1 COMMUNICATIONS NET

Assume that there are 3 ways to assemble a communications system. Each uses switches and channels in different combinations. Each has its own cost and productivity. See Table below.

<table>
<thead>
<tr>
<th>System Concept</th>
<th>Switches</th>
<th>Channels</th>
<th>Output (M msg/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Satellite&quot;</td>
<td>100</td>
<td>200</td>
<td>60</td>
</tr>
<tr>
<td>&quot;Cellular&quot;</td>
<td>100</td>
<td>50</td>
<td>30</td>
</tr>
<tr>
<td>&quot;Central Office&quot;</td>
<td>200</td>
<td>250</td>
<td>80</td>
</tr>
<tr>
<td>$300/day each</td>
<td>$120/day each</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) Define an isoquant (3)

Locus of equal output on production function, all points on isoquant are technically efficient.

b) Plot isoquant for 60 M msg/day (6)

\[
\begin{array}{ccc}
\text{Switches} & \text{Channels} & \text{Budget Line} \\
200 & 50 & \\
(100, 100) & & \\
(100, 100) & & \\
(50, 50) & & \\
(0, 0) & & \\
\end{array}
\]

\[
\begin{array}{ccc}
S & C & O \\
8 & 100 & 200 \\
C_0 & 200 & 100 \\
C_0 & 150 & 180 \\
\end{array}
\]

c) Define "marginal rate of substitution" (MRS) (3)

The rate at which marginal increases in one output must substitute for marginal decreases in another output so that the total product remains constant.

d) Determine MRS as completely as feasible from above data (6)

see plot for question b)

Section I: \( \text{MRS}_{SC} = 0 \)

Section II: \( \text{MRS}_{SC} = -1 \)

Section III: \( \text{MRS}_{SC} = 0 \)

e) Define and Contrast Economic and Technical Efficiency (5)

Technical efficiency represents the maximum product that can be obtained from any given set of resources.

Economic efficiency represents the best design economically.

Economic efficiency informs technical efficiency, while technical efficiency does not necessarily lead to economic efficiency.
What is the price for switches that justifies the use of the "Cellular" concept? Assume cost of channel capacity is fixed.

As seen in the plot for part (d), the slope of the budget line must be less than \(-1\) to justify the usage of the "Cellular" concept. (budget line and the isocost first intersect at point \((200,100)\) when the budget line moves outward.)

\[
\text{or } \quad -\frac{\text{Cost of Channels}}{\text{Cost of Switch}} < -1
\]

\[
120 \quad \frac{\text{Cost of Switch}}{120} > 1
\]

\[
\text{or } \quad \text{Cost of Switch} < 120
\]
2. OPTIMIZING LAYOUT (25)

Assume that there are 3 ways to assemble a communications system. Each uses switches and channels in different combinations. Each has its own cost and productivity. See Table below.

<table>
<thead>
<tr>
<th>System Concept</th>
<th>Switches</th>
<th>Channels</th>
<th>Output (M msg/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Satellite&quot;</td>
<td>100</td>
<td>200</td>
<td>60</td>
</tr>
<tr>
<td>&quot;Cellular&quot;</td>
<td>100</td>
<td>50</td>
<td>30</td>
</tr>
<tr>
<td>&quot;Central Office&quot;</td>
<td>200</td>
<td>250</td>
<td>80</td>
</tr>
</tbody>
</table>

$300/day each $120/day each

Your company needs to distribute 80 M msg/day over the holidays. They have available 100 switches and 100 channels, and must pay for these in any case. The company could (if it wanted to) buy up to 700 more switches and 500 additional channels. They wish to minimize their costs.

a) What are the decision variables? (5)

- \( x_1 \): # of units of "Satellite" concept
- \( x_2 \): # of units of "Cellular" concept
- \( x_3 \): # of units of "Central Office" concept

b) Formulate as a linear programming problem. (10)

\[
\begin{align*}
\text{Min} & \quad S_a \cdot P_s + C_a \cdot P_c \\
\text{S.t} & \quad 60x_1 + 30x_2 + 80x_3 \geq 80 \\
& \quad 100x_1 + 100x_2 + 300x_3 \leq 100 + S_a \\
& \quad 200x_1 + 50x_2 + 250x_3 \leq 200 + C_a \\
& \quad x_1, x_2, x_3, S_a, C_a \geq 0
\end{align*}
\]

c) Explain why this problem could (or could not) be formulated as a linear program if the cost of additional channels were less than the normal rate of $120 per day. (4)

Current answer, the problem could be solved. Because 100 switches and channels need to be paid any way, so they can be removed from the objective function.

Guess the problem needs to be specified as: For switches under 100 and channels under 100, they can get it at normal rate. But if the number is bigger than 100, they can get prices different than the normal rate.

Explain why this problem could (or could not) be formulated as a linear program if the cost of additional switches were more than the $300 per day. (4)

see above.
1) Suppose the company now recognizes that, to implement the "central office" concept, they have to buy a plan that would cost $1,000/day plus $100 per 1 M msg/day. Could a linear program handle this possibility? Explain reasoning. (4)

It is a fixed charge problem. And a linear program usually cannot handle this possibility.

For this problem, we can divide it into two problems, one with "central office" concept, another without. Compare the optimal solutions for the 2 problems, choose the one with lower cost. This method only works for trivial fixed charge problems.
3. SYSTEM DESIGN CHANGE (20)

You company produces lawnmowers. You find that the California Environmental Protection Agency requires new standards for the lawnmowers your company sells in that state. Your problem is how to modify this product to maintain your market position.

Your engineers tell you that you have 2 choices. One involves additional insulation using known technology, and should meet the California requirements. The other pioneers fuel cell technology that might fail to meet the California requirements but could, if successful, position you to jump ahead of your competition significantly. See Table below.

<table>
<thead>
<tr>
<th>Approach to System Design</th>
<th>Possible Results of Design Approach</th>
<th>probability and financial return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Failure</td>
<td>Good Product</td>
</tr>
<tr>
<td>Standard</td>
<td>0.25</td>
<td>0.75</td>
</tr>
<tr>
<td>Fuel Cell</td>
<td>2/3</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Loss of $24 million</td>
<td>$16 million profit</td>
</tr>
</tbody>
</table>

a) Draw decision tree for company's choice, showing all outcomes and probabilities.

\[
\begin{align*}
\text{Good Product} & \quad 16 M \\
\text{Failure} & \quad -24 M \\
\text{Great Success} & \quad 60 M \\
\end{align*}
\]

b) Which approach is the better? Assume company will base decision on expected profit.

Standard.

c) Define the concept of "expected value of perfect information" (EVPI)

Perfect information represents the test results that would come from a hypothetical process that identifies exactly which event is true or what will happen. EVPI is the increase in expected value to be obtained from a situation due to the perfect information, without regard for the cost of obtaining it.

d) Calculate the EVPI for perfect information on the outcomes of the "Fuel Cell" approach

\[\text{EVPI} = 28 - 6 = 22 M\]
4. FUEL CELL TESTS (25)

You could pay $1 million to a lab research that would run tests on the Fuel Cell technology. You believe, based on your experience with this lab, that the lab anticipates future outcomes correctly 80% of the time (that is, for example, the lab will report "failure" 4/5 of the times when failure occurs). With their information, you would revise your prior estimate of the probability of success for the Full Cell technology, displayed in the Table from Problem 3:

<table>
<thead>
<tr>
<th>Approach to System Design</th>
<th>Possible Results of Design Approach – probability and financial return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Failure</td>
</tr>
<tr>
<td>Standard</td>
<td>0.25</td>
</tr>
<tr>
<td>Fuel Cell</td>
<td>2/3</td>
</tr>
<tr>
<td></td>
<td>Loss of $24 million</td>
</tr>
</tbody>
</table>

a) Explain what Bayes' formula does. (4)

The Bayes theorem is a process for revising estimates of probabilities given more information.

b) Write Bayes' formula (4)

\[ P(E|O) = \frac{P(O|E)P(E)}{P(O)} \]

where \( P(O) = \sum P(O|E_i)P(E_i) \)

c) What is the estimated probability of failure for the "Pioneer" approach, if the lab predicts failure? Show calculations that justify this conclusion (8)

\( E = \text{FAILURE} \quad O = \text{PREDICT FAILURE} \)

\[ P(E) = \frac{2}{3} \quad P(O|E) = 0.8 \]

\[ P(O) = P(O|E)P(E) + P(O|\overline{E})P(\overline{E}) = 0.8 \cdot \frac{2}{3} + 0.2 \cdot \frac{1}{3} = \frac{3}{5} \]

\[ P(E|O) = \frac{\frac{2}{3} \cdot 0.8}{\frac{3}{5}} = \frac{8}{9} \]

d) What is the estimated probability of "great success" for the "Pioneer" approach, if the lab predicts great success? Show calculations that justify this conclusion (4)

\( E = \text{GREAT SUCCESS} \quad O = \text{PREDICT GREAT SUCCESS} \)

\[ P(E) = \frac{1}{3} \quad P(O|E) = 0.8 \]

\[ P(O) = P(O|E)P(E) + P(O|\overline{E})P(\overline{E}) = 0.8 \cdot \frac{1}{3} + 0.2 \cdot \frac{2}{3} = \frac{2}{3} \]

\[ P(E|O) = \frac{1}{3} \cdot \frac{0.8}{\frac{2}{3}} = \frac{2}{3} \]
e) Define "expected value of sample information" (EVSI) (5)

Sample information is a set of actual observations as obtained in practice. It contains errors. EVSI is the increase in expected value to be obtained from a situation due to the sample information, without regard for the cost of obtaining it.

EXTRA CREDIT

Calculate the EVSI for the process prototype activity. (10)

\[ P(\text{PREDICT SUCCESS}) = P(\text{PREDICT SUCCESS} | \text{SUCCESS}) \cdot P(\text{SUCCESS}) + P(\text{PREDICT SUCCESS} | \text{FAILURE}) \cdot P(\text{FAILURE}) \]

\[ = 0.8 \cdot \frac{1}{3} + 0.2 \cdot \frac{2}{3} = \frac{3}{5} \]

\[ P(\text{PREDICT FAILURE}) = 1 - P(\text{PREDICT SUCCESS}) = \frac{2}{5} \]

\[ \text{EVSI} = 21.4M - 6M = 15.4M \]
5. CONCEPTS (21)

a) Valuations issues: What is the meaning of diminishing marginal value? How does the "threshold effect" influence the valuation function? What is risk aversion? (9)

**DIMINISHING MARGINAL VALUE**: THE FACT THAT PEOPLE COMMONLY ATTACH LESS AND LESS VALUE TO EACH ADDITIONAL UNIT OF A BENEFIT THAT THEY MIGHT RECEIVE.

**THRESHOLD EFFECT**: THRESHOLD WHERE A PERSON'S RISK ATTITUDE SWITCH BETWEEN "POSITIVE" AND "AVERSE".

**RISK AVERSE**. WHEN A PERSON'S CERTAINTY EQUIVALENT TO A LOTTERY IS LESS BENEFICIAL THAN THE CERTAIN AMOUNT EQUAL TO EXPECTED VALUE.

b) Describe the CAPM. What concept of value justifies this model? (8) OF THE LOTTERY.

**CAPM**: \[ R_p = R_f + \beta_p (R_m - R_f) \]

- \( R_p \): return of the portfolio
- \( \beta_p \): beta of the investment to be valued
- \( R_m \): expected return for market portfolio
- \( R_f \): risk-free rate.

The concept of "risk averse" justifies this model because people need risk premium.

c) Describe the lattice method for representing the evolution of a future state of some variable (such as the price of a stock). What are the computational advantages of this approach to representing future states? (8)

**LATTICE METHOD** reproduces uncertainty over time to simulate actual sequence of possibilities it approximates price changes as sequence of increases and decreases over stages.

Path independence permits implicit enumeration of paths so that we don't need to examine all paths to find the solution and effort of analysis is linear in number of stages.
6. WHAT'S IT WORTH? (37)

A pipeline company is designing an undersea pipeline to an off-shore field. It anticipates the possibility that this line could be used in a neighboring field. The basic design thus calls for a Y shaped layout, where the single line from the shore has a branch to the second field.

The company could install the branch now at an extra cost of $18 million. If the oil prices and the quantity of crude justify the full exploitation of the second field, the branch pipeline would be worth $40 million. However, there is a good chance, maybe 2 in 3, that the oil companies would only develop part of the second field, in which case the branch pipeline would worth only $10 million.

Alternatively, the company could lay a line to a single field, and enable the possibility of the branch link to the second field by creating a T-junction in the pipeline. If the oil companies decide to develop the second field, the pipeline company would then have to pay $20 million to lay the branch line.

Calculate the value of the T-Junction so that the Chief Designer for the Pipeline can decide whether this addition to the basic system is worthwhile.

You may assume for this exercise that:
- It will take 2 years to know if the second field will be developed and thus the value of pipeline;
- The pipeline could be sold to other companies at the stated values;
- The discount rate for the company is 15% annually, and
- The risk-free rate is 5% annually.

a) Define an option (4)
   - A right, but not obligation
   - to take some action now or in the future for a pre-determined price.

   It is a formal way to define flexibility.

b) What kind of option is does this situation suggest? Explain your answer. (4)

   Call option
   A right to build, not an obligation though. The pipeline company could build the branch or abandon it now or in the future for the price of $20 million.

c) Draw a diagram showing the payoffs from the option in this venture. (4)

\[ \text{Payoff} \]

\[ \text{20M} \quad \text{Value of Pipeline} \]
d) Calculate the value of the option using decision analysis

Tree with option:

\[
\begin{array}{c}
\text{Option} \quad \text{Good Case} \quad D \quad \text{Bad Case} \quad D \quad \text{Abandon} \quad \rho
\
\text{Value of Option} = 6 \frac{2}{3} - 2 = 4 \frac{2}{3}
\end{array}
\]

Tree without option:

\[
\begin{array}{c}
\text{Good Case} \quad \frac{2}{3}
\
\text{Bad Case} - 8
\end{array}
\]

Value of Option = 6 \frac{2}{3} - 2 = 4 \frac{2}{3}

(4)

e) Define the concept of arbitrage enforced pricing of options.

**Arbitrage Enforced Pricing**

The possibility of setting up a risk-free portfolio to balance option absolutely defines prices of option.

(5)

f) Calculate the arbitrage enforced price for the option on this project.

<table>
<thead>
<tr>
<th>Period</th>
<th>Start</th>
<th>End</th>
<th>End</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset Price</td>
<td>18</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Buy Stock</td>
<td>18</td>
<td>10</td>
<td>40</td>
</tr>
<tr>
<td>Borrow Money</td>
<td>(\frac{10}{1+i})</td>
<td>-10</td>
<td>-10</td>
</tr>
<tr>
<td>Net</td>
<td>(-18 + \frac{10}{1+i})</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>Buy Call</td>
<td>0</td>
<td>0</td>
<td>20</td>
</tr>
</tbody>
</table>

\[
\text{Buy Call at Strike (20) } = -C
\]

\[
\text{Value of Option} = -18 + \frac{10}{1+i} = -18 + \frac{10}{1.05} = -18 + 9.52
\]

\[
C = (18 - 9.52) \times \frac{2}{3} = 6.55
\]

(15)

g) Discuss and explain the similarity or difference you get between the value of the option given by decision analytic and by the arbitrage enforced pricing analysis.
7. **USE OF REAL OPTIONS** (25)

In your field of professional interest, define a significant "real option" that might be worth using for improving the expected performance of the a system you are professionally interested in.

b) Describe this example (5)

BUILDING DAMS ON A RIVER BASIN

d) Is this option: a put or a call? European or American? "In" or "on" the system? (5)

CALL, AMERICAN
IT IS "IN" THE SYSTEM.

e) Identify the cost of acquiring the option (3)

NO SIGNIFICANT COSTS
THEY ARE "NATURAL" OPTIONS.

d) Define the cost of exercising the option (3)

INVESTMENT IN THE DAM

e) Define the conditions that would make it worthwhile to exercise (3)

ELECTRICITY PRICE HIGH.
1) Outline how you might go about analyzing the value of this option, indicating the factors that would lead you to prefer one approach over another. (5)

*BINOMIAL TREE + INTEGER PROGRAMMING*

Because of the complexity and interdependency of options, *"arbitrage enforced pricing" hard to use because the illiquidity of the assets.*