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In This Issue: Real Options and Corporate Strategy

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Real Options Analysis: Where Are the Emperor's Clothes?

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The term "real options" was coined by Stewart Myers in 1977. It referred to the application of option pricing theory to the valuation of investments in non-financial or "real" assets where much of the value is attributable to flexibility and learning over time. Two examples of such investments are multi-stage R&D and modular manufacturing plant expansion.

The topic attracted moderate, primarily academic, interest in the 1980s and 1990s, and a number of articles were published on theory and applications. But beginning in the mid-1990s, interest in the concepts underlying option value and the techniques of valuation increased substantially. Real options began to attract attention from industry as a potentially important tool for valuation and strategy. Beginning principally in the oil and gas industry and extending to a range of other industries, management consultants and internal analysts began to apply real option tools intermittently, and in some cases regularly, to major corporate investments. An annual real options symposium for both academics and practitioners was first organized in 1996, and continues to this day. Several practitioner books on the topic have appeared, and more are in the works. Most mainstream academic finance texts now mention real options prominently. Conferences on the topic, attracting industry as well as academic participants, are held regularly. The number of academic articles on real options is now matched by a number of stories in popular publications like *BusinessWeek* and *USA Today*. All in all, real options has made the transition from a topic of modest, specialized interest to one that now receives active, mainstream academic and industry attention.

Despite this transition, the state of the topic at the analytical level from a potential practitioner's point of view is problematic. There is a great deal of agreement about the appeal of the underlying concepts. However, a variety of contradictory approaches have been suggested for implementing real options in practice. The assumptions underlying these different approaches and the conditions that are appropriate for their application are typically not spelled out. Where they are spelled out or can be inferred, they differ widely from approach to approach. The difficulties in implementing the approaches are rarely discussed, and the pros and cons of alternative approaches are not explained.

This situation leaves potential practitioners in troubling circumstances. In principle, the concept seems valuable and appealing. But given the current state of practice, there is a good chance that one could either apply an unsound approach or make inappropriate use of a sound one. The result is not simply a lack of theoretical precision, but mistaken investment decisions and lost value.

The purpose of this article is to help remedy this situation by describing, contrasting, and. . . yes. . . criticizing the major proposed analytic approaches to applying real options. The emphasis is not on details, but on three fundamental issues surrounding each proposed approach:

1. Applicability: what does the calculated real option value represent, and when is it appropriate to use this calculation?

2. Assumptions: when applied appropriately, what are the notable assumptions underlying the approach, and what is the evidence regarding the validity of these assumptions?

3. Mechanics: provided the assumptions are valid, what steps are involved in applying the approach, and what are the associated difficulties?

To elucidate these points and contrast the approaches, this article uses a single example of a firm evaluating an investment in which uncertainty is perceived to come primarily from two sources: the size of the business that the investment will create and the profitability of that business. Both uncertainties are dynamic in the sense that information about them becomes available and thus learning occurs over time. The firm can commit to the investment immediately, reject it, or invest in an opportunity (acquire an option) to learn more before making the commit/reject decision in the future. For simplicity, we assume that the costs associated with creating the business—specifically the cost of the investment and the option—are not a source of uncertainty.

For the purposes of illustrating the mechanics of the approaches, we rely on an oil and gas version of this example. However, it would be just as easy to create a pharmaceutical, high tech, or other industry version. In the oil and gas version, the investment is the possible acquisition of an undeveloped natural gas field in the Western U.S. Uncertainty about the size of the business is represented by the amount of natural gas in the field. Proved reserves are currently (May 2003) estimated by the firm at 100 billion cubic feet (BCF). Uncertainty about the profitability of the business is represented by the future price of natural gas. The current (May 2003) Henry Hub price of natural gas is $5.25 per thousand CF.

The firm can purchase and develop the field now for $175 million, decide never to purchase and develop the field, or acquire an option for $20 million to purchase and develop the field in two years for $175 million. For simplicity, development is assumed to be instantaneous so that revenues begin immediately after the acquisition and development decision. The option will expire if not exercised at the end of two years. Over the two-year period, the value of the field may change as uncertainty evolves regarding the amount and price of natural gas. Information about the amount of reserves may be obtained through geologic testing and test drilling, while information about the price may be obtained through market observation. Again for simplicity, the cost of information collection is assumed to be included in the $20 million option price.

After describing the various approaches and illustrating them with this example, the article concludes with observations about the relative strengths and weaknesses of the proposed approaches.

The Classic Approach (No Arbitrage, Market Data)

The best place to start in describing real options approaches is with what can be termed the "classic" approach, which is the direct application of classic option pricing from finance theory to nonfinancial or real investments. The most complete exposition of this approach appears in a book by Martha Amram and Nalin Kulatilaka published by the Harvard Business School Press in 1999 (hereafter referred to as Amram and Kulatilaka [1999]).[1] Like much of the well-known work in this field, the book is mainly conceptual and contains relatively few details about mechanics.[2]

Applicability

Real options approaches have at their core the calculation of value. In the case of the classic approach, these calculated values represent "financial market value" or, more generally, "internal valuations of strategic business opportunities that are aligned with valuations in the financial markets."[3] The calculated value of an investment thus represents an estimate of the shareholder wealth created by the investment or, said another way, what the investment would trade for in the capital markets.

When evaluating an investment from the point of view of a firm, this concept needs to be couched in terms of strategy or decisions. Although not stated explicitly, the calculated value can be viewed as a decision threshold. That is, the investment should be pursued if it can be acquired for less than the estimated value or sold if it can be marketed for more than the estimated value. Such a decision increases the overall risk-adjusted wealth of the firm's shareholders. The classic approach is thus applicable to investment decisions made from the point of view of diversified shareholders.

Assumptions

The classic approach makes the standard assumption of financial option pricing that a portfolio of traded investments can be constructed to replicate the returns of the real option in question, and therefore that the option can be valued based on standard no-arbitrage arguments. Although it is not emphasized, the classic approach also generally assumes that the traded replicating portfolio behaves in standard ways. In particular, the approach assumes that asset price movements can be described by geometric Brownian motion (GBM), an assumption that in turn allows the use of standard financial tools such as the well-known Black-Scholes option pricing model.[4]

This approach, and its underlying assumptions, are also endorsed in the second edition of the well-known valuation book by Tom Copeland, Tim Koller, and Jack Murrin, published in 1994 and widely regarded as the standard in this field. The authors observe that "the option-pricing approach . . . combines the desirable features of both the NPV and DTA [decision tree analysis] approaches. From the NPV approach, it borrows the idea that we must find a comparable (perfectly correlated) security to correctly evaluate risk, and from the DTA approach it uses decision nodes. . . to model flexibility."[5]

Mechanics

Given the strong, standard replicating portfolio/no-arbitrage assumptions of this approach, application is straightforward.[6]

1. Identify the replicating or tracking portfolio, and calculate its price and volatility.

2. Size the investment relative to the replicating portfolio.

1. For full citations to all references in this paper, see the References section at the end.

2. Among the first articles to present this approach were Brennan and Schwartz (1985) and Trigeorgis and Mason (1987). Other works that recommend this method are Trigeorgis (1999) and Copeland, Koller, and Murrin (1994).

3. Amram and Kulatilaka (1999), p. 30.

4. Although Amram and Kulatilaka wrote the first real options book that relied heavily on the replicating portfolio assumption, this assumption figures prominently in earlier articles or book chapters by several other authors. For example, in their pioneering application of real options, Brennan and Schwartz (1985) state explicitly (p. 137) that "the cash flows from the project can then be replicated by a self-financing portfolio of riskless bills and futures contracts," and later (p. 154) that "The explicit analysis rests on the assumption that such portfolios may be formed by trading in futures contracts in the output commodity." The explicit use of replicating portfolios is also used to justify the approach taken by Trigeorgis and Mason (1987) in using real options to value a manufacturing plant (see p. 15), and by Trigeorgis (1999) in a simple oil exploration and production example (p. 10).

5. Copeland, Koller, and Murrin (2nd edition, 1994), p. 453. As I discuss later, however, these authors have changed their views about the importance of a comparable security or replicating portfolio.

6. Kulatilaka and Amram (1999), p. 99.

3. Apply standard financial option pricing tools, typically Black-Scholes.

Kulatilaka and Amram (1999) provide an example valuing a start-up, which they view as an option on a duplicate of a comparable established firm.[7] The replicating portfolio in this example is identified clearly as the established firm with available current price and historical volatility data. The investment is sized relative to this established firm based on relative sales. Then Black-Scholes is applied.

In our oil and gas example, the first step is to find and characterize a replicating portfolio (or single replicating asset) that tracks the value of the Western U.S. natural gas investment in question. There are many publicly traded oil and gas firms that concentrate on natural gas in the Western U.S., although none whose fortunes are tied specifically to the field in question. After studying these firms, we concluded that Newfield (NYSE: NFX) is the closest analogue to the investment in question.

At the time of this writing (May 2003), Newfield was priced at $36.15 with a market cap of $1.88 billion (according to www.cbsmarketwatch.com). Short-term options (out seven months) are available for Newfield for use in estimating volatility, but no long-term options are available. Historical annual stock price volatility is in the 20-25% range, whereas implied annual volatility is in the 25-30% range (according to www.ivolatility.com). Consequently, 25% appears to be a good volatility figure.

The second step is to size the investment relative to the replicating portfolio, Newfield. Newfield is currently estimated to have proved reserves of 1,200 BCF (according to www.hoovers.com). The investment under consideration is estimated to have proved reserves of 100 BCF. Consequently, the $175 million investment in our example can be thought of as representing 1/12, or 8.33%, of Newfield. The option in our example can thus be thought of as a two-year option on 8.33% of the shares of Newfield with an exercise price of $175 million.

Using Black-Scholes, the parameters are as follows:

- Current value: 8.33% of $1.88 billion or $157 million
- Time to maturity: 2 years
- Risk free (five-year Treasury) interest rate: 3%
- Volatility: 25%
- Exercise price: $175 million

The resulting Black-Scholes option value is $19 million.

The interpretation of this result is straightforward. Based on the value of equivalent traded assets, the investment under consideration is worth $157 million. Acquiring it for $175 million thus causes an $18 million loss in value for shareholders. Based on the value of equivalent traded assets, the option to acquire/develop the investment in two years for $175 million is worth $19 million. It's a closer call, but acquiring the option for $20 million will also cause a loss in shareholder value, in this case $1 million. The appropriate value-maximizing strategy is to acquire neither the investment nor the option.

The rationale behind using the $157 million and $19 million figures for decision-making comes directly from the replicating portfolio/no-arbitrage assumptions. In this example, the approach presumes that an identical underlying investment and option—with the same financial outcomes in all states of the world—are available to the firm's shareholders at prices of $157 million and $19 million, respectively. If these opportunities are available at these prices, there is no reason to pay the higher prices, in this case $175 million and $20 million. Doing so would clearly destroy shareholder value.

Discussion

The primary difficulty with the classic approach is the lack of substantiation, from either theoretical principles or empirical data, for the replicating portfolio argument—namely, the contention that a traded replicating portfolio of financial assets exists for a typical corporate investment in real assets. There has been considerable attention paid to covariances between individual stocks and the overall market, which are reflected in betas. However, there has been relatively little attention paid to covariances among individual stocks, and even less to covariances between real and financial assets. Proponents of the classic approach make no mention of any such information. Furthermore, there is little reason to believe on principle that an individual corporate investment, such as a specific manufacturing plant or R&D project, would be highly correlated with a particular stock or collection of stocks. In fact, given that most firms typically oversee numerous investments of various types in various stages (in a way, that is the whole idea of a firm), the correlation is likely to be low. In the absence of arguments based on principle or evidence, the replicating portfolio assumption is difficult to accept.

Interestingly, in the most recent (2000) edition of their best-selling corporate finance textbook, Richard Brealey and Stewart Myers state that replicating portfolio/no-arbitrage arguments cannot in fact be used to justify the application of real options: "When we introduced option pricing model. . . , we suggested that the trick is to construct a package of the underlying asset and a loan that would give exactly the same payoffs as the option. . . . But many assets are not freely traded. This means that we can no longer rely on arbitrage arguments to justify the use of option models."[8]

Furthermore, proponents of the classic approach have seen fit to apply it in cases where the replicating portfolio argument appears on the basis of simple economic reasoning to be decidedly weak. For example, Amram and Kulatilaka (1999)

7. Ibid., p. 143.

8. Brealey and Myers (6th edition, 2000).

apply the approach to a company's investment in a specific textile mill.[9] In this example, reference is made to a "tracking" or replicating portfolio of textile mill stocks. But while there may be some relationship between the value of a specific mill and this portfolio, the portfolio in no way "replicates" the risk of the specific mill. The risk of the specific mill investment is likely to be much greater than the proposed tracking portfolio or even, in the extreme, negatively correlated with the portfolio. Another application involves an investment in a specific piece of vacant land where the proposed tracking portfolio is a basket of REITs. Once again, there is very little reason to believe that the value of a specific piece of vacant land is highly correlated with the value of a basket of REITs. In neither case is the argument in favor of the replicating portfolio concept made based on principle or evidence.

Amram and Kulatilaka acknowledge difficulty with the tracking portfolio concept, and recognize the existence of "tracking error" due in part to what is termed private or untracked risk. But they provide no substantive information about the importance of this tracking error,[10] and they offer no guidance on how to take private risk into account and how to deal with tracking error in the calculation of value. In their examples, the authors treat all risks in the same manner using standard option pricing methods.

Assumptions aside, as our oil and gas example illustrates, implementation of the classic approach is remarkably straightforward. The main tasks are quite manageable: (1) identifying and characterizing the replicating portfolio, and (2) sizing the investment relative to that portfolio.

The Subjective Approach (No Arbitrage, Subjective Data)

The subjective approach is also based on replicating portfolio/no-arbitrage arguments and the use of standard option pricing tools from finance. But in place of the explicit identification of a replicating portfolio, this approach uses entirely subjective estimates of inputs. The best exposition of this approach is contained in three *Harvard Business Review* articles by Tim Luehrman.[11]

Applicability

Luehrman discusses a variety of investment types and valuation approaches. He proposes that one class of investments, those he calls "opportunities," should be approached using real options. These are staged investments where the initial investment typically does not provide cash flow but rather the right to make further investments.

Luehrman argues that investment valuation is central to firm performance. Although firm performance is not explicitly defined, it is clear from the context that he means market performance—that is, return to shareholders. Taking this view, real options analysis of staged investments provides a yardstick whereby decisions can be made to maximize (diversified) shareholder value.

Assumptions

The assumptions underlying the subjective approach are essentially the same as those underlying the classic approach, although they are mentioned even less prominently. This approach assumes the existence of a replicating portfolio and therefore the applicability of no-arbitrage arguments. In addition, it assumes that GBM describes the dynamics of the value of this replicating portfolio.

Luehrman acknowledges but puts very little emphasis on these assumptions. In referring to option value calculations, he states, "The Black-Scholes option-pricing model that generated the numbers in the table makes some simplifying assumptions of its own. They include assumptions about the form of the probability distribution that characterizes project returns. They also include assumptions about the tradability of the underlying project assets; that is, about whether those assets are regularly bought and sold. And they include assumptions about the ability of investors to continually adjust their investment portfolios. When the Black-Scholes assumptions fail to hold, this framework still yields qualitative insights but the numbers become less reliable."[12] Examples are provided but no attempt is made to justify the applicability of these assumptions in the examples. As is the case with Amram and Kulatilaka (1999), no information is provided to quantify the degree of error introduced when the assumptions are less than perfect.[13]

Mechanics

Although the key assumptions behind the subjective approach are effectively identical to the classic approach, the mechanics are different in one very important way: data collection. This makes the mechanics even simpler. The two steps in this approach are:

1. Subjectively estimate the value and volatility of the underlying investment.

9. p. 42.

10. There is one stylized graph that shows "market-priced risk" representing say 70% of the total change in value over a short time interval. The remaining 30% is divided into "leakages" (e.g., dividends), basis risk, and private risk. The unstated but obvious implication is that private risk represents perhaps 10% of the total risk of a typical corporate investment.

11. The subjective approach is also recommended in the real options book by Sydney Howell and six colleagues (hereafter Howell et al.).

12. Luehrman (1998a), p. 14.

13. Howell et al. treat assumptions similarly. They pay little attention to assumptions, and only mention them and their appropriateness late in their book. In a section titled "Using real options when we shouldn't," they note that key assumptions are that "the risks of an option can be hedged away" [replicating portfolio/no arbitrage] and "the relevant uncertainties are random walks [GBM price dynamics]. . ." (Howell et al., 2001, pp. 193-4). However, in the many examples in this book, no attempt is made to identify the replicating portfolio or defend the random walk assumption. Nor is there any discussion about the impact of divergence from these assumptions.

2. Apply standard financial option pricing tools, particularly Black-Scholes.

In applying this approach to our oil and gas example, we make no attempt to identify a replicating portfolio. Instead, we subjectively estimate the current value and volatility of the underlying investment.

In our example, we adopt the view that the subjective data should be based on general industry experience. At the time of this writing, proved natural gas reserves in production were valued at around $2.25 per thousand CF or $2.25 million per BCF. Consequently, we estimate the value of the underlying investment to be $225 million (excluding the cost of acquiring and developing it). The stock price volatility of major oil and gas firms is currently around 25% (according to ivolatility.com) but is projected to increase; so we adopt a figure of 30%.

Using these subjective estimates, the inputs to Black-Scholes are:

- Current value: $225 million
- Time to maturity: 2 years
- Risk free interest rate: 3%
- Volatility: 30%
- Exercise price: $175 million

The resulting Black-Scholes value is $71 million.

As with the classic approach, the interpretation of this result is straightforward. Based on subjective inputs regarding an unspecified but equivalent traded asset, the underlying investment under consideration is worth $225 million. Acquiring it for $175 million will create $50 million in value for shareholders. The option on the investment is worth $71 million. Acquiring the real option for $20 million will create $51 million in value for shareholders. Both the investment and the option are highly valuable, but the appropriate value-maximizing strategy is to acquire the option and increase shareholder wealth by an estimated $51 million.

While both the classic and subjective approaches use the same basic computational tool, the results are decidedly different. The classic approach valued the underlying investment at $157 million and the option at $19 million, and the value-maximizing strategy was to reject both the investment and the option. The subjective approach values the underlying investment at $225 million and the option at $71 million, and the value-maximizing strategy is to acquire the option. Perhaps stating the obvious, different inputs lead to different outputs.

Discussion

The major difficulty with the subjective approach is the odd and inconsistent combination of a reliance on the replicating portfolio/no-arbitrage assumption and the use of subjective inputs completely detached from that portfolio. No attempt is made to justify the use of these subjective assessments as appropriate proxies for traded market values.

Suppose, for example, that one is considering a corporate investment with an obvious replicating portfolio; at the extreme, the corporate investment could even be a share of traded stock. In this case, it seems extremely unwise to rely on subjective assessments, rather than on direct market data, for the value of this investment. On the other hand, suppose for example that extensive analysis cannot find a replicating portfolio. In this case, there is no choice but to rely on subjective inputs. However, the assumptions underlying this approach regarding the existence of a replicating portfolio are then violated and we are limited to treating the output as at best "qualitative."

If there is a replicating portfolio, the classic approach using market data will provide the correct answer. If there is no replicating portfolio, the meaning and relevance of an option value calculated on the basis of no arbitrage using such a portfolio are highly questionable. Consider our oil and gas example. If there is an analogous traded investment, we should presumably accept the results ($157 million and $19 million) and recommendation (no investment, no option) of the classic approach. There does not appear to be any reason to accept instead the higher results ($225 million and $71 million) and different recommendation (acquire the option) of the subjective approach. If there is no analogous traded investment, we have numerical results and an investment recommendation but little if any guidance on how to interpret these outputs.

Assumptions aside, the mechanics of the subjective approach are remarkably straightforward. This is presumably the basis of its appeal.

The MAD Approach (Equilibrium-Based, Subjective Data)

The classic approach is firmly connected to standard option pricing. It is based on the existence of a traded replicating portfolio, and built on data from that replicating portfolio. The subjective approach takes a half-step away from standard option pricing. It is based on the existence of a traded replicating portfolio, but built on data that is subjectively assessed (although the use of this data is not explicitly justified). The Marketed Asset Disclaimer (or MAD) approach extends this progression. It takes a full step away from standard option pricing, and it justifies this step explicitly.

Specifically, the MAD approach does not rely on the existence of a traded replicating portfolio. Instead, the same, weaker assumptions used to justify the application of net present value (or discounted cash flow) to "fixed" corporate investments are used to justify the application of option value (or real options analysis) to flexible corporate investments. Furthermore, the same source of input data for the valuation calculations—namely, subjective assessment—is recommended. Tom Copeland and Vladimir Antikarov's real options book, published in 2001, provides the most complete description of this approach, and names it after the key assumption: Marketed Asset Disclaimer, or MAD, which asserts that the replicating portfolio is unnecessary.[14]

Applicability

Like Amram and Kulatilaka and like Luehrman, Copeland and Antikarov view the maximization of shareholder value as the overriding corporate objective and, hence, as the goal of real options analysis. They also endorse the idea that valuation in general, and real options analysis in particular, provides a yardstick for determining if the marginal return of an investment exceeds the marginal cost for shareholders.[15] Consequently, we can interpret the output of the MAD approach as an estimate of value created for the firm's (potentially) diversified investors and a guide for decision-making with shareholder value in mind.

Copeland and Antikarov also argue that traditional NPV, the dominant approach for valuing investments, "systematically undervalues every investment opportunity" because of its failure to incorporate management flexibility. They argue that "real options will replace NPV as the central paradigm for investment decisions" within ten years because of its superior treatment of flexibility.[16] Clearly, these authors believe that their approach to real options is applicable to nearly all major corporate investment decisions where value maximization is the goal.[17]

Assumptions

As mentioned earlier, the second edition of the valuation text by Copeland, Koller, and Murrin (1994) makes a strong case for the classic approach and the importance of finding a traded replicating portfolio. However, by the time of the third edition, published in 2000, their thinking had shifted in favor of the MAD approach with no need for a traded replicating portfolio. In their words,

The option pricing approach gives the correct value because it captures the value of flexibility correctly by using an arbitrage-free replicating portfolio approach. But where does one find the twin security? We can use the project itself (without flexibility) as the twin security, and use its NPV (without flexibility) as an estimate of the price it would have if it were a security traded in the open market. After all, what has better correlation with the project than the project itself? And we know that the DCF value of equities is highly correlated with their market value when optionality is not an issue. We shall use the net present value of the project's expected cash flows (without flexibility) as an estimate of the market value of the twin security. We shall call this the marketed asset disclaimer.[18]

This is a truly remarkable departure from a heavy reliance on capital market data.

Copeland and Antikarov reiterate this point in their real options book: "We are willing to make the assumption that the present value of the cash flows of the project without flexibility (i.e., the traditional NPV) is the best unbiased estimate of the market value of the project were it a traded asset."[19] And hence the option value based on this NPV is assumed to be the best unbiased estimate of the market value of the option (were it also a traded asset). Copeland and Antikarov justify these assumptions based on a comparison with the NPV method. They note that the use of NPV to value a corporate investment assumes that there are traded assets of comparable risk (same beta), and they argue that "MAD makes assumptions no stronger than those used to estimate the project NPV in the first place."[20]

Consistent with this argument, the calculated NPV of the "fixed" corporate investment is an estimate of the value that the asset would have if it were traded. The only market data used in this calculation is the risk-adjusted cost of capital or discount rate. Similarly, the calculated real option value of the "flexible" corporate investment is an estimate of the value that the flexible asset would have if it were traded.[21] Again, the only market data used in this calculation is the risk-adjusted cost of capital.

14. Copeland and Antikarov (2001). A similar view is taken by Trigeorgis (1999) and by Richard Brealey and Stewart Myers in the 2000 edition of their classic corporate finance text.

15. Ibid. p. 56.

16. Ibid., p. v, vi.

17. Trigeorgis (1998, p. 24) takes a similar view. NPV provides inadequate guidance in the firm's effort to maximize value because of its inability to value flexibility appropriately. Real options analysis expands the concept of NPV to include these factors, and provides a more accurate estimate of value for virtually all corporate investments.

18. Copeland, Koller, and Murrin (2000), p. 406.

19. Copeland and Antikarov (2000), p. 94.

20. Copeland and Antikarov (2001), p. 67. Trigeorgis and Mason (1987) also use this "MAD" argument to justify their method. They argue that the existence of a 'twin security' is implicitly assumed in traditional NPV analysis for purposes of estimating the required rate of return on a project. That is, they argue that the assumptions underlying the application of option pricing to real assets are no stronger than the assumptions underlying the application of NPV to real assets.

Moreover, Trigeorgis repeats and expands this argument in his 1998 book in a section called *Justification of the Options Analogy*. "Can the standard techniques of valuing options on the basis of no-arbitrage equilibrium, using portfolios of traded securities to replicate the payoff to options, be justifiably applied to capital projects where projects may not be traded? As Mason and Merton (1985) point out, the answer is affirmative if we adopt the same assumptions used by standard DCF approaches – including NPV – which attempt to determine what an asset or project would be worth if it were to be traded. Recall that in DCF analysis we identify for each project a twin security with the same risk characteristics [note that the term 'twin security' in the context of DCF typically refers to the same beta, whereas the term in the context of option pricing typically refers to the same return in all states of the world] which is traded in the financial markets, and use its equilibrium required expected rate of return – typically by estimating the project's co-variability with the market from the prices of a twin security and applying the CAPM – as the appropriate discount rate. The 'correctness' of using NPV (value maximization) rests, of course, on the assumption of market completeness." (p. 127)

The current view of Brealey and Myers is essentially the same: "When we value a real option by the risk-neutral method, we are calculating the option's value if it could be traded. This exactly parallels standard capital budgeting… a DCF calculation of project NPV is an estimate of the project's market value if the project could be set up as a mini-firm with shares traded on the stock market. The certainty equivalent (i.e., risk-neutral) value of a real option is likewise an estimate of the option's market value if it were traded." (Brealey and Myers, 2000, p. 636-637)

21. Copeland and Antikarov (2001) state explicitly that the data used in this approach is entirely subjective, with the exception of the discount rate. Trigeorgis (1998) is less clear, but his examples make extensive use of subjectively assessed base values and volatilities, so one can only surmise that his approach is akin to Copeland and Antikarov. Like Amram and Kulatilaka, Trigeorgis uses real estate in one example (p. 347). Unlike Amram and Kulatilaka, he makes no effort to identify a twin security. Instead, he simply applies option pricing to the subjectively assessed base value and volatility of the investment.

In addition to MAD, Copeland and Antikarov make the usual assumption that underlying asset values follow GBM or a similar random walk process.[22] Rather than using this assumption to justify the use of the Black-Scholes approach, they rely on a closely related and somewhat more general option-pricing tool—the binomial lattice.[23] In this lattice, the starting NPV is an estimate of the value of the asset if it were traded now. The GBM behavior of the asset is a projection of the future behavior of the asset value if it were traded. And the real option value obtained using the lattice is an estimate of option value if the underlying asset were traded and exhibited GBM behavior.

Copeland and Antikarov argue that the MAD approach makes real options applicable to a much broader range of investments than approaches based on a replicating portfolio argument:

Most asset option-pricing applications are limited to those situations where the option value depends on the market price of a world commodity, such as oil, coal, copper, nickel, gold, or zinc. By using the marketed asset disclaimer and a lattice approach, however, it is possible to solve a much wider set of problems than ever before, in a way that is fairly easy to understand from a manager's point of view.[24]

The range of applications is wider, but the more important question is whether these applications are likely to be correct.

Mechanics

The steps in the MAD approach, following Copeland and Antikarov, are as follows:

1. Build a spreadsheet cash-flow model of the underlying investment using subjectively estimated inputs, and calculate its NPV using a CAPM-based discount rate.

2. Subjectively estimate the uncertainty associated with inputs to this model, and conduct a Monte Carlo simulation of the model.

3. Use the resulting distribution to build a risk-neutral binominal lattice (based on GBM), and estimate the option value using this lattice.

In our example, the two uncertainties are the amount and price of natural gas. Let us assume that the firm's geologists have studied the proposed acquisition extensively. They believe that the amount of gas is distributed lognormally with a mean of 145 BCF and a standard deviation of 30 BCF. There is a 5% chance that the amount is 100 BCF or less (proved reserves) and a 5% chance that they are 200 BCF or more. Our geologists assume that this uncertainty will be entirely resolved after one year of intense geologic study and test drilling. The second uncertainty is the price of natural gas. The Henry Hub price is currently $5.25 per thousand CF, and the firm's economists believe that annual natural gas price changes are normally distributed with a mean of 2% and a standard deviation of 4%. Variable operating costs are estimated to be $2.00 per thousand CF.

Following the procedure outlined above, our first step is to build a spreadsheet model of the value of the underlying investment (excluding any costs for acquiring the investment or the option) based on our subjective inputs. Production is assumed to go on for ten years, with 10% of the reserves produced each year. These amounts are discounted based on the weighted average cost of capital for oil and gas firms, which we assume to be 13%. Given the base assumptions, our spreadsheet model indicates that the value of the underlying investment if it were acquired and developed today is $322 million. Assuming that production starts in 2005 (if the option is acquired and exercised), the value is $268 million.

We then use Monte Carlo simulation to vary the inputs to this model, and derive information on the expected value and volatility of the annual return. The result is a 14% mean annual return with a standard deviation of 25%. To determine the value of the option, this information is used to build a binomial lattice of the value of the underlying investment (assuming production starts in 2005) over two years as shown below. The probability of up (the greater value) is 0.492 and the probability of down is 0.508. The current value of the underlying investment (with production starting in 2005) is $268 million. In two years, it could be as high as $450 million and as low as $159 million.

		$450.43
	$347.30	
$267.79		$267.79
	$206.48	
		$159.21

Solving this binomial lattice using a risk-neutral approach, we generate the following lattice of option values. The current value of the option is $113 million. In two years, it could be as high as $275 million or as low as $0. The appropriate option exercise policy is to wait in the first two years, to exercise the option if the final value is $92

22. Copeland and Antikarov (2001), p. 219.
23. Copeland and Antikarov build their argument on Samuelson's proof that "properly anticipated prices fluctuate randomly" and therefore that change in asset value follows a random walk even if underlying processes affecting cash flows do not follow random walks (i.e., are mean-reverting, cyclical, or the like). "Even the most complex set of uncertainties that may affect the cash flows of a real options project can be reduced to a single uncertainty—the variability of the value of the project through time. Samuelson's proof that properly anticipated prices fluctuate randomly implies that no matter how strange or irregular the stochastic pattern of future cash flows may be, the value (wealth relative) of the project will follow a normal random walk through time with constant volatility."
24. Copeland and Antikarov (2001), p. 415.

million or $275 million, and to allow the option to expire if the final value is $0.

		$275.43
	$182.72	
$113.17		$92.79
	$45.69	
		$0.00

Based on this analysis, acquiring and developing the investment now for $175 million will generate $147 million ($322 million less $175 million) in shareholder value. Acquiring the option for $20 million will generate $93 million ($113 million less $20 million) in shareholder value. The value-maximizing decision is to make the investment now. These values, and the associated recommendation, differ greatly from those obtained using the approaches described earlier.

Importantly, the interpretation of the value calculations with this approach is different than in the classic and subjective approaches since we no longer make any reference to a traded replicating portfolio. The $322 million figure is our subjective estimate of the current equilibrium value of the underlying investment if traded. The $113 million figure is our subjective estimate of the current value of an option on this underlying investment. If the option can be acquired for $20 million, then we estimate that $93 million in shareholder value would be created in equilibrium. However, the $113 million estimate is based entirely on subjective data, such as the natural gas price forecast. Furthermore, the $93 million gain cannot be realized in a risk-free transaction involving an equivalent traded asset, and this value is in no way tied to current market data other than the discount rate.

Discussion

There appear to be two fundamental conceptual difficulties with the MAD approach. The first stems from the MAD assumption itself; that is, that the value of the underlying investment (or the "fixed" corporate investment) should be assessed entirely subjectively. This assumption appears to ignore the possibility that there might be a replicating portfolio for the corporate investment in question, or at least that important elements of the investment in question, such as input or output commodity prices, might have market equivalents. As a result, market information on the value of the corporate investment or important elements of that investment is completely ignored (other than using a market-based discount rate). The use of option pricing may ensure that the "Law of One Price" is maintained internally between the investment and the option. However, the use of subjective data for all inputs other than the cost of capital means that arbitrage opportunities may be available between the corporate investment and traded investments if any related traded investments are available. Ironically, this implies that this finance theory-based approach is really appropriate only where the corporate investment has absolutely no financial market analogues.

The second difficulty stems from the GBM assumption. While there may be good arguments for GBM with respect to equilibrium prices in highly liquid, widely accessible markets, there is no reason to believe that subjective assessments (which are the basis of the MAD approach) of the value of the underlying investment should follow GBM. In fact, the assessed value of the underlying investments may be driven by specific events in specific time periods in a manner that looks nothing like "random drift."

These two difficulties are clearly evident in our example. Despite the fact that natural gas is traded and therefore valued in the capital market, no effort is made to tie assessments of this key driver to market valuations. In general, the MAD approach is the same if the underlying investment is entirely separate from the market (e.g., a coin flip), entirely tied to the market (e.g., IBM stock), or somewhere in the middle (e.g., an undeveloped natural gas field). In our example, we rely on our subjective assessment that natural gas prices will rise at 2% per year, whereas the capital market may currently be trading natural gas futures at entirely different values. With this approach, the natural gas investment and option may be priced consistently but they may both be mispriced relative to the market. As far as the amount of natural gas is concerned, we assume that the amount is lognormally distributed and that the uncertainty about amount is resolved quickly and definitively in one year. A GBM-based binomial lattice is incapable of reflecting this particular evolution of value in the underlying investment, which can have a dramatic effect on the value of the real option.

In addition to these two conceptual difficulties, the MAD approach also presents the practical challenge of developing a cash flow model and the associated subjective inputs.

The Revised Classic Approach (Two Investment Types)

The classic, subjective, and MAD approaches are all examples of alternative "one-size-fits-all" views; their proponents argue that they are applicable in the same basic fashion to all types of corporate investments. The revised classic approach, on the other hand, is based on the view that there are two different types of corporate investments, each requiring its own approach.

Applicability

In remarkable contrast to the MAD approach, the revised classic approach states explicitly that the assumptions underlying real options analysis are quite restrictive. They suggest that the classic finance-based real options analysis approach should be applied when these assumptions are appropriate, and that management science-based approaches such as dynamic programming and decision analysis should be applied otherwise. In particular, real options analysis should be used when investments are dominated by market-priced or public risks, and dynamic programming/decision analysis should be used when investments are dominated by corporate-specific or private risks.

In the former case, the meaning of the calculated real option value is precisely the same as in the classic case. The calculated value represents shareholder value, and it can be used as an objective, no-arbitrage-based threshold for maximizing shareholder wealth. In the latter case, the meaning of the calculated value is somewhat more complicated. Investments in this case are dominated by private risks, and subjective assessments are used to evaluate those risks. If the value is calculated using managerial time and risk preferences, the calculated value represents the investment's certain equivalent based on management's utility function. On the other hand, assuming time and risk preferences are handled in a manner consistent with finance theory (a very key assumption), the calculated value represents an estimate of shareholder value (for a potentially diversified investor) using management estimates of the inputs.

This view was elaborated most extensively by Avinash Dixit and Robert Pindyck in their 1994 book *Investment Under Uncertainty*. But it is also the view taken recently—in an article published in this journal in 2000—by Amram and Kulatilaka, earlier proponents of the classic approach. Consequently, we use the term "revised classic" approach and view this essentially as a replacement for the classic approach.

Assumptions

Amram and Kulatilaka (2000) make their argument using the term "tracking." Real options analysis should be applied when an investment's behavior can be reasonably tracked by traded assets. This form of analysis is relevant only to "the subset of strategic options in which the exercise decision is largely triggered by market-priced risk."[25] Decision analysis is applied when an investment cannot be tracked by market risks, or presumably when decisions are not driven by such risks.[26]

Amram and Kulatilaka do not address two important implementation issues. Although they discuss several examples of investments in the two categories, they provide no guidance (other than the use of judgment) on how to assign investments to one category or the other. Perhaps most importantly, they provide no information about the appropriate discount rate when using decision analysis for "untracked" investments.

Mechanics

The revised classic approach involves the following steps:

1. Determine if the investment in question is dominated by public or private risks.
2. If public risks, apply the classic approach.
3. If private risks, apply decision analysis.

a. Build a decision tree representing the investment alternatives.

b. Assign probabilities and values to the risks based on subjective judgment.

c. Apply a spreadsheet cash-flow model at each tree endpoint, and calculate NPV using the appropriate discount rate.

d. "Roll back" the tree to determine the optimal strategy and its associated value.

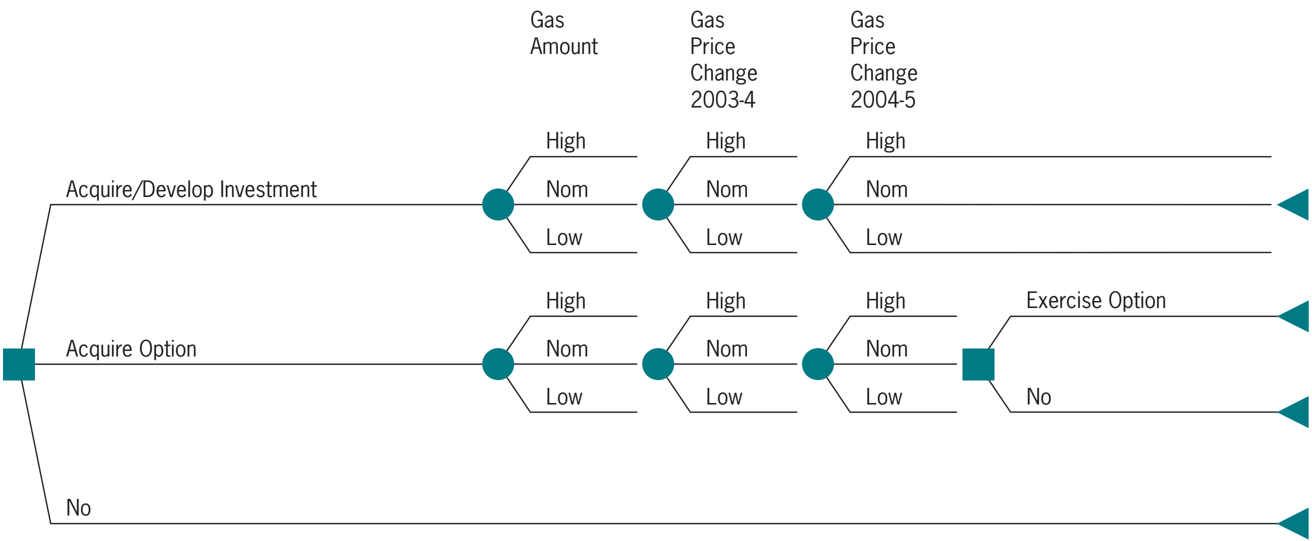
Amram and Kulatilaka (2000) use pharmaceutical R&D as an example of an investment dominated primarily by private risk—namely, project-specific scientific uncertainty. They use oil and gas development as an example of an investment dominated primarily by public risk—namely, market-based oil and gas prices. Although our example is also oil and gas, the value of this investment is driven primarily by the scientific uncertainty surrounding the amount of gas in the field. Given this observation, we determine that the investment is dominated by private risk and should be evaluated using decision analysis.

Following the decision analysis approach, we set up a decision tree that shows the investment decisions and the evolution of the two major uncertainties (amount and price) over time. As shown in Figure 1, the uncertainty about the amount of gas is resolved definitively in the first year while gas price uncertainty evolves incrementally over the two-year period. The tree indicates that there are three fundamental alternatives. First, the investment can be acquired now and the field developed. The realized value will depend on the gas price and amount. Second, the option can be acquired. In this case, information about the gas price and amount will be obtained over a two-year period, and then a decision must be made whether to let the option expire or to acquire and develop the field. Third, both the investment and option can be rejected.

25. Amram and Kulatilaka (2000), p. 17.

26. Dixit and Pindyck (1994) refer to option pricing with the more general term "contingent claims analysis." They state explicitly, "The use of contingent claims analysis requires one important assumption: stochastic changes in V [the value of the project in question] must be *spanned* by existing assets in the economy. Specifically, capital markets must be sufficiently 'complete' so that, at least in principle, one could find an asset or construct a dynamic portfolio of assets . . . the price of which is perfectly correlated with V. This is equivalent to saying that markets are sufficiently complete that the firm's decisions do not affect the opportunity set available to investors. The assumption of spanning should hold for most commodities, which are typically traded on both spot and futures markets, and for manufactured goods to the extent that prices are correlated with the values of shares or portfolios. However, there may be cases in which this assumption will not hold; an example might be a project to develop a new product that is unrelated to any existing ones, or an R&D venture, the results of which may be hard to predict" (Dixit and Pindyck, 1994, p. 147). Based on this view, they propose (p. 121) the use of dynamic programming where spanning is not a reasonable assumption and contingent claims analysis where it is.

Figure 1



The data for the two uncertainties is based on subjective assessment as in the MAD approach. The amount of gas is assessed to be log-normally distributed with a mean of 145 BCF and a standard deviation of 30 BCF. In the tree, this uncertainty is represented by a discrete distribution of high, nominal, and low states with the appropriate moments. This uncertainty is resolved in 2004. The annual change in the price of gas is assessed to be normally distributed with a mean of 2% and a standard deviation of 4%. Again, this uncertainty is represented by a discrete distribution of high, nominal, and low states with the appropriate moments.

As noted above, the proponents of the revised classic approach suggest the use of decision analysis for investments dominated by private risk. In this example, the dominant risk is the amount of gas in this particular field. Given the definition of private risk, this risk is not closely correlated with any individual investment or combination of investments in capital markets. Thus, it would be reasonable to conclude that this risk shows no correlation with the market as a whole—that is, it has zero beta. Consequently, for the purposes of this example, we conclude that discounting with the risk-free rate is appropriate for capturing the effect on shareholder wealth.

Solving the tree, we determine that the value of the investment is \$474 million, or \$299 million net of the \$175 million acquisition/development cost. The value of the option is \$300 million, or \$280 million net of the \$20 million acquisition cost. It is a close call between the investment and the option, but the value-maximizing decision is to make the investment rather than acquire the option. These values are very different, and considerably higher, than the values obtained using the approaches described earlier, including the classic approach initially advocated by Amram and Kulatilaka. Some of the difference is due to the subjective uncertainty assessments, but much of the difference is actually due to the use of a risk-free rate directly in the decision tree. The use of the risk-free rate is based on the viewpoint that the dominant risk in this example is private.

Discussion

The major difficulty with the revised classic approach is its “black and white” or “all or nothing” nature. In the world view represented by this approach, all investments are either public-risk dominated or private-risk dominated. Instead, most corporate investments involve a mix of public and private risks, and substantial error can be introduced by assuming that they are entirely in one or the other category.

There is a further potential problem with this approach, at least as recommended by Amram and Kulatilaka. The authors indicate that decision analysis should be used when private risk dominates. However, as noted above, they provide no guidance regarding what discount rate is appropriate to reflect shareholder risk preferences under these circumstances. Common practice in decision analysis is to use the weighted-average cost of capital (WACC) for firms in the industry in question (e.g., the WACC for oil and gas firms for an oil and gas investment). This is designed to reflect shareholder access to investments of comparable (but not identical) risk; that is, investments with the same beta. However, it is unclear in applying the revised classic approach whether the discount rate should be designed to reflect the shareholders' view of the overall risk of the investment or just the private risk, since the private risk is viewed as the dominating factor. Presumably, the WACC for a truly private risk with no market correlation is the risk-free rate. Although it is not spelled out explicitly by the authors, we adopted this latter, risk-free, view in the example above. This led in turn to extremely high values for the private-risk-dominated investment and option.

If one accepts the separation of investments into two cases, the implementation difficulties faced are those of the classic approach in one case and those of standard decision analysis in the other. In the latter case, the major problem is developing the subjective inputs, but there is the added complication of selecting a discount rate that allows one to interpret the results appropriately in terms of shareholder value.

The Integrated Approach (Two Risk Types)

The four approaches described so far originated with practitioners in finance looking to expand to real as opposed to financial investments. The integrated approach, on the other hand, originated with practitioners in management science looking to incorporate capital market considerations, and shareholder value in particular, into their evaluation of corporate strategy.

The integrated approach begins by noting that there are two types of risk associated with most corporate investments: public (or market) and private (or corporate). But unlike the classic or revised classic approaches, the integrated approach neither views private risk as a source of error (as does the former) nor does it assign investments entirely to one of two categories (as does the latter). Instead, it acknowledges that most investment problems encountered in practice have both kinds of risk—and it is designed to address that very situation.

This approach was first described in depth in a 1995 article by James Smith and Robert Nau and in a 1998 article by Smith and Kevin McCardle. Both articles refer specifically to their approach as the "integration" of option pricing and decision analysis, not as a real options analysis approach per se.[27]

Applicability

Given its origin in management science rather than finance, the integrated approach is based on a somewhat different philosophy than the other approaches. This approach recognizes that firms have a variety of stakeholders, owners and managers in particular. It then establishes a goal of making investment (and financing) decisions to maximize the utility of these owners and managers.

How does this goal compare to the "maximize the wealth of diversified shareholders" objective associated with the real options approaches above? As Smith and Nau write, "When markets are complete, investment decisions can be made solely on the basis of market information and all owners, regardless of their beliefs and preferences, will agree on appropriate project values and management strategies."[28] Under these conditions, then, maximizing the utility of the owners and managers is the same as maximizing the wealth of the owners.

The situation is somewhat more complicated when markets are incomplete because the beliefs (probability assessments) and preferences (risk attitudes) of the individual stakeholders may be important. There are many ways that these beliefs and preferences can be reflected. In practice, the view adopted by the integrated approach is completely consistent with the view taken by the other real options analysis approaches. In particular, the integrated approach takes as its goal the maximization of the wealth of diversified shareholders (as do the other approaches), where these shareholders agree that the probability assessment of private risks will be made by firm managers (not owners) as the agents of the owners. The risk tolerance of firm managers is ignored, and the risk neutrality of diversified owners with respect to private risks is adopted.

Assumptions

According to Smith and McCardle, "the basic idea of the integrated valuation procedure is to use option pricing methods to value risks that can be hedged by trading existing securities and decision analysis procedures to value risks that cannot be hedged by trading."[29] For any investment, their approach is to "mark to market" the portion of value driven by public risks, and to rely on judgment for the portion of value driven by private risks. Their fundamental assumption is that financial markets are "partially complete"—that is, complete only with respect to the set of risks, known as public risks, that can be hedged in any state of the world. The integrated approach makes relatively few assumptions about the form of these risks (for example, there is no need to assume GBM), although there are some limiting assumptions involving the relationship between public and private risks.

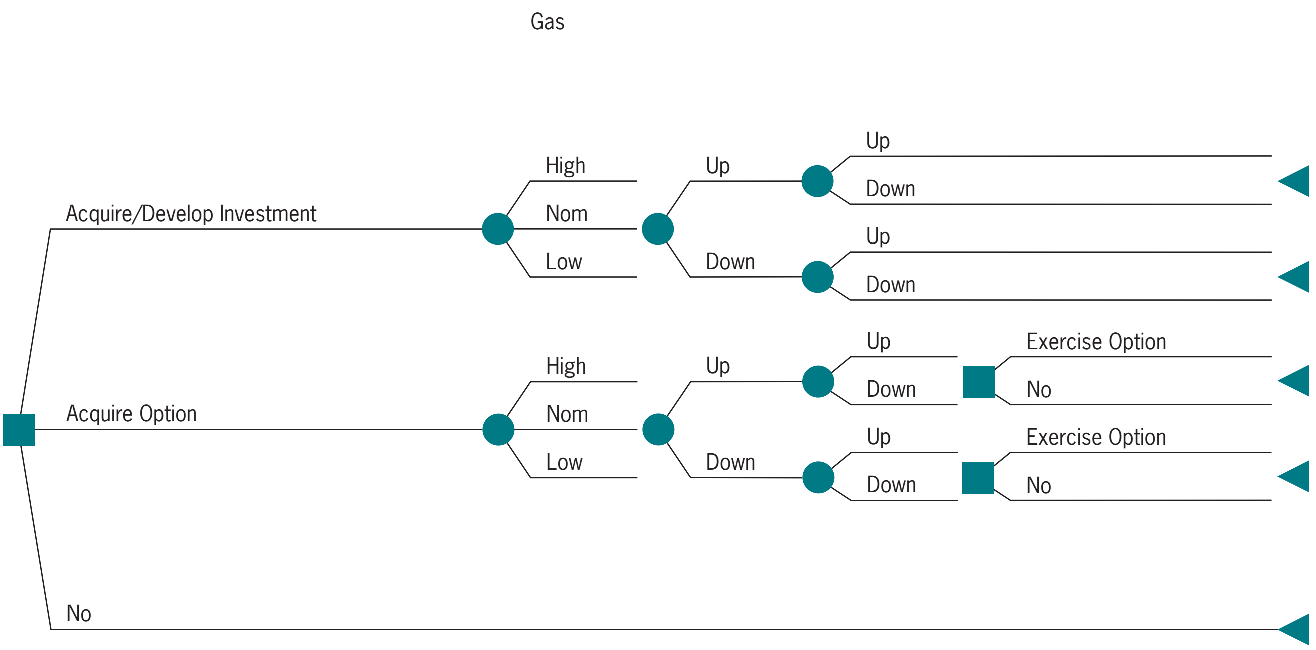
To implement this approach, Smith and Nau use what may be termed a "risk-adjusted decision tree," in which public and private risks are identified explicitly, opportu-

27. Other authors have discussed the integration of public/market and private/corporate risks less extensively either in a general context (see Constantinides [1978], Luenberger [1998]) or in specific applications (see Laughton [1998], McCormack and Sick [2001]).

28. Smith and Nau (1995), p. 812.

29. Smith and McCardle (1998), p. 199.

Figure 2



nities for hedging public risks are assumed available, and opportunities for arbitrage are removed.[30]

Mechanics

The integrated approach involves the following steps:

1. As with the revised classic approach for private risks, build a decision tree representing the investment alternatives.
2. Identify each risk as either public or private.
3. For public risks, identify the replicating portfolio and assign "risk-neutral" probabilities.
4. For private risks, assign subjective probabilities.
5. Apply a spreadsheet cash-flow model at each tree endpoint, and calculate NPV using the risk-free rate.
6. "Roll back" the tree to determine the optimal strategy and its associated value.

In our example, we build a decision tree similar to the one developed above in the revised classic approach. However, in the case of the integrated approach, we explicitly identify "amount of gas" as a private risk and "price of gas" as a public risk. As a private risk, the treatment of "amount of gas" remains the same as in the revised classic approach. For the price of gas, however, we do not use subjective assessments, but instead identify the replicating portfolio and impose no-arbitrage conditions. This is reflected in the tree (in Figure 2) where the gas-amount private risk is represented as a three-branch discrete distribution and the gas-price public risk is represented as a binomial lattice with up-down movements in each of two years.

The replicating portfolio for gas price is clearly gas price. The current price of natural gas is $5.25. There is an extensive market for gas price futures and options through 2009. Gas price options imply an annual volatility of around 19%, and previous studies have indicated a convenience yield of near 7%. Using this information, we develop the following binomial model of gas price changes for the periods 2003-4

30. Luenberger develops a similar approach using binomial lattices (Luenberger, 1998, p. 458). Much earlier, Constantinides (1978) outlined a related approach. He argued that corporate "projects" can be valued by first adjusting for market risk by adjusting the drift (essentially a risk-neutral approach) and then discounting the cash flows at a risk-free rate. This is quite similar in philosophy to the integrated approach, although Constantinides makes a variety of restrictive assumptions, such as limiting asset price movements to a Wiener process.

Interestingly, Copeland and Antikarov (2001) devote a chapter titled "Keeping Uncertainties Separate" to an alternative to the MAD approach that is quite similar to the integrated approach. They acknowledge (p. 270) that investments can be driven by multiple uncertainties, and that these uncertainties may not be well represented by GBM. They mention two types of uncertainty, one correlated with the market and one not, and build a decision or event tree with both these types of uncertainties. Furthermore, they indicate that market-correlated uncertainties should be evaluated using a replicating-portfolio approach and market-uncorrelated uncertainties should be assessed subjectively and discounted at the risk-free rate. In their examples, however, it appears that the authors still rely on the GBM assumption for the individual uncertainties whether market or not (producing a quadranomial not binomial lattice for two uncertainties). Furthermore, in these examples, there is no evidence that the authors use market data (as opposed to subjective assessment) to evaluate the market-correlated uncertainties.

and 2004-5. The probability of up is 0.53 and the probability of down is 0.47.

		$6.8229
	$5.985	
$5.25		$4.5486
	$3.99	
		$3.0324

As indicated in Figure 2, we incorporate this binomial model of gas prices in the tree, together with the three-state model of gas amount, and roll the tree back using the risk-free rate. The result is that the investment is worth $225 million, or $50 million net of the $175 million acquisition/development cost; the option is worth $125 million, or $105 million net of the $20 million acquisition cost. Both are good investments, but the value-maximizing decision is to acquire the option. The gas price risk can be effectively hedged in the capital markets, leaving shareholders exposed only to the private risk regarding the amount of gas. These results differ considerably from the results obtained using the approaches described earlier.

Discussion

The integrated approach appears to be the only one that takes the view that corporate investments typically involve a mix of public and private risks, and that an accurate valuation depends on addressing both. It presents an approach that covers the range of corporate investments as a continuum from traditional decision analysis at one extreme (all private risks) to finance theory or option pricing at the other (all public risks). Perhaps because this approach stands so squarely between the separate camps of finance theory and decision theory, it seems to be remarkably unfamiliar to both the finance and management science academic communities.

On a practical level, the major difficulty with this approach is that it is more work and harder to explain. The integrated approach requires conceptualizing, data gathering, and modeling both from the finance and management science point of view.

Summary of the Different Approaches

Applicability

The real options approaches described above have considerable differences. However, at a philosophical level, they are quite similar. They are all intended to help firm management select investments that maximize the wealth of the firm's shareholders. They each provide a number or "real option value" that provides the threshold buy/sell price for the investment being evaluated.

In some cases, the meaning of this threshold value is unambiguous. For the approaches built around the replicating portfolio concept—namely, the classic and subjective approaches and the revised classic approach applied to market-dominated investments—the value represents the price of an exactly equivalent asset currently available to the firm's investors. Consequently, the recommendation constitutes a risk-free increase in wealth for shareholders. In the case of the classic approach and the revised classic approach applied to market-dominated investments, this risk-free increase is identified with specific traded assets. In the case of the subjective approach, such traded assets are assumed to exist but are not identified. The risk-free increase is in this sense a management estimate.

For the MAD approach and for the revised classic approach applied to private-dominated investments, the value represents management's subjective estimate of the equilibrium value of the investment and its associated option if and when traded. For the integrated approach, it represents the combination of the market value of the market-driven components of the investment and management's subjective estimate of the equilibrium market value for the private-driven components of the investment.

There are important differences in the types of investments for which each approach is considered appropriate. The classic and subjective approaches are applicable to those investments where a suitable replicating portfolio exists. (In the extreme, one may believe that all investments have such a replicating portfolio, but the point still holds.) The other approaches are applicable to any investment.

Assumptions

Although the approaches share a common goal, they differ markedly in their assumptions. The key assumptions are in two areas: the nature of capital markets and the source of data.

The classic approach assumes that capital markets are complete, and therefore that all corporate investments have equivalents in the capital markets and can be effectively hedged through a traded tracking portfolio. The data for valuing the investment is market data from that traded portfolio, and the value calculated is the "no-arbitrage" value of the investment. In my view, this is a consistent, but not fully accurate or relevant, world view. In the case of our oil and gas example, there are major differences between the risks affecting the value of a particular undeveloped natural gas property and the risks affecting the value of any individual company or combination of companies. Presumably, one major reason that corporate investments are made is that they are not duplicated in the capital markets, and it is important for a valuation process to reflect this reality.

The subjective approach also assumes that capital markets are complete, but draws the data for establishing a "no-arbitrage" value for the investment from subjective judgment. This eliminates the task of finding the replicating portfolio, and thereby substitutes a lack of consistency for the lack of accuracy

Table 1

Approach	Net Value of Investment	Net Value of Option	Value-Maximizing Recommendation
Classic	(18)	(1)	Neither
Subjective	50	51	Option
MAD	147	93	Investment
Revised Classic	299	280	Investment
Integrated	50	105	Option

in the classic approach. Suppose our oil and gas example had actually involved acquiring a portion of a publicly traded natural gas company whose name was unknown but could be uncovered. Knowing that such a "replicating portfolio" existed, it would clearly make sense to find the name of the company and use the available market data to value the investment and the option using the classic approach. Instead, while appealing to the replicating portfolio argument, the subjective approach ignores data on that replicating portfolio and relies on subjective judgment informed only by general industry information.

The MAD approach effectively assumes that corporate investments do not have close equivalents in the capital markets and then calculates a subjective estimate of the value that the investment, both fixed and flexible, would have if traded. This, in my opinion, is a consistent but not fully relevant or accurate approach. Suppose, for the sake of argument, our example had involved drilling not for an unknown amount of gas, but an unknown amount of IBM stock certificates. Would it make sense to estimate the value of the stock certificates by building a spreadsheet cash flow model of IBM's business based on subjective inputs? Or would it make sense to obtain that value directly from the capital markets? The MAD approach would have you do the former, where the latter clearly makes more sense when taking a shareholder point of view.

The revised classic approach assumes that some corporate investments have market equivalents and that others do not. For investments in the former category, it applies the classic approach using market data. For investments in the latter category, it applies decision analysis using subjective judgment. Although reasonably consistent and relevant, this method is likely to lack precision. In our oil and gas example, the value of the investment was affected by both a private risk, the amount of gas, and a public risk, the price of gas. It seems artificial to describe this investment as being dominated by either one or the other; it is simply not that black and white. And the valuation obtained from placing this investment wholly in one camp may be far off the mark. Given the time and resources, analysts would be better served by reflecting the fact that investments come in shades of gray with respect to the relative significance of public and private risks.

The integrated approach also assumes that some risks have market equivalents and other risks do not; but unlike the revised classic approach, it assumes that *each* corporate investment may be affected by *both* types of risks. For public risks, "no-arbitrage" option pricing is applied and market data is used. For private risks, decision analysis is applied and subjective judgment is used. In my opinion, this approach is consistent, relevant, and reasonably accurate.

Mechanics

The different assumptions underlying the various approaches lead to different mechanics. The mechanics of the classic and subjective approaches are perhaps the simplest, given their use of the powerful Black-Scholes algorithm. The mechanics of the classic approach are only slightly more involved than the subjective approach, since it involves the added task of actually finding the replicating portfolio. The MAD and revised classic approaches both require a moderate amount of effort in spreadsheet modeling and option pricing. The integrated approach requires perhaps the most effort, since spreadsheet modeling is required and each individual risk must be evaluated and modeled separately using either option pricing or decision analysis.

The results obtained from the different approaches following these mechanics are summarized in Table 1. This table tells a very important story. Specifically, the differences discussed above as far as applicability, assumptions, and mechanics are not simply a matter of semantics or academic correctness. Instead, they lead to dramatic differences in valuation—that is, differences in orders of magnitude and even in sign. And they lead to dramatic differences in strategy; each of the three alternatives (investment, option, neither) is the recommended alternative on the basis of at least one approach.

Conclusion

There is considerable confusion in the current state of real options analysis. While the various approaches are philosophically similar, they differ in fundamental and contradictory ways and, when applied, they provide fundamentally different and contradictory results. In the face of this situation, it would be hard to fault potential practitioners who simply respond with "a pox on all your houses" and conclude that little if anything of value can be obtained from conducting any analysis under the "real options" banner. But such a response would be equally unfortunate, given the importance of making good corporate investment decisions and the potential contribution that real options analysis can make. An alternative response is to evaluate each approach critically, as I have attempted to do here, and then to determine which approach to apply under what conditions.

The classic, subjective, and MAD approaches all have significant problems with inaccurate and inconsistent assumptions that make them effectively unacceptable for practical use in valuation or strategy applications involving corporate investments. Their relative ease of use does not counterbalance this effect. The revised classic approach is based on an overly simplistic world view, but may be useful when approximate results are acceptable and resources are limited. If one accepts this world view and is willing to assign a particular corporate investment either to the "entirely market-dominated" or "entirely private-dominated" categories, and if there is insufficient time or need for more accurate results, then this approach is appropriate. The integrated approach is based on the most accurate and consistent theoretical and empirical foundation, but it requires more effort as a result. For applications where quality and credibility are important, this approach is appropriate.

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