

Engineering Systems Analysis for Design**Mid-Semester Quiz****October 25, 2005**

This is a closed book exercise. Computers and other wireless enabled devices for communication with web and outside are not allowed.

You may use old-fashioned, non-communicating calculators (if you have them).

Grade Tables

There are 90 points possible for the regular test. Points associated with each question correspond to the estimated time it might take to answer them. .

Item	Score	
	Max	Yours
Your Name (provided we can read it)	1	
Concepts	22	
What's the best design?	18	
Static valuation of projects	19	
Decision Analysis and Value of Information	12	
Garage Example	18	
Total	90	

I have completed this test fairly, without copying from others, a book, or the web.

Please sign your name legibly _____ (1 point)

Feedback voluntary question (no credit):

Suggest a CD for classroom enjoyment in second half of semester:

Concepts (22 points -- 2 points per part)**Note: Full marks only for conceptually precise responses**

Write a short definition or description explaining the following:

Production Function

The functional relationship between a set of inputs and the maximum output. Commonly modeled for the one-output form via the Cobb-Douglas function.

Criterion for Technical Efficiency

The maximum production level for a given set of inputs that can not be exceeded without adding more of some inputs.

Economic Efficiency

$$MP_x/MC_x = MP_y/MC_y$$

Isoquant

A collection of technically efficient input combinations that yield the same level of output.

Increasing Returns to Scale

The relationship when the same change in all input levels leads to a greater corresponding change in the output level.

Economies of Scale

The relationship between a change in production level and the corresponding increase in cost. When the production costs increase less rapidly than the level of production.

$$C = a \cdot y^b \quad \text{where } b < 1.0$$

Expansion Path

Cost minimizing sets of inputs (isoquant isocost tangencies) along a set of isoquants. Locus of inputs that define economically efficient design.

Output Cost Function

Cost minimizing inputs for a given level of production.

$$C = f(Y)$$

Relationship between minimal cost and production level

Discount Rate

Risk-adjusted required rate of return for a given (or marginal) project/asset.

WACC

Weighted Average Cost of Capital – pretty much what it says.

Historical, proportionally weighted rate of return (from equity & debt obligations)

CAPM

Capital Asset Pricing Model – relative risk weighted linear model of returns

Linear risk-return extrapolation model based on asset's correlation with the market returns

$$R_i = R_f + \text{Beta} * (R_{\text{market}} - R_f)$$

What's the best design? (18 points)

You are given a production function: $4 R^{0.6} S^{0.3}$

And the cost of the resources as: $2 R^{0.4} + 0.5 S^{0.8}$

[Note: $a^{(b)}$ means "a" raised to the power of "b"]

Note: In calculating answers, you may leave exponents in fractional form rather than estimating numbers in decimal form. For example, (.4) exp (2/3) would be acceptable.

- a) What can you say immediately, by inspection, about the returns to scale? The economies of scale? Explain answer (3 points)

Sum of exponents in the production function = $0.9 < 1 \Rightarrow$ decreasing R.S.
Economies of scale \Rightarrow too early to tell \Rightarrow see part (d).

- b) What is the optimal relationship between the resources R and S? (6 points)

$$\frac{MP_R}{MP_S} = \frac{MC_R}{MC_S} \Rightarrow \frac{2.4 R^{-0.4} S^{0.3}}{0.3 S^{-0.2}} = \frac{0.8 R^{-0.6}}{0.4 S^{-0.2}}$$

$$\frac{S}{R} = \frac{S^{0.2}}{R^{0.6}}$$

$$S^{0.8} = R^{0.4}$$

$$S = R^{0.5} \Leftrightarrow \boxed{R = S^2}$$

- c) What is the associated cost function? (6 points)

$$P = 4 \cdot S^{2 \times 0.6} \cdot S^{0.3} = 4 \cdot S^{1.5} \Rightarrow S = \left(\frac{1}{2.5} \cdot C\right)^{1.25}$$

$$C = 2 \cdot S^{2 \times 0.4} + 0.5 \cdot S^{0.8} = 2 \cdot S^{0.8} + 0.5 \cdot S^{0.8} = 2.5 \cdot S^{0.8} \Rightarrow S = \left(\frac{1}{2.5} \cdot C\right)^{1.25}$$

$$= 2.5 \left(\frac{1}{4} P\right)^{1.5 \times 0.8} = 2.5 \cdot \left(\frac{1}{4}\right)^{1.2} \cdot P^{1.2}$$

$$= 1.19 * P^{0.53}$$

- d) What do you now say about the economies of scale? (2 points)

Yes \Rightarrow increasing economies of scale

P \Rightarrow C
[2*x] [1.48*x]

Sum of exponents < 1.0

Static Valuation of Projects (19 points)

Consider the project with the following revenues and costs:

	Year				
	0	1	2	3	4
Revenues		200	400	600	600
Costs	600	90	158	200	240
Net Revenues	-600	110	242	400	360
$(1+r)^{\text{exp}N}$	1	1.1	1.21	1.331	1.4641
Present value	-600	100	200	300	246

Assume a discount rate of 10%.

a) Define Net Present Value and calculate it for this case (6 Points)

$$\begin{aligned} \text{NPV} &= \text{Differences between the discounted (Present Values) of Total Benefits and the discounted Total Costs} \\ &= -600 + 846 = 246 \end{aligned}$$

b) How would you calculate the benefit-cost ratio? What are the major advantages and disadvantages of the Benefit/Cost ratio as a criterion of evaluation? (7)

$$\begin{aligned} \text{B-C ratio} &= \text{PV (All Benefits)} / \text{PV (All Costs)}. \\ \text{When NPV} \geq 0, &\Rightarrow \text{B-C ratio} \geq 1. \end{aligned}$$

Advantages: simple to calculate and rate the alternatives; unit-less; $\text{BC} > 1 \Rightarrow$ good project
Disadvantages: can not take to the bank; biased against recurring costs

b) Define and Calculate the Pay Back Period. What are the major advantages and disadvantages of this criterion of evaluation? (6 points)

$$\text{Pay Back Period} = \text{Cost} / (\text{Undiscounted annual Cash Flows}), \text{ or the time period that would be required to recoup undiscounted initial investment} =$$

$$= 2 \text{ years} + 246/400 = 2.615 \text{ years}$$

Advantages: useful when short turn-around is a priority; really simple; avoids the difficulty of picking the proper discount rate; avoids speculative long-term forecasts;
Disadvantages: ignores cash flows after the initial pay-back period (that could turn negative), useless for ranking projects with lives beyond the shortest pay-back period

Effect of Uncertainty on Design (15 points)

a) Under what circumstances is the value of a project, when calculated on the basis of the most likely forecasts, the same as its value when calculated for the range of possible scenarios that lead to those most likely forecasts? (3)

IFF: $f[E(x)] = E[f(x)]$ i.e.: Jensen's inequality does NOT hold.
 \downarrow
 $n=1$ or $x_1 = x_2 = \dots = x_n$
 $f(x)$ is linear & convex & continuous

b) Therefore, how likely is it that the valuation based on the most likely forecasts are correct? (2)

Very unlikely

c) What are the advantages of a staged approach that allows the designer to achieve a capacity through several increments, instead of one? (2)

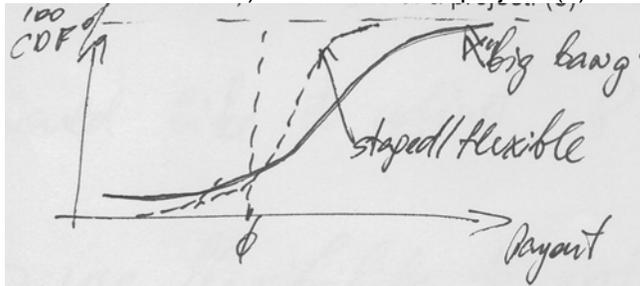
Flexibility attained by delaying investment commitments until:

- Forecasts become more precise by incorporating more up-to-date data
- Uncertainty is mitigated with time
- Ability to terminate investments into the project if the business plan does not pan out (value of the Put option)

d) What are the disadvantages of the staged approach? (2)

- Likely higher upfront/fixed costs to reserve the flexibility (cost of the Put option)
- Additional design/modeling complexity

e) Illustrate how a staged approach could affect the VAR (Value at Risk or the Cumulative Distribution Function) of the value of a project. (4)



Minimizes downsides
 Minimizes distribution of outcomes (risk)
 Can take advantage of the upsides.

Decision Analysis (39 points)

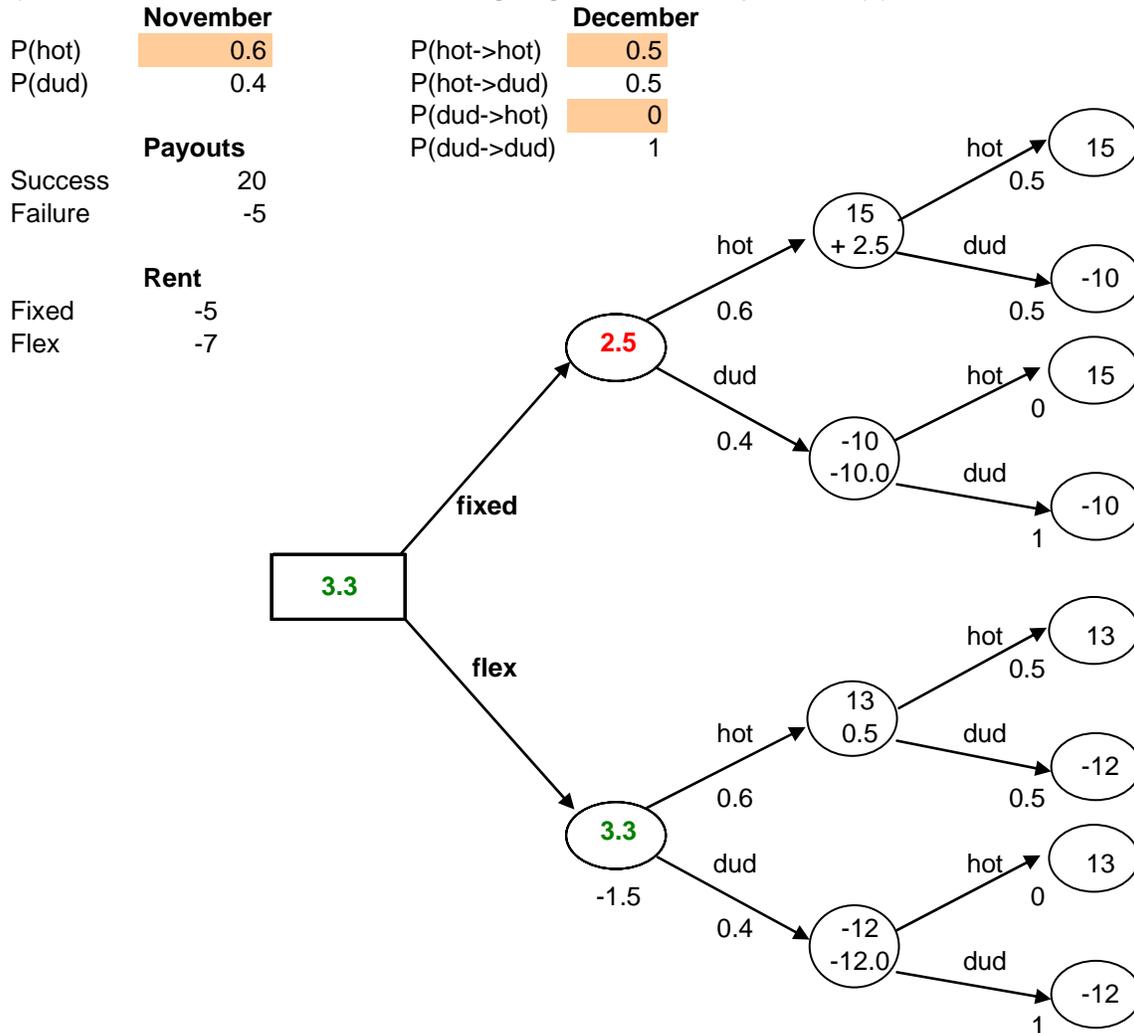
You are advising friends who are going to invest in the production of their invention: some electronic holiday accessory. They estimate they have a 60% chance of a “hot” product which will get them 20K in November (before the cost of space). If the product is “hot” in November, there is a 50% chance it will also be “hot” in December, for another 20K (before the cost of space). If the product is not hot in November, it is and remains a “dud”, and they will lose 5K a month if they continue to produce it (in addition to any cost of space).

These friends face two possibilities for getting space:

- Sign a definite two month lease for production space. This will cost them a rental of 5K a month for both November and December. (A total of 10K for sure)
- Lease space one month at a time at a cost of 7K per month. After they see how their product is received in November, they can decide if they want to continue production and to lease this space for December also.

In any case, they can decide whether to continue production in December. If they decide not to, they will neither make nor lose money from their operations.

a) Draw the decision tree for this choice, giving all information provided. (4)



b) Which is the best initial choice? (4)

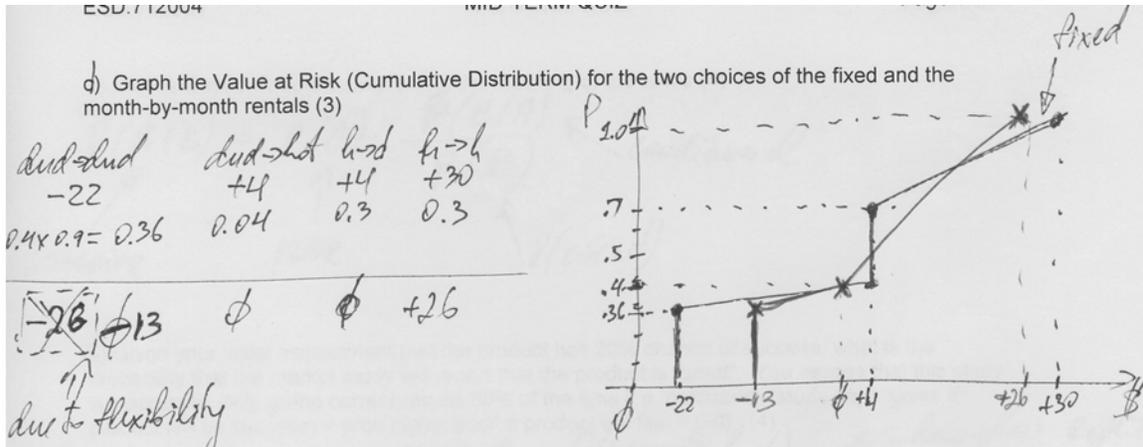
Go with flexible rent.

c) Define the optimal strategy over the two months (2)

Be flexible:

- If "dud" => stop production.
- If "hot" => keep going

e) Graph the Value at Risk (Cumulative Distribution) for the two choices of the fixed and the month-by-month rentals (3)



f) Your friends are concerned about the uncertainty associated with whether the product will be a "hit" or a "dud" in November, and are thinking about getting some extra information that would help you make your choice. Define the concept of the Expected Value of Perfect Information, and calculate its value for this case. (5)

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Scenario	Probability	Payoff
If: "hot → hot" ⇒ go fixed = +15 + 15 = +30	0.6 × 0.5 = 0.3	9
"hot → dud" ⇒ go flex = max(+4, 43) = +13	0.6 × 0.5 = 0.3	3.9
"dud → hot" ⇒ go fixed = +4	0.4 × 0.1 = 0.04	0.16
"dud → dud" ⇒ go flex, then stop = -13	0.4 × 0.9 = 0.36	-4.68
EVPI = (Best Possible) - (Expected) = 8.38 - 2.6 = 5.78	1.00	8.38

g) Define Bayes' Theorem, explaining the meaning of the terms (4)

$$P(A | B) = P(A) * P(B | A) / P(B)$$

↑
↑
↑
↑

 Posterior Prior Conditional Likelihood of additional info

h) The market study has a history of accurately predicting correct outcomes 60% of the time [i.e.: study says "great" if product is hot]. The same study has a documented of producing "great" and "poor" assessment equally frequently. Given your initial assessment that the product has 60% chance of success, what, if anything, can you learn from this study?

product will be success) = prob (says 'poor' if product will fail) = 0.6] (4)

$$P(\text{hot} | \text{"great report"}) = P(\text{hot}) * \frac{P(\text{"great"} | \text{hot})}{P(\text{"great"})} = \leftarrow \text{law of total prob} \leftarrow \text{frequency of event}$$

$$= 0.6 * \frac{0.6}{0.5} = 0.72$$

j) Calculate how the test results would change your initial assessment of success? (8)

EV-fixed (using Bayesian probability updates) = 7.0

EV-flex (using Bayesian probability updates) = 6.4

k) What is the expected value of the sample information? Is it worthwhile to buy the market survey? (6)

EV (Best Outcome with Bayesian probability updates) – EV (Best Outcome before) = 7.0 – 3.3 = 3.7

Note:

EVPI > EVSI > EV

10.5 > 7.0 > 3.3

EVPI = 0.6 * (Outcome if knew it was going to be hot) + 0.4 * (If knew it's a 'dud') =
= 0.6 * 17.5 + 0.4 * 0 (If knew it's a dud => don't even start) = 10.5