

**Engineering Systems Analysis for Design****Mid-Semester Quiz****October 23, 2007**

**This is a closed book exercise. Computers and other wireless enabled devices for communication with web and outside are not allowed.**

**You may use old-fashioned, non-communicating calculators that are not miniature computers with extensive memory (if you have them! And if so, show me first...)**

## Grade Table

There are 90 points possible for the regular test. Points associated with each question correspond to the estimated time it might take to answer them. .

Item	Score	
	Max	Yours
Your Name (provided we can read it)	1	
Concepts	26	
What's the best design?	21	
Static valuation of projects	11	
Effect of Uncertainty on Design	13	
Decision Analysis	18	
Total	90	

I have completed this test fairly, without copying from others, a book, or the web.

**JOHN Q. SOLUTION**

Please sign your name legibly \_\_\_\_\_ (1 point)

**Feedback voluntary question (no credit):**

Suggest a CD for classroom enjoyment in second half of semester:

**Concepts (26 points)****Note: Full marks only for conceptually precise responses**

Write a short definition or description explaining the following:

**Technical Efficiency (2 points)**

The maximum production level or amount of output **(1 pt.)** that can be achieved for a given **(1 pt.)** set of inputs.

**Production Function (2 points)**

...is the functional relationship between a set of inputs **(1 pt.)** and the maximum or technically efficient **(1 pt.)** output. Commonly modeled for the one-output form via the Cobb-Douglas function **(0 pts.)**.

**Economic Efficiency – define and contrast with Technical Efficiency (2 points)**

Economic efficiency is defined by combination of technical efficiency (production function) **(1 pt.)** and economic (in general, value) **(1 pt.)** relationships. Defining economic efficiency as the most economic output was not acceptable; you need to define economic! The difference between economic and technical efficiency is that the former maximizes value **(1 pt.)**. Two points max.

**Criterion for Attaining Technical Efficiency (2 points)**

Combination of inputs **(1 pt.)** such that the highest level of output is reached **(1 pts.)** produces a technically efficient design.

**Isoquant (2 points)**

The set of all input combinations that yield the same output **(1 pt.)** in a technically efficient **(1 pt.)** process.

**Increasing Returns to Scale (2 points)**

Occur in a production function when the same change in scale in **all** input levels **(1 pt.)** leads to a greater corresponding change in the scale of the level of output **(1 pt.)**.

**GRADING NOTE:** If the element of efficiency is left off of both IRTS and EOS, only one point was deducted.

Economies of Scale – define and contrast with IRTS **(3 points)**

Occur in production when the cost does not increase as fast as the production level given an optimal combination of inputs **(1 pt)** required to produce that output level (in a technically efficient process.) **(1 pt.)**. Definition and contrast with IRTS as defined above (1 pt).

Expansion Path **(2 points)**

The set of inputs that minimize cost **(1 pt.)** for all levels of production **(1 pt.)** or the locus of input combinations that define economically efficient design

Output Cost Function -- define and compare with input cost function **(3 points)**

Total cost as a function of the output **(1 pt.)**, assuming technical and economic efficiency **(1 pt.)**. The input cost is the relationship between the inputs that create the budget cost available **(1 pt)**.

Discount Rate -- define and compare with interest rate **(3 points)**

Rate used to evaluate the cash flows of a project over time **(1 pt.)**.  
The discount rate represents the opportunity cost of the resources used for the project **(1 pt.)**  
It differs from the interest rate in that it adds a return premium representing the risk of the project **(1 pt.)**  
Two points max.

WACC – define concept and discuss advantages and disadvantages for use as discount rate **(3 points)**

Weighted Average Cost of Capital **(1 pt.)**  
... the rate of return achieved by an organization on previous projects having similar risk **(1 pt.)**  
... or as average rate required by lenders and investors external to the project. **(1 pt.)**  
... often used as a measure of the discount rate that should be used for an average project **(1 pt.)**  
... In financial terms, this is  $R = R_e \times E/(E+D) + R_d \times D/(E+D) \times (1-t)$  **(1 pt.)**  
Two points max.

**What's the best design? (21 points)**

You are given a production function:  $Z = R^{0.3} S^{0.6}$

And the cost of the resources as:  $C = 1.5 R^{0.8} + 2 S^{1.2}$

**Note: In calculating answers, you may leave exponents in fractional form rather than estimating numbers in decimal form. For example,  $0.4^{2/3}$  would be acceptable.**

- a) What can you say immediately, by inspection, about the returns to scale? The economies of scale? Explain answer **(3 points)**

RTS:  $\alpha_R + \alpha_S = 0.9 < 1$                       Therefore DRTS                      **(2 pts.)**  
EOS: it is too early to tell                      **(1 pt.)**

We need the output cost function and look at how cost behaves as we increase the output level, which is why it is too early to say anything.

- b) What is the economically efficient relationship between the resources R and S? **(6 points)**

$$\frac{MP_R}{MP_S} = \frac{MC_R}{MC_S} \Rightarrow \frac{S}{2R} = \frac{1}{2R^{0.2}S^{0.2}} \quad \textbf{(4 pts.)}$$

$$S^{1.2} = R^{0.8} \Rightarrow S = R^{2/3} \quad \textbf{(2 pts.)}$$

- c) What is the associated cost function? **(8 points)**

Express R and S as a function of Z

$$Z = R^{0.3} S^{0.6} = R^{0.3} (R^{2/3})^{0.6} = R^{0.7} \quad \textbf{(4 pts.)}$$

$$\Rightarrow R = Z^{1/0.7}, S = (Z^{1/0.7})^{2/3} = Z^{2/2.1}$$

And plug into the input cost function

$$C = 1.5(Z^{1/0.7})^{0.8} + 2(Z^{2/2.1})^{1.2} = 1.5Z^{0.8/0.7} + Z^{2.4/2.1} \quad \textbf{(4 pts.)}$$

$$\Rightarrow C^* = 3.5Z^{8/7}$$

- d) What can you now say about the economies of scale? Explain why this is so **(4 points)**

Expressing  $C^*$  as a function of  $Z^*$  shows that  $C^*$  increases with a factor of  $(8/7)$  of the increase in  $Z^*$ , or  $a=(8/7)>1$  **(2 pts.)**. This case shows diseconomies of scale. **(2 pts.)**

**Static Valuation of Projects (11 points)**

Consider the project with the following revenues and costs:

	Year				
	0	1	2	3	4
Revenues		60	100	220	380
Costs	190	12	28	47	70
Net Cash Flow	<b>-190</b>	<b>48</b>	<b>72</b>	<b>173</b>	<b>310</b>
(1+r) <sup>N</sup>	<b>1</b>	<b>1.2</b>	<b>1.44</b>	<b>1.73</b>	<b>2.07</b>
Present value	<b>-190</b>	<b>40</b>	<b>50</b>	<b>100</b>	<b>150</b>

Assume a discount rate of 20%. Use no more than 3 significant figures

Answers that used approximate values were accepted.

a) Define Net Present Value and calculate it for this case **(5 Points)**

Net present value is the sum total of net cash flows **(1 pt.)** appropriately discounted to the present time **(1pt.)** using appropriate discount factor.

Here, NPV = 150. **(3 pts.)**

b) How would you calculate the benefit-cost ratio? What are the major advantages and disadvantages of the Benefit/Cost ratio as a criterion of evaluation? **(3 points)**

The benefit-cost ratio is the ratio of the present values of all benefits to all costs. **(1 pt.)** That is, while the NPV can simply discount the net cash flow for each year, the B-C ratio discounts each separately.

Formula for benefit-cost ratios:

$$\frac{\text{Benefit}}{\text{Cost}} = \frac{PV \text{ all benefits}}{PV \text{ all costs}} = \frac{PV B}{PV C_k + C_r}$$

$$\frac{\text{Net Benefit}}{\text{Cost}} = \frac{PV \text{ net benefits}}{PV \text{ investments}} = \frac{PV B - C_r}{PV C_k}$$

B is the benefit (revenue), C<sub>k</sub> is capital investment, and C<sub>r</sub> is recurring cost.

**NOTE: THIS PROBLEM DOES NOT ASK YOU TO CALCULATE THE B-C RATIO**

Advantages: simple to calculate; allows a rating of the alternatives; unit-less **(1 pt.)**

Disadvantages: biased against recurring costs **(1 pt.)**; does not give a definitive ranking; may disagree with NPV

In high recurring costs projects, C<sub>r</sub> is high compared to C<sub>k</sub> because it is not a capital-intensive project. Given two projects, one a capital-intensive one and one non capital-intensive, having the same C<sub>k</sub>, the ratio will be higher for the capital intensive project because C<sub>r</sub> is smaller, even though the NPV of the recurring cost project might be higher. This causes the bias against recurring projects with benefit-cost ratio approach.

c) Define and Calculate the Pay Back Period. What are the major advantages and disadvantages of this criterion of evaluation? **(3 points)**

Pay Back Period is the time period that will be required to recuperate the initial investment as based upon undiscounted **(1 pt.)** future cash flows

Formula for payback period:

$$\text{Payback Period} = \frac{\text{Initial Investment}}{\text{Annual Net Undiscounted Benefits}}$$

But this formula assumes the same annual net undiscounted benefits every year!

Here, we have different undiscounted net benefits every year. So we need to add up the benefits every year until they get to 190, which is the initial investment. The first two years contribute  $48 + 72 = 120$  towards this. So we already have 2 years counted. The third year, we only need  $190 - 120 = 70$  out of the 173 available, which represents  $70/173 = 40\%$  of the year. So we count 2 years + 0.4 year = 2.4 years total **(1 pt.)**.

$$PBP = 2 \text{ years} + \frac{190 - (48 + 72)}{173} = 2.4 \text{ years} \quad \text{(1 pt.)}$$

Advantages: useful when short turn-around is a priority; really simple; avoids the difficulty of picking the proper discount rate; avoids speculative long-term forecasts **(1 pt)**

Disadvantages: ignores cash flows after the initial pay-back period (that could turn negative), projects could become very profitable later on!; useless for ranking projects with lives beyond the shortest pay-back period; does not account for the time value of money **(1 pt.)**

Three points max, but must calculate the PBP correctly and have partial answers for each part in order to receive full credit.

**Effect of Uncertainty on Design (13 points)**

a) Under what circumstances is the value of a project, when calculated on the basis of the most likely forecasts, the same as its value when calculated for the range of possible scenarios that lead to those most likely forecasts? **(3 points)**

$$E[f(x)] = f(E[x])$$

If and only if  $f(x)$  is linear and continuous.

**(3 pts.)**

Partial credit given for something about Jensen's inequality

b) So, how likely is it that the valuation based on the most likely forecasts are correct? **(2 points)**

Since most engineering systems behave in a non-linear manner, this is very unlikely **(2 pts)**.

c) What are the advantages of a staged approach that allows the designer to achieve a capacity through several increments, instead of one? **(2 points)**

Flexibility may provide:

Lower initial cost

The possibility to defer investment to later, lowering present value of cost

More time to gather information and up-to-date data for a more informed decision

Uncertainty is mitigated with time – that is, investment decisions involve less guesswork, which makes them more appropriate for the given conditions

Ability to terminate investments into the project if the business plan does not pan out

**(1 pt. each, max 2)**

d) What are the disadvantages of the staged approach? **(2 points)**

Flexibility may have an upfront cost (the purchase cost of the option), which can increase the overall cost per unit.

Possible loss of economies of scale not related to the option

Additional design/modeling complexity

“Larger initial cost” is **not** accepted unless students fully explained what they mean. Total initial cost, including the cost of flexibility, may be less for some initial designs (e.g. parking garage)

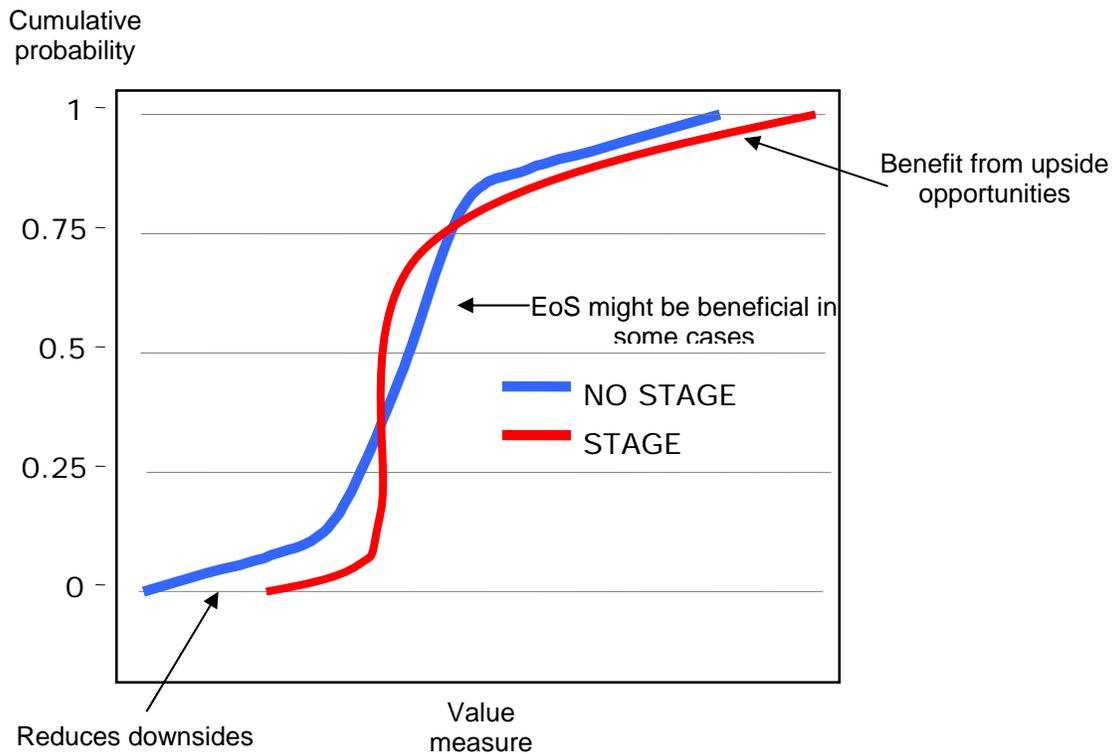
e) Illustrate how a staged approach could affect the Value at Risk and Gain of the value of a project. **(4 points)**

- Draw VARG chart (1 pt.)
- Labels and readability (1 pt.)
- Chart indicates benefits (1 pt.)
- Benefits are identified correctly (1 pt.)

If the diagram is confusing or has incorrect statements, points may have been deducted.

For examples:

The diagram below shows a decrease in the downsides (lower left) and a gain of upsides (upper right). In some cases, the inflexible approach might benefit from economies of scale (EoS) in some cases, which explains why the staged approach curve can be to the left of the unstaged one for a number of scenarios.



**Decision Analysis (18 points)**

As the systems engineer for the Copper Cable Company (CUCACO) you need to establish their production plan for a new 3C product over the next 2 years. A key decision concerns the purchase of Copper: should CUCACO buy copper on a

- fixed contract for 2 years, or
- flexible contract that will change at the end of the first year?

The fixed contract is stable. However, the flexible contract might save CUCACO money if the price goes down or if CUCACO decides to close operations. The fixed contract will cost \$5M a year for 2 years. The flexible contract costs \$6M for the first year, but is equally likely to rise to \$10M in the second year, or to fall to \$1M.

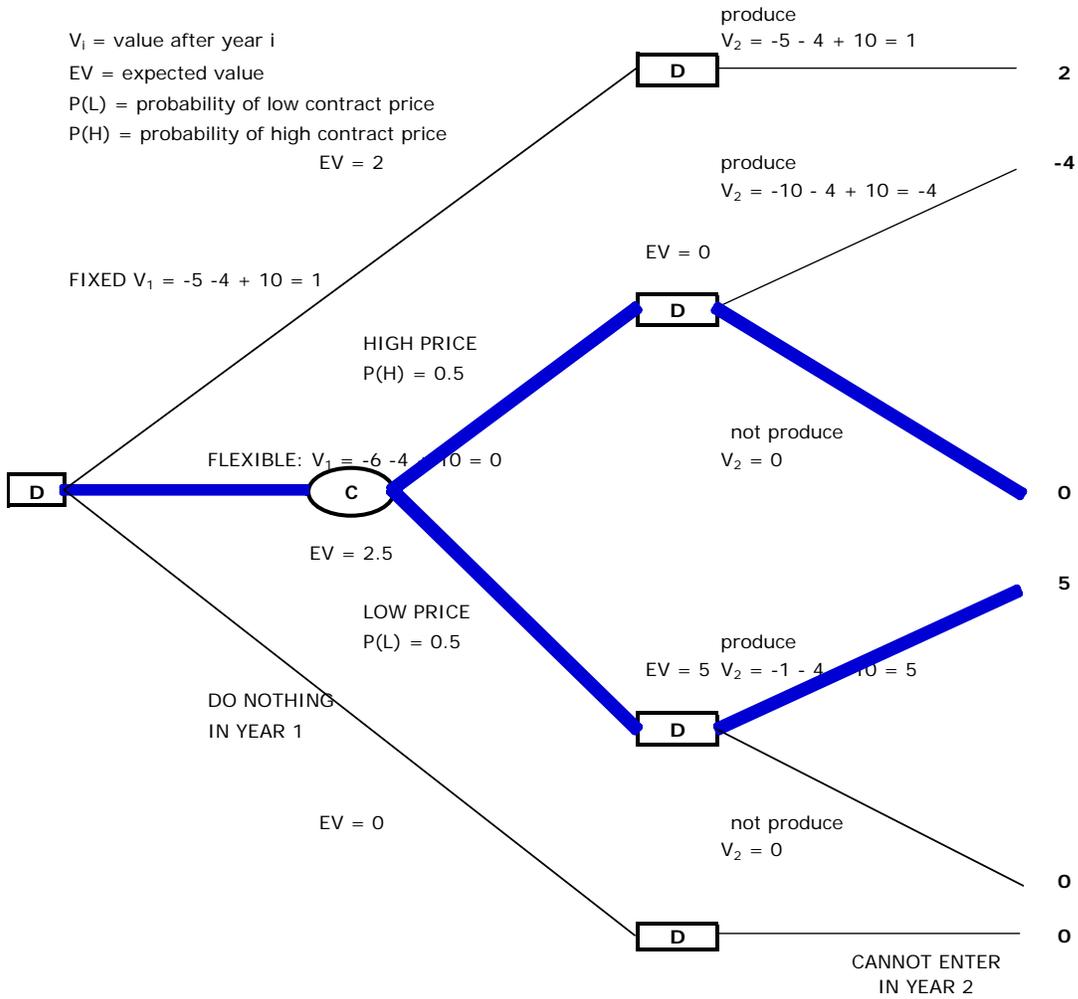
Note the following economic context: CUCACO

- Requires \$4M/year to turn copper into finished product
- Can sell the product for \$10m/year
- Under the flexible contract, CUCACO can decide not to enter the 3C business, or to exit the business at the end of year 1. It cannot enter the business in the second year.

a) Draw the decision tree for this choice, giving all information provided. **(6 points)**

For full credit in this section, the three main branches of the decision tree **(3 pts. each)** need to be shown, including the value of each outcome and probabilities for the chance branch.

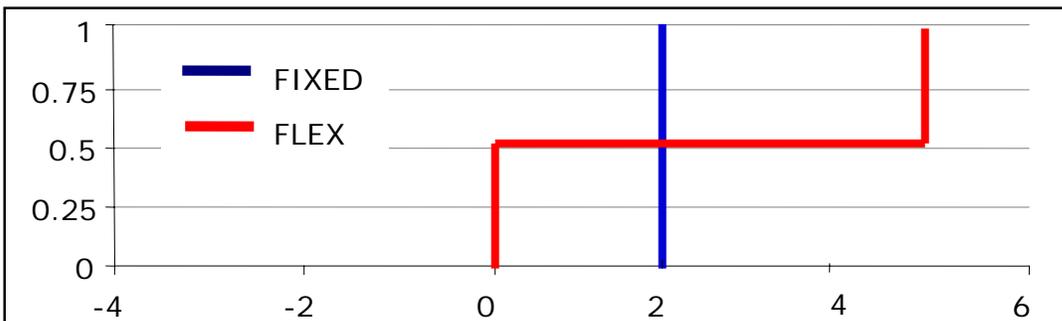
See diagram below.



b) Define and calculate the value of the optimal strategy over the 2 years (3 points)  
**NOTE: for ease of calculation, you may assume no discounting for this quiz exercise.**

The value of the optimal strategy is 2.5. **2pts.**  
 The strategy is to choose the FLEXIBLE contract. If the price of the contract is high, choose not to produce in the second year. If the price is low, choose to produce in the second year. **(1 pt.)**

c) Graph the Value at Risk and Gain (VARG) for the two production plans (3 points)



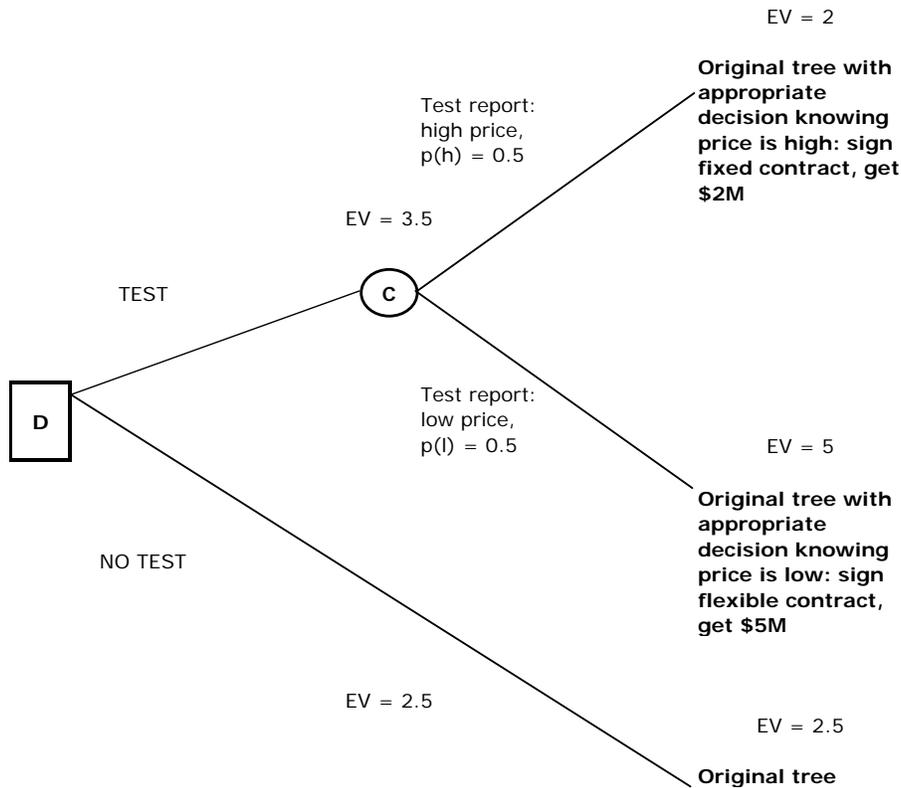
**1 pt.** each for correct shape (step function), **1 pt.** for correct numbers

d) CUCACO is concerned about the uncertain price of copper for the second year. Before deciding about the first year, it is thinking about getting extra information that would help make the choice between the fixed and flexible contracts.

Define the concept and use of the Expected Value of Perfect Information. Calculate its value for information at time zero concerning the contract price for year 2. **(6 points)**

EVPI is the change in value of our situation (decision) if we could know a priori what will occur – we can think of it as the maximum willingness to pay for knowledge of the future before it happens. In this case, we are considering knowledge of the price in year 1.

This analysis can be represented with the following tree (see next page):



To calculate the EVPI, perform a test before the original tree, which outcomes can be either that contract price will be high in year 2 or low. If test says price will be high in year 2, sign fixed contract and obtain \$2M. If test says price will be low in year 2, sign flexible contract and obtain \$5M. Since there is a 50-50 chance of low and high contract prices, so is the probability of occurrence of the above scenarios because the test has a 100% reliability **(1 pt)**. If it says the price will be low, it will be low, and vice-versa. The difference between the optimal expected value of the decision tree with the test and the expected value of our original decision tree is EVPI **(2 pts.)**. Hence:

$$EV(\text{with perfect test}) = 0.5 \times \$5M + 0.5 \times \$2M = \$3.5M \text{ (1 pt)}$$

$$EV \text{ (without test)} = \$2.5M$$

$$EVPI = \$3.5M - \$2.5M = \$1.0M \text{ (2 pts)}$$