Structure of the Exam
The questions refer to various aspects of a hypothetical renewal energy project. You will want to start at the beginning and carry on through.

Precision of Answers
Recognizing the great uncertainties associated with future situations, and the dubious value of meaningless significant figures, you will:

   **GIVE RESULTS USING TWO-DECIMAL PLACE ACCURACY**

Grading
The concepts are the focus of the exam. You will earn most of your points by demonstrating that you know what the correct procedures are, and how to use them. You will do this by ‘running the numbers’. However, in this exam the numbers themselves are not too important – points will be deducted for mistakes, but neither the course nor the exam is focused on arithmetic.

Because the focus is on the concepts, you should clearly indicate how you going about each part of the exam so that we can give you the credit you deserve.

If the numbers are wrong and you have not explained your analysis, we will not be able to give you credit.

   **BEST WISHES!**
1. Future Demand for Power (17 pts)

You are now investigating the possible future demand for power that you might produce from a prospective renewable energy farm supplying a resort on a Caribbean island.

Your consultants estimate that the demand will:
- Start at the rate of 1,000,000 kWh during the first year, based on negotiations with the resort operator;
- Grow at an average rate of 7% per year;
- With a standard deviation of 35%.

To cover a reasonable future without getting into excessive detail, you decide to consider 2 blocks of 4-year periods.

Useful formulas:
\[ p = 0.5 + 0.5 \left( \frac{v}{\sigma} \right) \sqrt{\Delta T} \]
\[ u = e^{\sigma \sqrt{\Delta T}} \]
\[ d = \frac{1}{u} \]

a) Using the data table on next page, what are the values for \( p, u, d \)? (7 pts)

b) Fill in the Tables (10 pts)

**Table of Probabilities at nodes**

<table>
<thead>
<tr>
<th></th>
<th>Year 0</th>
<th>Year 4</th>
<th>Year 8</th>
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**Table of Demand at end of each 4-year period (in millions of kWh)**

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Image of Data Table defining formula

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*Table of values for $e^{\sigma \sqrt{\Delta t}}$*
2. **Power Economics** (35 points)

According to your suppliers, there your renewal energy equipment comes in 2 sizes:
- **Big**: for 3,000,000 kWh /year
- **Small**: for 1,500,000 kWh /year

Their corresponding costs of building the plants and producing power are:

<table>
<thead>
<tr>
<th></th>
<th>CAPEX $</th>
<th>Operating cost $/kWh</th>
</tr>
</thead>
<tbody>
<tr>
<td>Big Plant</td>
<td>700,000</td>
<td>0.02</td>
</tr>
<tr>
<td>Small Plant</td>
<td>350,000</td>
<td></td>
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</tbody>
</table>

a) Define returns to scale.
What is a suitable test to know if a technology exhibits increasing returns to scale?
Does the renewable energy technology represented by the above cost data demonstrate increasing returns to scale?  **(8 points)**

b) Define economies of scale.
What is a suitable test to know if a production situation exhibits economies of scale?
Does the renewable energy technology case represented by the above cost data demonstrate increasing economies scale?  **(7 points)**
c) Calculate the costs of meeting the demand using Big and Small alternatives
Remember that each plant has a maximum capacity (10 points)

Table of Demand at end of each 4-year period (in millions of kWh) -- from previous analysis

<table>
<thead>
<tr>
<th>Year 0</th>
<th>Year 4</th>
<th>Year 8</th>
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</tbody>
</table>

Costs ($ Thousands), at end of 4 year period, for each size of plant

<table>
<thead>
<tr>
<th>Big Plant (up to 3M kWh)</th>
<th>Small plant (up to 1.5M kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>700</td>
<td>350</td>
</tr>
</tbody>
</table>


d) Calculate the net revenue from operations for each plant at the end of the appropriate period (10 points)

Assume that the revenues from power will equal $ 0.10 / kWh (close to current US national cost)

Net Revenues from Operations, in last year of 4 year period, for each size of plant

<table>
<thead>
<tr>
<th>Big Plant</th>
<th>Small plant</th>
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<tbody>
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<td></td>
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</table>
3 Decision Analysis (40 points)

Being under pressure to determine which of the two systems should be purchased, you set out to do a decision analysis of the choice between the big and the small plant.

a) Draw the decision tree describing the problem (8 points)

b) Enter your information on the probabilities and net revenues at the end of each period (10 points)
c) Now, estimate the net revenues over each 4 year period associated with each end state of power demand. **For simplicity, omit any discounting of revenues over time.** Use the formula below:  
\[(12\text{ points})\]

Estimated net revenue over period \(= 2 \times [(\text{Revenue at start of period}) + (\text{Revenue at end of Period})] \]

<table>
<thead>
<tr>
<th>Path</th>
<th>Net Revenues</th>
<th>Path</th>
<th>Net Revenues</th>
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<tbody>
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<td>up</td>
<td></td>
<td>Up – up</td>
<td></td>
</tr>
<tr>
<td>down</td>
<td></td>
<td>Up – down</td>
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<td></td>
<td></td>
<td>Down - up</td>
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<td>Down - down</td>
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</table>

This is for Big plant

<table>
<thead>
<tr>
<th>Path</th>
<th>Net Revenues</th>
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</thead>
<tbody>
<tr>
<td>up</td>
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<tr>
<td>down</td>
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</table>

This is for Small plant

<table>
<thead>
<tr>
<th>Path</th>
<th>Net Revenues</th>
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<td>Up – up</td>
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<tr>
<td>Up – down</td>
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<td>Down - up</td>
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d) Which technology choice maximizes Expected Net Revenues?  \(10\text{ points}\)
4. **Value at Risk and Gain (VARG)** (30 pts)

a) Sketch the VARG diagram for each plant. Label axes and grid and draw accurately. **(13 pts)**

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b) Fill in the Table to compare the two Technologies according to the criteria indicated below. **(10 points):**

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Big Plant</th>
<th>Small Plant</th>
<th>Which better?</th>
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</thead>
<tbody>
<tr>
<td>Expected Value</td>
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<tr>
<td>CAPEX</td>
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<td>( \frac{B/C}{E(V)/CAPEX} )</td>
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<td>Minimum Value</td>
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<td>Other you Define</td>
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Note: for simplicity, omit any discounting of revenues over time

d) Which do you feel is better? Discuss and justify your response. **(7 points)**
5. **Value of Information (29 points)**

You recognize that your preferred choice could lead to problems:
- If you choose the big plant, it may be unprofitable if demand does not grow;
- If you choose the small plant, you may miss out on upside opportunities.

You therefore plan to get additional information that could reduce your uncertainty. To help you organize this, you will use the concept of EVPI.

a) Your boss asks: “What’s this ‘perfect information’ stuff?”? “Nobody’s perfect!”
You then explain the concept of EVPI using the following words: (5 pts)

b) In this specific case, how might perfect information about whether the growth in the second 4 year period is “up” (if not, it is “down”) change your perception of the decision problem you analyzed in part 3? A descriptive rather than a numerical answer is required here. (4 points)

c) Now draw the diagram showing the decision tree associated with the decision to engage in process that gives perfect information about whether the growth in the second 4 year period is “up” (if not, it is “down”) (8 pts)

c) What is your estimate of EVPI in this case? (8 pts)

d) What is the most money you might justify spending to improve your information? Explain this choice. (4 pts)
6. **Option Value** (28 points)

The small plant can be expanded! Now let's look into what it might be worth to add another 1,500,000 kWh unit at the end of the first 4 years. 

For this expansion, the CAPEX of the second small plant is much less, since you have carefully arranged the site and done a deal with the manufacturer. The CAPEX for the second small increment is now only $200,000.

a) Under what circumstances would this have any value? Explain reasoning. (6 points)

b) What might be the operating costs of adding this increment and running it over the period? (5 points)

c) What might be the extra revenues of adding this increment? (4 points)

d) What is the expected value of having the increment? (3 points)

e) Is the option worth its strike price? (2 points)
f) How does option to double the installation of the small plant change the VARG you had previously for the small plant? *(8 points)*

THANK YOU FOR YOUR PARTICIPATION IN THE CLASS. ALL BEST WISHES!!!