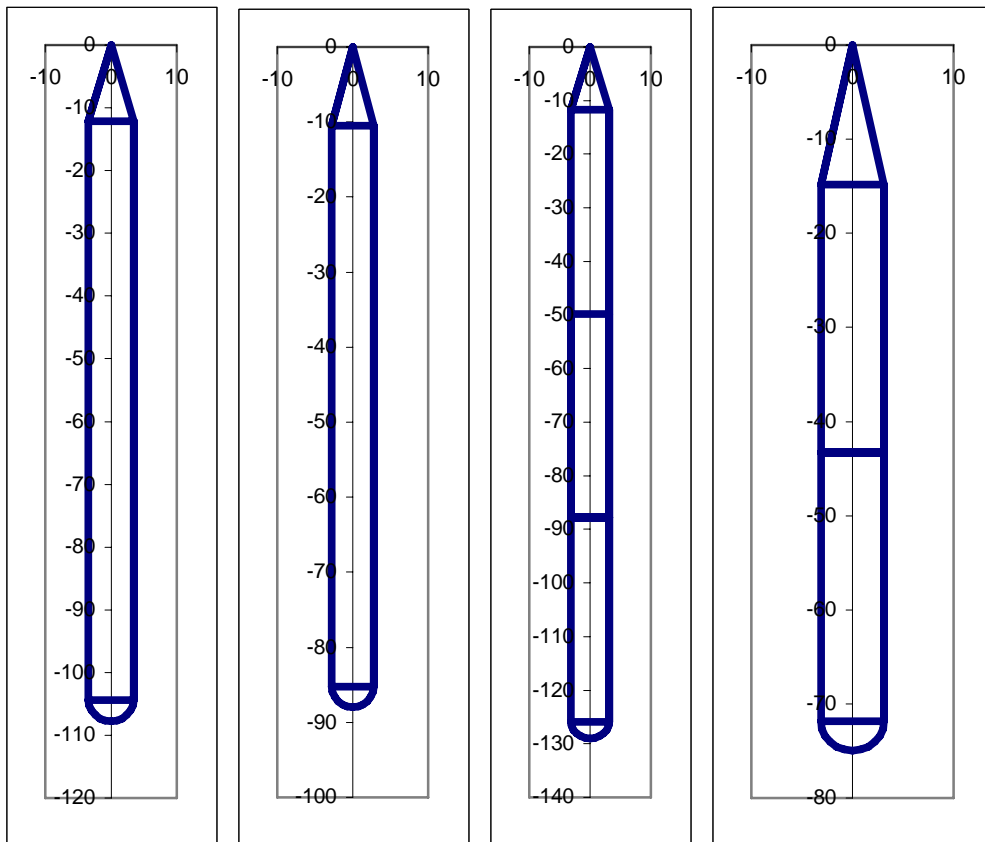


Figure 6: Four designs: large fixed, small fixed, large flexible, small flexible

The flexible design is a segmented tank. In this design, both the small and large versions are constrained to have the same radius, and to use the same hemisphere and cone portions of the tank. Figure 6 shows that the flexible small tank is shorter and fatter than the fixed tank, and that the flexible large tank is longer and thinner than the

equivalent fixed tank. This is due to the constraint that both flexible tanks have the same radius.

The design variables of the fixed tank are chosen to maximize the profit for the upper end of the payload range (15000 kg for the small tank and 30000 kg for the large tank). For the flexible tank, an average profit of light and heavy payloads is maximized, weighted at 70% light and 30% heavy.

3.1. Modeling of costs and profits

The costs of the system are analyzed by the spreadsheet program. The major costs of the tank manufacturing are the aluminum and the seams. The flexible design is more costly to manufacture than the fixed designs because of the extra seams.

Another factor taken into account in the analysis is the change in aerodynamic drag with the design of the tank. The flexible design has a larger drag than the fixed design, because the fixed design is optimized for the specific payload and the flexible design has more constraints (specifically that the large and small flexible designs use the same radius and the same cone). This aerodynamic drag is modeled as an “extra payload” the tank must accommodate.

The revenue for the system is the payload multiplied by \$21000/kg. This is approximately what NASA currently pays. The profit is the revenue minus the costs.

4. Two-stage decision analysis

A two-stage decision analysis was performed to determine the optimal strategy for the choice of the tank design. In this model, it is assumed that once a fixed design is chosen, a fixed design must always be used. It is possible to move from a flexible segmented design to a fixed design in either of two sizes, but it is not possible to move from a fixed to a flexible design.

4.1. Scenario Descriptions

Because the decision tree method requires discrete states, the continuous distribution of the percentage of heavy flights is reduced to three possible scenarios, described below. Each scenario describes the percentage of heavy flights in a year.

4.1.1. Scenario A

In this scenario, 90% of the of the flights fall into the light category, and 10% fall into the heavy category. This is assumed the most probable initial scenario, with a 50% probability.

4.1.2. Scenario B

This scenario has a more balanced distribution: 70% of the flights will be light, and 30% will be heavy. There is an assumed 30% chance of this scenario in the first period.

4.1.3. Scenario C

In the third scenario, 50% of the flights are light, and 50% are heavy. Scenario C has a probability of 20% in the first period.

4.1.4. Second time period

It is assumed that the first period gives us some, but not all, information about the second period. That is, it is always quite likely that the second period will have the same distribution as the first. Therefore, if the first period is Scenario A, there is an 85% chance that the second period will also have Scenario A. There is a 10% chance it will move to Scenario B and a 5% Chance of moving to Scenario C. Similarly, if the first period is Scenario B, it is most likely, 80%, to stay in B, and there is a 15% chance of it moving to scenario A and a 5% chance of moving to scenario C. If the first period is Scenario C, it is 75% probable that it will stay there, 15% that it will move to Scenario B, and 10% that it will move to Scenario A. These values are summarized in Table 4.

	%light	%heavy	Initial chance	Chance A next	Chance B next	Chance C next
Scenario A	90%	10%	50%	85%	10%	5%
Scenario B	70%	30%	30%	15%	80%	5%
Scenario C	50%	50%	20%	10%	15%	75%

Table 4: Scenario Descriptions and Probabilities

The average profit per flight for each combination of scenario and tank design is shown in Table 5.

	Average Profit Scenario A	Average Profit Scenario B	Average Profit Scenario C
Large fixed	\$ 516,508,880	\$ 520,508,880	\$ 524,508,880
Small fixed	\$ 639,706,468	\$ 497,549,475	\$ 355,392,482
Flexible	\$ 586,830,750	\$ 550,937,538	\$ 515,044,326

Table 5: Average profit per flight for each combination of design and scenario

4.2. Decision analysis

The decision tree is shown on the next four pages.

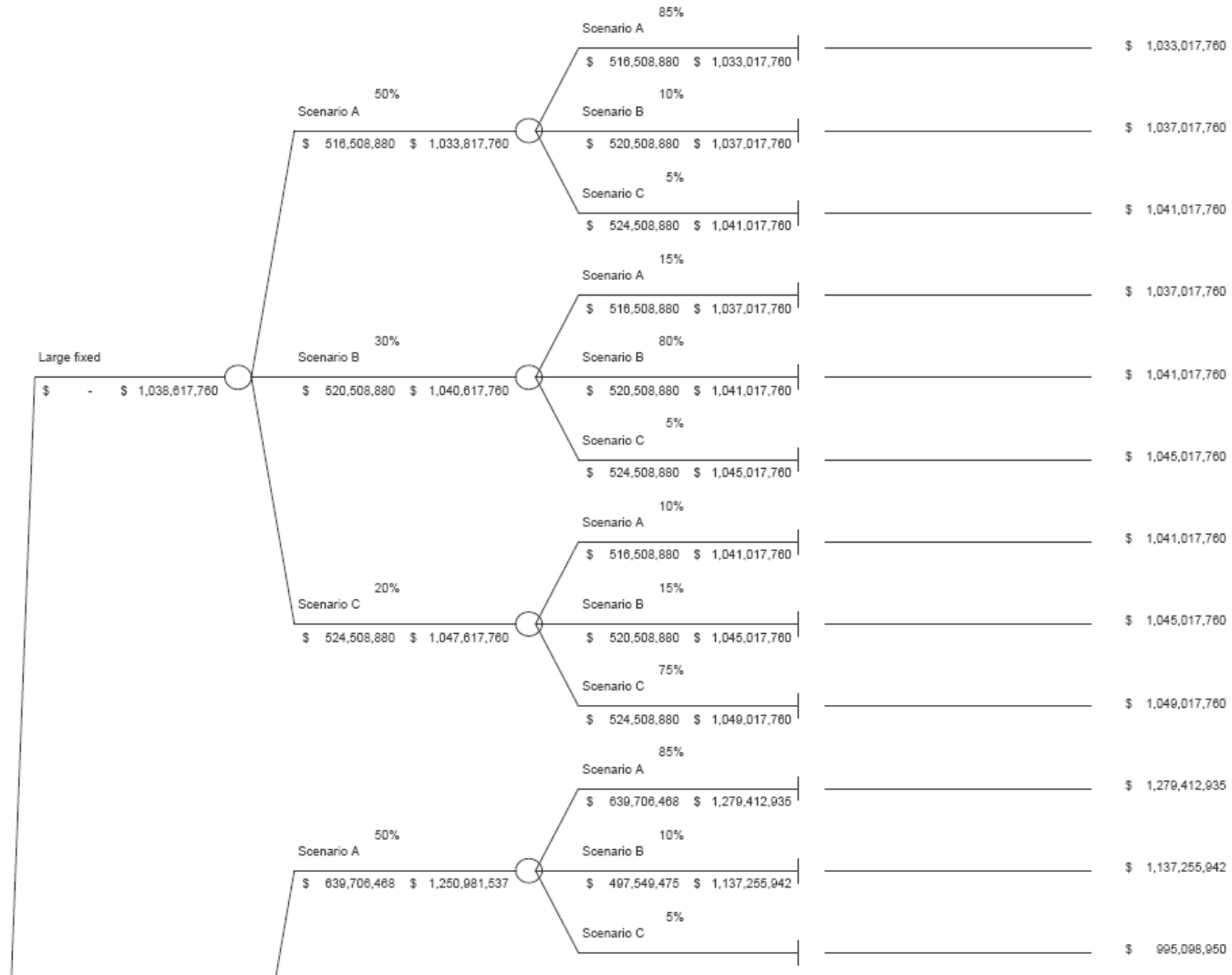
4.3. Optimal Strategy

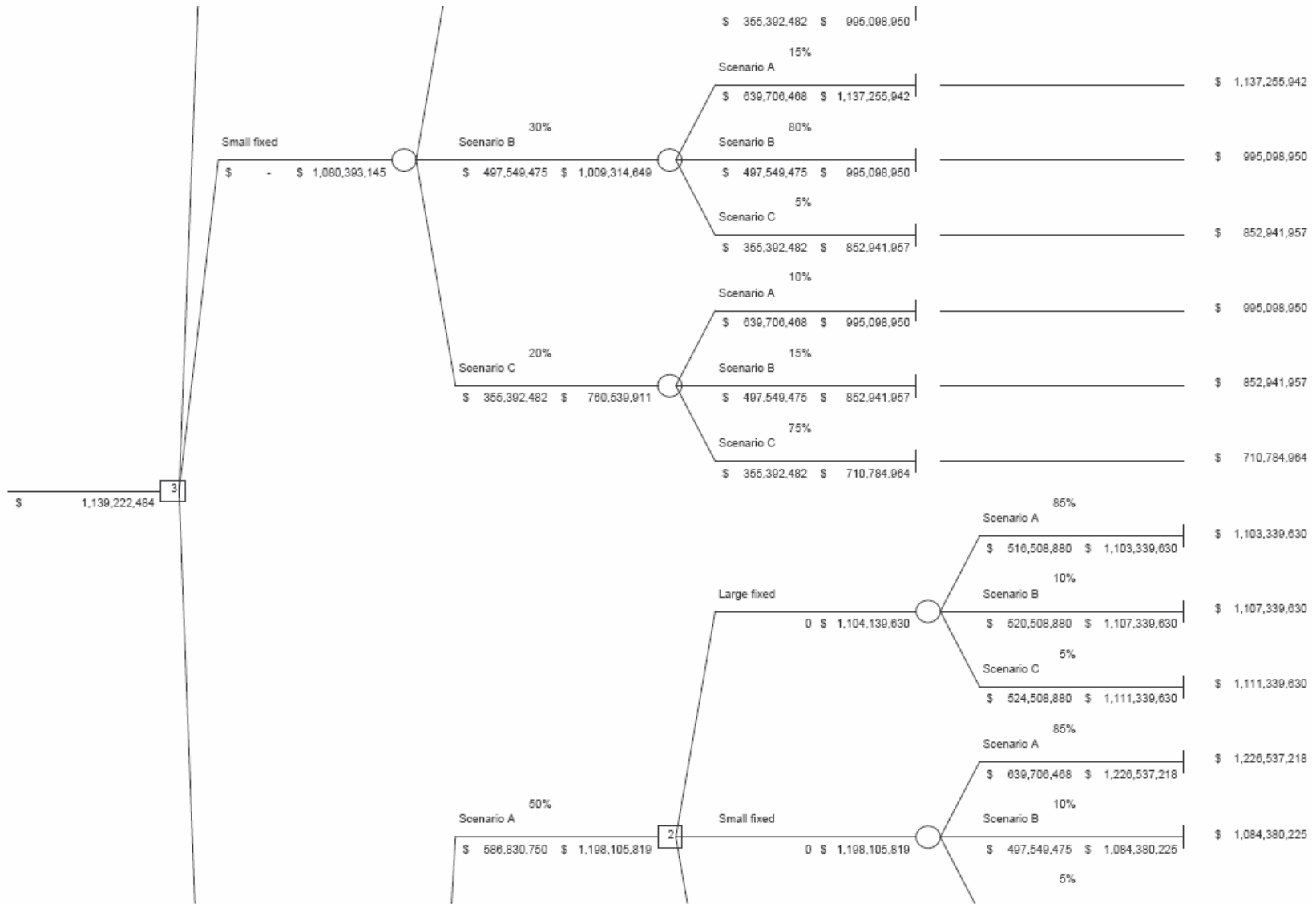
The optimal decision in the first period is to use the flexible design. After that, the decision is based on the scenario. If the first period is scenario B or C, the optimal decision is to use the flexible design in the second period also. However, if the first period is scenario A (which is 90% light launches, and predicts with 85% certainty that the next period will also be 90% light launches), the optimal second period decision is to switch to the small fixed design.

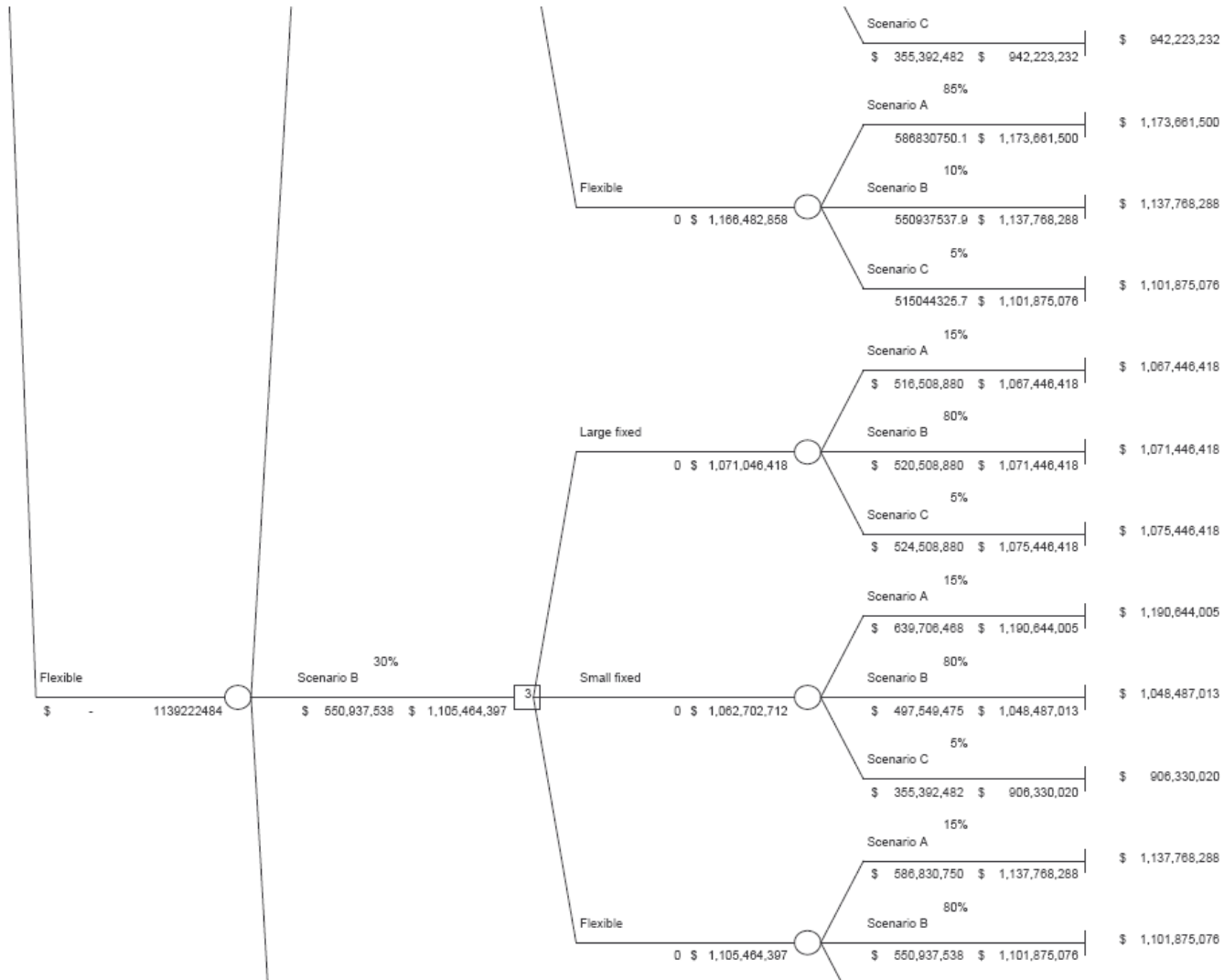
4.3.1. Expected Value

The expected value of the strategy is an average profit of \$1,139,222,484 per launch.

TreePlan (Tryout Version)







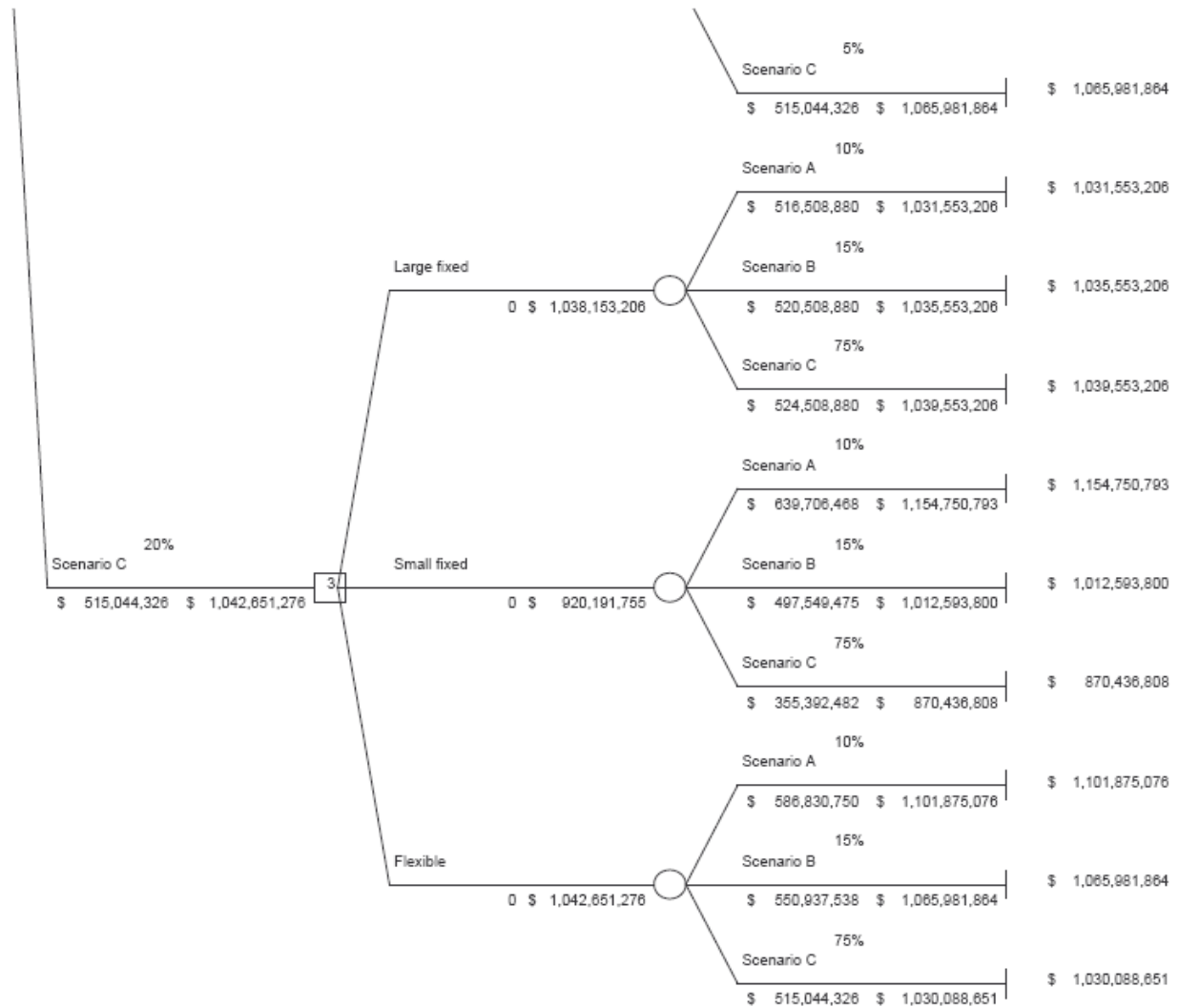


Figure 7: Decision tree

5. Lattice analysis of evolution of payload

The time period considered has a length of five years, divided into yearlong stages. At time zero, 20% of the flights are heavy, the rest are light.

The lattice model assumes that the uncertain parameter can be modeled using geometric Brownian motion. This assumption is invalid in this case: the predicted number is a percentage, and should not exceed 100%, while the geometric Brownian model allows it to grow without bound. Nevertheless, since the purpose of this work is to learn the standard lattice method, the model will be used. Note, however that extending the model beyond the given number of periods can result in nonsensical results.

In this model, the variation in the percentage of heavy flights is assumed a function of volatility alone; there is no predicted growth rate. Therefore, in calculating the lattice parameters, the growth $v=0$. The volatility due to an uncertain political, economic, and scientific climate is assumed to have a standard deviation of $\sigma=28\%$. The lattice parameters are therefore $u=e^{\sigma}=1.32$, $d=1/u=0.76$, and $p=0.5+0.5v/\sigma=0.5$. The outcome and probability lattices are shown in Table 6.

OUTCOME LATTICE: FRACTION OF SHUTTLE FLIGHTS IN HEAVY CATEGORY

0.200	0.265	0.350	0.463	0.613	0.811
	0.151	0.200	0.265	0.350	0.463
		0.114	0.151	0.200	0.265
			0.086	0.114	0.151
				0.065	0.086
					0.049

PROBABILITY LATTICE

1.000	0.500	0.250	0.125	0.063	0.031
	0.500	0.500	0.375	0.250	0.156
		0.250	0.375	0.375	0.313
			0.125	0.250	0.313
				0.063	0.156
					0.031

Table 6: Outcome and probability lattices

6. Decision analysis using lattice

Recall that it is assumed possible to move from a flexible segmented design to a fixed design in either of two sizes, but not possible to move from a fixed to a flexible design.

Therefore, at each period, it is possible do one of the following:

- Use a small fixed design, and continue to do so for all remaining periods.
- Use a large fixed design, and continue to do so for all remaining periods.
- Use a flexible design, and do the best possible design for all remaining periods (small fixed, large fixed, or flexible)

The average profit at a particular state is the percentage of heavy flights times the profit from a heavy flight with that particular tank plus the percentage of light flights (equal to 1 minus the percentage of heavy flights) times profit from a light flight with that particular tank. The profit for each combination of tank size and payload weight was calculated in Section 3.1 and shown in Table 3 on page 8.

PROFIT: LARGE FIXED

\$ 518,508,880	\$519,801,399	\$521,511,570	\$523,774,348	\$526,768,297	\$530,729,680
	\$517,532,015	\$518,508,880	\$519,801,399	\$521,511,570	\$523,774,348
		\$516,793,716	\$517,532,015	\$518,508,880	\$519,801,399
			\$516,235,722	\$516,793,716	\$517,532,015
				\$515,813,999	\$516,235,722
					\$515,495,268

PROFIT: SMALL FIXED

\$ 568,627,971	\$522,692,809	\$461,914,626	\$381,497,200	\$275,094,507	\$134,309,931
	\$603,345,020	\$568,627,971	\$522,692,809	\$461,914,626	\$381,497,200
		\$629,583,601	\$603,345,020	\$568,627,971	\$522,692,809
			\$649,414,294	\$629,583,601	\$603,345,020
				\$664,402,010	\$649,414,294
					\$675,729,481

PROFIT: FLEXIBLE

\$ 568,884,144	\$557,285,977	\$541,940,097	\$521,635,505	\$494,769,894	\$459,223,203
	\$577,649,850	\$568,884,144	\$557,285,977	\$541,940,097	\$521,635,505
		\$584,274,828	\$577,649,850	\$568,884,144	\$557,285,977
			\$589,281,879	\$584,274,828	\$577,649,850
				\$593,066,126	\$589,281,879
					\$595,926,199

Table 7: Profits for combinations of states and designs

The procedure for solving the decision analysis was the following:

- Calculate the expected NPV at every state for each of the large and small fixed designs. The recursive equation is:

$$\begin{aligned}
 &[\text{Expected NPV, fixed}] = [\text{Profit for this state and year, fixed}] \\
 &+ (p * [\text{Expected NPV, fixed, for next year if increase}] \\
 &+ (1-p) * [\text{Expected NPV, fixed, for next year if decrease}]) \\
 &/ (1 + [\text{discount rate}])
 \end{aligned}$$

where the profits are calculated for the given fixed design. The discount rate used in this analysis is 12%.

- For each state in the last year, choose the best design, whether large fixed, small fixed, or flexible. Call the profit from that design in that state the “Best NPV” for that state and year.
- From a flexible design, it is always possible to choose the best design in the next year. Working backwards, at each year before the last one, calculate the NPV of the flexible design in terms of the NPV of the best future designs, calculated in the next step:

$$\begin{aligned}
 &[\text{Expected NPV, flexible}] = [\text{Profit for this state, flexible}] \\
 &+ (p * [\text{Expected NPV, best for next year if increase}] \\
 &+ (1-p) * [\text{Expected NPV, best for next year if decrease}]) / (1 + [\text{discount rate}])
 \end{aligned}$$

- To get the best design, at each year before the last one, choose the best course of action to maximize NPV:

$$\begin{aligned}
 &[\text{Expected NPV, best for this state}] = \max([\text{Expected NPV, large fixed}], \\
 &[\text{Expected NPV, small fixed}], \\
 &[\text{Expected NPV, flexible}])
 \end{aligned}$$

Expected NPV, large

\$ 2,389,275,477	\$ 2,100,172,653	\$ 1,775,403,880	\$ 1,409,900,519	\$ 997,529,024	\$ 530,729,680
	\$ 2,090,344,523	\$ 1,764,627,727	\$ 1,398,818,254	\$ 987,393,600	\$ 523,774,348
		\$ 1,758,472,291	\$ 1,392,487,964	\$ 981,604,154	\$ 519,801,399
			\$ 1,388,872,044	\$ 978,297,171	\$ 517,532,015
				\$ 976,408,191	\$ 516,235,722
					\$ 515,495,268

Expected NPV, small

\$ 2,559,476,045	\$ 2,055,107,684	\$ 1,524,816,483	\$ 993,522,391	\$ 505,365,548	\$ 134,309,931
	\$ 2,404,392,001	\$ 1,907,792,837	\$ 1,387,377,769	\$ 865,570,880	\$ 381,497,200
		\$ 2,126,552,401	\$ 1,612,351,530	\$ 1,071,323,430	\$ 522,692,809
			\$ 1,740,858,582	\$ 1,188,851,152	\$ 603,345,020
				\$ 1,255,984,052	\$ 649,414,294
					\$ 675,729,481

Expected NPV, flexible

\$ 2,649,540,238	\$ 2,256,277,649	\$ 1,847,434,507	\$ 1,424,352,173	\$ 965,530,621	\$ 459,223,203
	\$ 2,401,247,725	\$ 1,958,306,838	\$ 1,499,955,305	\$ 1,024,556,313	\$ 521,635,505
		\$ 2,081,243,628	\$ 1,593,665,088	\$ 1,087,022,982	\$ 557,285,977
			\$ 1,680,726,166	\$ 1,143,542,379	\$ 577,649,850
				\$ 1,184,648,169	\$ 589,281,879
					\$ 595,926,199

Expected NPV, best

\$ 2,649,540,238	\$2,256,277,649	\$1,847,434,507	\$ 1,424,352,173	\$ 997,529,024	\$530,729,680
	\$2,404,392,001	\$1,958,306,838	\$ 1,499,955,305	\$1,024,556,313	\$523,774,348
		\$2,126,552,401	\$ 1,612,351,530	\$1,087,022,982	\$557,285,977
			\$ 1,740,858,582	\$1,188,851,152	\$603,345,020
				\$1,255,984,052	\$649,414,294
					\$675,729,481

Table 8: Expected NPVs at each state for each design

Best Decision (if arriving in a state with a flexible design)

1	2	3	4	5	6
flexible	flexible	flexible	flexible	large	large
	small	flexible	flexible	flexible	large
		small	small	flexible	flexible
			small	small	small
				small	small
					small

Table 9: Best decisions from each state

The expected NPV for the flexible design is \$2.65 million, compared to an expected NPV of \$2.56 million for the best (small) fixed design. The option to move from a flexible to either a large or a small fixed design is therefore worth \$0.9 million.

7. Applicability and learning outcomes

It was challenging to apply the flexible design concepts to this problem. It was not immediately apparent how to incorporate flexibility into the design. For example, it does not make sense to change the design of the tank over the course of a single mission, since it is jettisoned once it has served its limited purpose. This showed me that the options approach is more valuable in some contexts than in others.

For example, physically disconnected groups of systems that have the capability to change over time, like constellations of satellites and fleets of aircraft, are ideal for this approach. Dr. Hassan and Professor De Weck have shown how satellite constellations can be made reconfigurable, allowing the owners to take advantage of shifts in usage and demand. As another application, aircraft fleets can be designed in such a way that many parts are interchangeable between different sizes of aircraft. The manufacturer can save costs on machinery (since one machine can produce many types of aircraft) and the operators (e.g. airlines) can save on spare-part purchase, transport, and storage costs, and on the cost of training mechanics (and possibly pilots).

This application has been useful in providing a coherent problem on which to test each new technique. It has allowed me to master the techniques in the context of a real-world problem with which I am familiar, and to understand the implications of the results. It has also taught me that some techniques are more useful than others are on certain problems, and that much of the work of engineering is choosing the correct tool to apply to the problem at hand. In addition, it has stressed that design is not a mechanical process, but that human ingenuity and guidance will always be necessary, even if computational tools exist to perform the analysis.