ENGINEERING SYSTEMS ANALYSIS FOR DESIGN

Mid-Semester Quiz, 2001

	Points	
Item	**Possible**	**Actual**
Your Name	1	
1	10	
2.1	6	
2.2	23	
3	31	
4	19	
TOTAL	90	

Your Name:

Solution____ [1]

Note: The points for each problem and sub-problem are marked in square brackets. They correspond to the amount of time you might spend on them.

You might want to use these as a guide for how you should spend your time. Don't spend 10 minutes on a 3-point problem, for example.

You may find it worthwhile to turn to the section that is easiest for you, and to do that section first. No need to respond to 5 main questions in the order presented.

DISCOUNTED CASH FLOWS

1 Explanations **[10]**

What is the proper basis for selecting a discount for an investment of government project? [3]

The discount rate is the opportunity cost of capital or shadow price on the budget (dollar increase in return for one dollar increase in the budget).

For a government that gets its budget from taxpayers, it should be on the same order as the discount rate used by private investors.

Explain the principal advantages and disadvantages of benefit-cost analysis. [4]

Advantages	Disadvantages
Simplicity (>or<1)	Benefit and cost must be expressed in monetary value
Useful in ranking	Sensitive to discount rate Low r → favor projects with long term benefits
Use the same principles as NPV (i.e. PV(benefits)/PV(costs)	BC is biased against projects with significant operating costs
--	ranking provided by BC is sensitive to r that is used

Explain the principal advantages and disadvantages of the payback period evaluation. [3]

Advantages	Disadvantages
Simplicity	No economic information about the value of the project
Provides direct useful information on when the investment starts to pay back	Difficult to value projects with different lifetimes and cash flow patterns
Independent of discount rate	

PRODUCTION FUNCTIONS

2.1 Definitions **[6}**

In your own words (not copied from text), explain the notion of
a) an isoquant [3]

Corresponds to all the <u>technically efficient</u> combinations of resources achieving an equal level of production.

b) returns to scale [3]

Describes how fast the output changes relative to the size of the production.
With the **important** specification that changes in the size of production are achieved by varying all inputs by the same factor. This is generally not the most efficient way to increase scale, of course.

2.2 Paper Mill [23]

Your company runs a paper mill that uses two kinds of wood: Fir and Balsam. Your chemical engineer tells you that the:

- marginal product of Fir is inversely proportional to the square root (~ $1/F^{0.5}$), and the
- marginal product of Balsam varies with the inverse ¼ power (~ $1/B^{0.25}$).

Note: these are inversed! As is usual….

Assuming a Cobb-Douglas production process:

a) what is the marginal rate of substitution? [6]

$$MRS_{FB} = -\frac{MP_B}{MP_F} \approx -\frac{F^{0.5}}{B^{0.25}}$$

Note: Inverse formulation also acceptable. Important to feature (-) sign.

b) what can you say about the returns to scale of the operation? [3]

Since marginal product of Fir is ~ $1/F^{0.5}$ and the marginal product of Balsam is ~ $1/B^{0.25}$, we can say that the power of F is ~0.5 and the power of B is ~0.75 in the Cobb-Douglas production function.

So, 0.5+0.75 = 1.25 > 1.0, we can say that the production process exhibits increasing returns to scale.

c) Assume that the costs of Fir and Balsam are each equal to their quantity to the 5/4th power (since more wood has to come from further away). Determine the expansion path for increasing production. [7]

First let us define the input cost function:

$$C = F^{5/4} + B^{5/4}$$

We can then deduce the marginal costs:

$$MC_F = F^{1/4}$$
$$MC_B = B^{1/4}$$

The optimality condition can be written as:

$$\frac{MP_F}{MC_F} = \frac{MP_B}{MC_B}$$
$$\Leftrightarrow F^{1/2}F^{1/4} = B^{1/4}B^{1/4}$$
$$\Leftrightarrow B^* = F^{*3/2} \text{ defines the expansion path}$$

d) Now calculate the cost function. [4]

The PF is Cobb-Douglas, so you know PF except for coefficient a_0.

$$Y = a_0 F^{1/2} B^{3/2} \quad \text{Production function}$$
$$C = F^{5/4} + B^{5/4} \quad \text{Imput cost function}$$
$$B^* = F^{*3/2} \quad \text{Expansion path}$$

- Stating Y in terms of F* using the expansion path:

$$Y = a_0 F^{*1/2} \left(F^{*3/2}\right)^{3/4}$$
$$Y = a_0 F^{*13/8}$$
$$\Rightarrow F^* = \left[\frac{1}{a_0} Y\right]^{8/13}$$

- Stating C in terms of F* using the expansion path:

$$C = F^{*5/4} + \left(F^{*3/2}\right)^{5/4}$$
$$C = F^{*5/4} + F^{*15/8}$$

- Replacing the expression of Y in terms of F* into the above expression leads to the cost function:

$$C = F^{*5/4} + F^{*15/8}$$
$$C(Y) = \left(\left[\frac{1}{a_0} Y\right]^{8/13}\right)^{5/4} + \left(\left[\frac{1}{a_0} Y\right]^{8/13}\right)^{15/8}$$
$$C(Y) = \left[\frac{1}{a_0} Y\right]^{10/13} + \left[\frac{1}{a_0} Y\right]^{15/13}$$

c) What can you say about the economies of scale for this plant? Explain reasoning. [3]

Economies of scale exist when the optimal cost per unit of output decreases when the level of production increases.

Second term of Y has exponent 15/13> 1, so C increases faster than Y, thus diseconomies of scale.

LINEAR PROGRAMMING

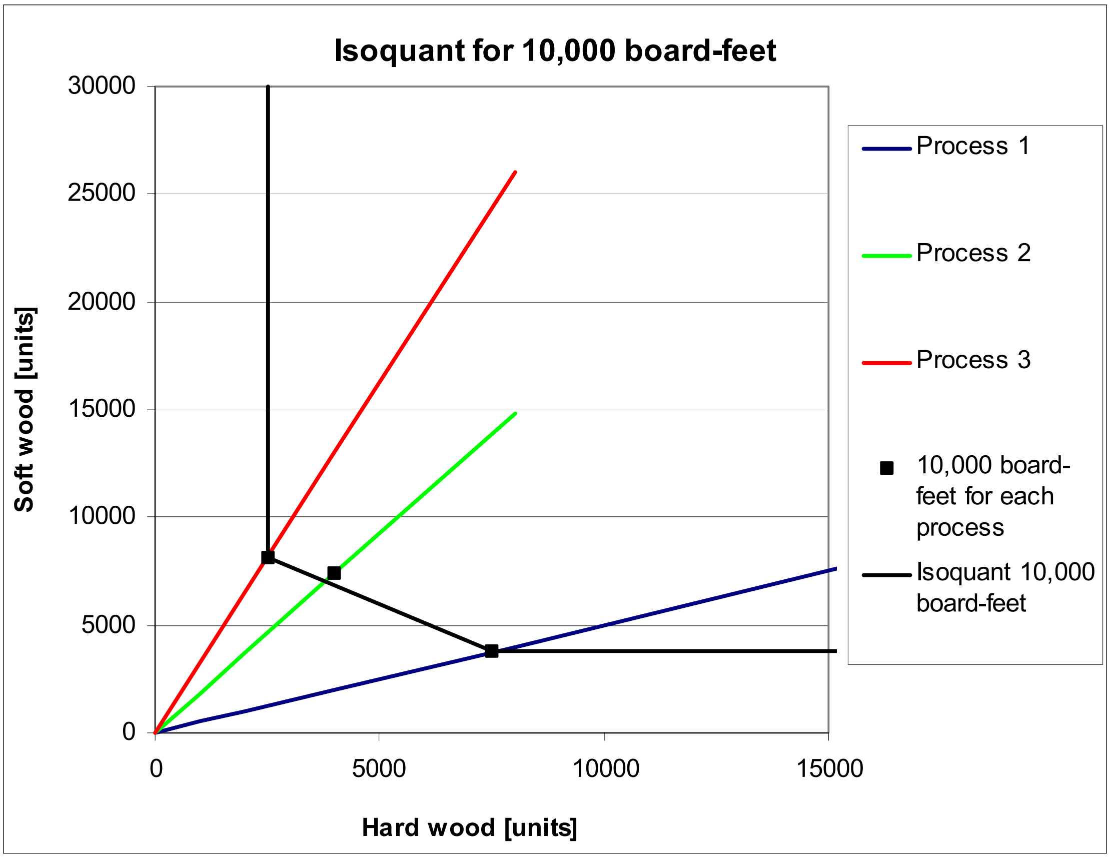
3 <u>Plywood Manufacture</u>

Charles "Chip" Bord makes plywood from hard and soft wood using any of three processes.

Process	Board-feet of Wood		
	Input		Output
	Hard wood	Soft Wood	Plywood
1	1000	500	1333
2	1000	1850	2500
3	1000	3200	4000

Graph the isoquant for 10,000 board-feet of plywood. [5]

Using the concept of activity (each process representing an activity), we can plot the isoquant for 10,000 boards of plywood in the resources space. The isoquant is obtained by a linear combination of Process 1 and 3. Process 2 is not optimum (i.e. it uses too much resources).



What is the marginal product for soft wood when Chip has 1000 board-feet of hard and 3200 board-feet of soft wood? [2]

The marginal product for soft wood is the unit change in output due to a unit change in soft wood. Adding one board-feet of soft wood when Chip has 1000 board-feet of hard and 3200 board-feet of soft wood will not generate any increase in output. So, the marginal product for soft wood is zero.

Chip finds out that process 2 is unavailable. Formulate the LP to maximize profits using processes1 and 3 if: [15]

- the price per board-foot is $5 for hard wood, $3 for soft wood , and 15$ for plywood (the sales price);
- Chip could get a maximum of 200,000 board-feet of hard wood and 320,000 board-feet of soft wood from his supplier
- His machines can only process 400,000 board-feet of wood in all
- He must buy at least $1million of wood from his supplier in order to maintain favorable relations.

One unit of process 1 yields $15 * 1,333 = $19,995 and costs $5 * 1,000 + $3 * 500 = $6,500.
Let us define X = [x_1, x_2] as the quantity of process 1 and 3, the problem can be formulated as:

$$\text{MAX} \quad \text{Profit} = \text{Revenues - Costs}$$

$$\text{MAX} \quad \text{Profit} = \begin{bmatrix} 19{,}995 & 60{,}000 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} 6{,}500 & 14{,}600 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$s.t. \begin{cases} 1 & 1{,}000(x_1 + x_2) \le 200{,}000 \quad \text{Maximum amount of hardwood from supplier} \\ 2 & 500x_1 + 3{,}200x_2 \le 320{,}000 \quad \text{Maximum amount of softwood from supplier} \\ 3 & 2{,}500x_1 + 4{,}200x_2 \le 400{,}000 \quad \text{Maximum board - feet that can be processed by the machines} \\ 4 & 5{,}000(x_1 + x_2) + 3(500x_1 + 3{,}200x_2) \ge 1{,}000{,}000 \quad \text{Minimum quantity of wood from supplier} \end{cases}$$

- Constraint 3 refers to the process and should be written in terms of inputs :

To run 1 unit of process 1, the machine processed 1,000 board - feet of hard wood and 500 board - feets of soft wood, that is 2,500 board - feet in total.

- If i*nterpreted* in terms of outputs, the answer is however acceptable, and the constraint 3 can be written as :

$$1{,}333x_1 + 4{,}000x_2 \le 400{,}000$$

Suppose that if the output obtained through process 1 exceeds 4000 board-feet of plywood, process 1 changes. In case A, it (process 1) doubles, in case B it is cut in half. Which cases (if any) can be appropriately included in the LP? Why? [3]

Doubling the output would define a non-convex feasible region, which cannot be treated by LP. Cutting production in half will still define a convex feasible region. The problem could be treated using piecewise linear approximation.

Show the equations you would use to incorporate the appropriate cases, if any. [6]

Let us define:

- X_{1A} units of process 1 before the output reaches 4,000
- X_{1B} units of process 1 after 4,000

The revenue associated with X_{1A} is $15 * 1,333 = $19,995
The revenue associated with X_{1B} is $15 * 1,333/2 = $9,997.50

The costs stay the same.

$$\text{MAX} \quad \text{Profit} = \text{Revenues - Costs}$$

$$\text{MAX} \quad \text{Profit} = [19{,}995 \quad 9{,}995 \quad 60.000]\begin{bmatrix} x_{1A} \\ x_{1B} \\ x_2 \end{bmatrix} - [6{,}500 \quad 6{,}500 \quad 14{,}600]\begin{bmatrix} x_{1A} \\ x_{1B} \\ x_2 \end{bmatrix}$$

$$s.t. \begin{cases} 1 \quad 1{,}000(x_{1A} + x_{1B} + x_2) \leq 200{,}000 \\ 2 \quad 500(x_{1A} + x_{1B}) + 1{,}850x_2 + 3{,}200x_3 \leq 320{,}000 \\ 3 \quad 2{,}500(x_{1A} + x_{1B}) + 4{,}200x_2 \leq 400{,}000 \\ 4 \quad 5{,}000(x_{1A} + x_{1B} + x_2) + 3(500(x_{1A} + x_{1B}) + 13{,}200x_2) \geq 1{,}000{,}000 \\ 5 \quad 1{,}333x_{1A} \leq 4{,}000 \quad \text{Limit of process 1 before output reaches 4,000} \end{cases}$$

Again, constraint 3 could be written as:

$$1{,}333(x_{1A} + x_{1B}) + 4{,}000x_2 \leq 400{,}000$$

SENSITIVITY ANALYSIS

4. Port Development

You represent a consultant for the government of Goldenland for a proposed investment in a new port in its country. The budget for the construction is supplemented by a loan in US dollars. Otherwise the budget is in terms of the national currency of "Goldens" (G) "M" indicates millions.

Your analyst used LP to maximize the benefits of the investment. She hands you the following results:

Constraint		Shadow Price(in G)	Range
Dollar loan	≤ $200 M	15G / $	$150 M ≤ b1 ≤ $220M
Probability of Oil Spill	≤ 0.1%	60,000,000G / %	0.08 ≤ b2 ≤ 0.7 %
Construction Time	≤ 5 years	0	3.5 ≤ b3
Local Workers	≥ 500 workers	20000 G / worker	420 ≤ b4 ≤ 700
Port Capacity	≥ 5 ships	10M G/ ship	4 ≤ b5 ≤ 6

Additionally, she reports that the Opportunity Costs for the tanker berth, the hydrofoil ferry and the pleasure marina are 150M, 30M and 20 M (in G), respectively.

Properly impressing officials from an international bank will increase the available budget by $10M. How many G's can the nation afford to spend on a "sales effort" directed at these officials? [4]

The maximum the Goldenland government should spend is 15 G/$ * $10M = 150M Goldens.

An oil spill would cost the tourist industry 100M G's. Would it pay to relax the environmental standards? [4]

The expected costs of a 0.1% increase in Probability of spill is 0.1 MG. Expected benefits of relaxing standards by 0.1% is: 60,000,000 G/% * 0.1= 6M Goldens, which is much greater than the expected cost of an oil spill for the tourist industry. Economically, it would pay to relax the environmental standards on the project.

Imported skilled labor could speed up construction, at some cost. Is this idea worth pursuing? [3]

The shadow price for the construction cost is zero, so the project has some slack in the schedule. It would therefore not pay to import skilled labor to speed up construction.

It has been suggested that the port be built to accommodate less than 5 ships and that service to excess ships be provided by barges costing 5M G per ship served. Should you design for 4 ships? For 3 ships? What additional information might you need to explore this issue? [5]

Lowering the capacity of the port to 4 ships would increase benefits (relax a constraint) by 10M G and cost 5M G to accommodate a fifth ship. Thus, I would design for four ships.

To know if lowering the capacity of the port to 3 ships is worthwhile, you need to know the shadow price in this range. ..and you don't, since you are lowering lower bound constraint, shadow price will drop and you cannot say anything about it, except that it is between 0 and 10G…

Under what new conditions might you agree to include "hydrofoil ferry" option in the design of the port? [3]

The hydrofoil option is not included because it costs too much. To be included in design, it would have to be cheaper by the amount of its Opportunity cost, i.e. 30M Goldens.