Please give name here so we are sure to have all your pages ____SOLUTIONS___

ENGINEERING SYSTEMS ANALYSIS FOR DESIGN Final Examination, 2006

I will complete this test fairly, without copying from others, a book, or the web.

Please sign your name legibly ____John Q. Solution____ **(1 point)**

Item	Points	
	Possible	Achieved
Your Name	**1**	**1**
1 Cost of Plant	**18**	**18**
2 Engrg Economy Valuation	**21**	**21**
3 Decision Analysis	**19**	**19**
4 Value of Information	**16**	**16**
5 Lattice Development	**27**	**27**
6 Value at Risk and Gain	**18**	**18**
7 Option Evaluation	**60**	**60**
TOTAL POSSIBLE	**180**	
TOTAL ACHIEVED		**180**
GRADE ON 100% (TOTAL/1.80)		**100%**

Structure of the Exam
The questions all tie to the design and justification of a proposed project. They take you through the material of the course from the beginning to the end.

The technological part is deliberately simple, to reduce the complexity of the calculations and focus on the concepts of the course.

Project
You are the chief engineer responsible for the design of an automobile assembly plant. As such, you need to justify the design to the decision-making board that can authorize the project.

GIVE RESULTS USING TWO-DECIMAL PLACE ACCURACY

Grading
The concepts are the focus of the exam. You will earn most of your points by demonstrating that you know what the correct procedures are, and how to use them. You will do this by "running the numbers'. However, in this exam the numbers themselves are not too important – points will be deducted for mistakes, but neither the course nor the exam is focused on arithmetic.

Because the focus is on the concepts, you should clearly indicate how you going about each part of the exam so that we can give you the credit you deserve.

If the numbers are wrong and you have not explained your analysis, we will not be able to give you credit.

BEST WISHES!

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1. Cost of Plant (18 points)

Your engineers have come up with a design for the plant. Their design capacity is 160,000 cars/year, based on an estimated initial production of 80,000 cars/year, that would double every 10 years (that is, would grow at about 7% /year compounded).

As you wonder whether if it might be better to roll out the capacity in a couple stages, your staff gives you their formula for estimating the cost of building assembly plants:

Cost of building and equipping plant = $16 + 0.01\,(\text{Capacity})^{3/4}$ **[Eqn 1]**

The cost is expressed in terms of millions of dollars; the capacity in terms of cars/per year.

a) Define "economies of scale". How do these differ from "increasing returns to scale" ? **(4 pts)**

Economies of scale occur in production when the production level increases faster than the cost of the **optimal** set of inputs **(1 pt)** required to produce that output level (in a technically efficient process.) **(1 pt.)**

Increasing returns to scale occur in a production function (technically efficient) when the production level increases faster than the **same** change in scale to **all** input levels. **(2 pts.)**

Student could also receive 1 pt for the IRTS answer for explaining the difference between EOS and IRTS – one considers size of output (IRTS) while the other considers value and cost (EOS). **(1 pt.)**

b) Does **[Eqn 1]** exhibit economies of scale? Increasing returns to scale? Explain each answer **(4 pts)**

Equation 1 exhibits economies of scale **(1 pt)**, as **¾ < 1 (1 pt)**, meaning that the average optimal cost per unit of capacity falls as capacity increases.

There is not enough information to determine the returns to scale **(1 pt).** The equation gives no information about the relationship between inputs and outputs (production function). **(1 pt).**

c) In general, if a cost function does NOT exhibit economies of scale, how would this fact affect your thoughts about developing a project in stages? **(4 pts)**

Without economies of scale, there are no cost benefits to developing high capacity early on. **(2 pts)** Therefore, staged development is preferred. **(2 pts)**

d) Define the concept of the "cost function" and its connection to the production function. **(4 pts)**

The cost function gives the **optimal least cost (1 pt)** of producing any level of product Y **(1 pt)**. It can be derived from the production function by substituting the inputs with their associated costs. **(2 pts)**

OR:

It is the total cost as a function of the output **(1 pt)** assuming **technical and economic efficiency (1 pt)**. It is derived by expressing the input cost function as a function of production, using the expansion path. **(2 pts)**

e) What would you call **[Eqn 1]**? **(2 pts)**

Equation 1 is an output cost function giving the cost of producing a certain level of output. **(2 pts)**

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2 Engineering Economy Valuation (21 pts)

Being pressed for time, you decide to see if you can justify the "base case" project proposed by your design staff. Therefore you do some traditional calculations.

As previously, you assume that the

- design is for Capacity = 160,000 cars/year
- cost formula is: Cost = 16 + 0.01 $(\text{Capacity})^{3/4}$ -- in millions of dollars
- initial production = 80,000 cars/year
- growth rate ~ 7 % /year (which means that it doubles every 10 years, which is the equivalent of increasing by √2 every 5 years)

You further assume that

- life of plant is 15 years
- net revenues (in millions) in any 5-year period
  = 5 x (production at end of the period in thousands) x ($0.2 per 1000 cars) **[Eqn 2]**
- revenues are paid in at the end of each period
- the company normally uses a discount rate ~ 15%, which is its estimated WACC

In keeping with 2 decimal point accuracy, use: (1 +0.15) exp(5) = 2

Not having a spreadsheet available, you will do your analysis by filling in the table below.

Base Case Project

Years	Units	0	1- 5	6 -10	10 -15
Production at period end	thousands	0	113	160	160
Net Revenue for 5 years	millions	----	**113**	**160**	**160**
Cost	millions	**96**	-----	-----	-----
Net Cash Flow	millions	**(96)**	**113**	**160**	**160**
Discounting Factor at 15%		**1**	**0.497**	**0.247**	**0.123**
Discounted Cash Flow	millions	**(96)**	**56.18**	**39.55**	**19.66**
Net Present Value	millions	**19.39**			

a) What is your estimate of the project NPV? **(10 pts)**

$19.39 MM ($20.5 MM was also accepted – based on 2 decimal point accuracy)

b) Define pay-back period. Estimate it for the project. **(4 pts)**

Pay back period is the amount of time that it will take for undiscounted **(1 pt.)** net revenues to equal the initial investment **(1 pt.)**.
PBP = 4.25 years **(2 pts.)**

c) Define internal rate of return. Guess at it for the project and give reasoning. **(3 pts)**

IRR is the discount rate for which the NPV of a project is $0 **(1 pt.)**, or 17.59%. Answers reasonably greater than 15% were accepted, IFF you provided justification by noting that increasing the discount rate reduces the positive contribution of production revenues in later years relative to initial costs **(2 pts.)**.

d) One of your engineers asks you to justify the discount rate, "…which is a lot more than the bank interest rate!" You therefore explain that the WACC is….. **(4 pts)**

Weighted Average Cost of Capital **(2 pts.)**; …the rates achieved by an organization on previous projects **(2 pts.)**; …rates set by project lenders external to the project **(2 pts.)**; ...the historical lending rates received by the firm **(2 pts.)**; …often used as a measure of the discount rate that should be used for an average project **(2 pts.)** **Four points max.**

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3 Decision Analysis (19 pts)

Wondering about the desirability of an alternate design that would develop the project in 2 phases, both to defer construction costs and provide for future production, you decide to do a simple decision analysis to test the following alternatives:

- Base Case: build 160,000 capacity now
- Phased Case: build for 80,000 at start, add 80,000 capacity at end of 5 years
- The cost of construction is as before: $16 + 0.01\ (\text{Capacity})^{3/4}$
  (Calculation help: (1/8)^(1/4) = 0.5946)
- When you expand in year 5, you immediately pay the construction costs and obtain the expanded project (so that the revenues for the expanded project are applied to the following 5-year period).

However, your tax consultants advise that there is a 1/3 chance that local authorities will give your company tax benefits if you expand (as opposed to no tax benefits if you choose to build the "base case"). The tax benefits would be realized as a doubling of net revenues per car **after** year 5.

Phased Project (with no tax benefit)

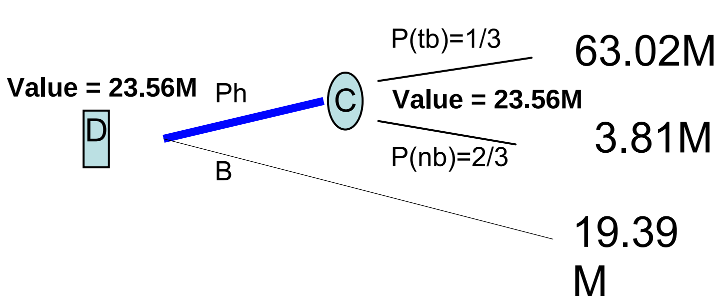
Years	Units	0	1-5	6-10	10-15
Production at period end	thousands	0	80	160	160
Net Revenue for 5 years	millions	----	**80**	**160**	**160**
Cost	millions	**63.57**	**63.57**	----	-----
Net Cash Flow	millions	**(63.57)**	**16.43**	**160**	**160**
Discounting Factor at 15%		**1**	**0.497**	**0.247**	**0.123**
Discounted Cash Flow	millions	**(63.57)**	**8.17**	**39.55**	**19.66**
Net Present Value	millions	**3.81**			

Phased Project (with tax benefit that doubles net revenues after year 5)

Years	Units	0	1-5	6-10	10-15
Production at period end	thousands	0	80	160	160
Net Revenue for 5 years	millions	----	**80**	**320**	**320**
Cost	millions	**63.57**	**63.57**	----	----
Net Cash Flow	millions	**(63.57)**	**16.43**	**320**	**320**
Discounting Factor at 15%		**1**	**0.497**	**0.497**	**0.123**
Discounted Cash Flow	millions	**(63.57)**	**8.17**	**79.1**	**39.32**
Net Present Value	millions	**63.02**			

a) Draw the decision tree. Fill in all relevant data. (**15 pts)**

Answers of $5.2M and $65.2M were also accepted.



tb: Tax benefit given
nb: No tax benefit given
Ph: Phased project
B: Base project

b) Which is the better choice? The base case or the phased project? **(4 pts)**

The phased project has a higher expected NPV.

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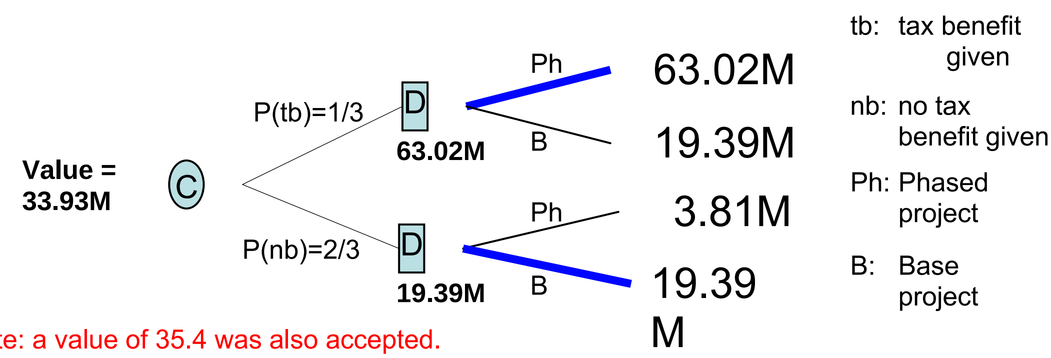
4 Value of Information (16 pts)

Thinking about the possibility that the phased alternative is attractive, you decide to estimate whether if would be worthwhile to gather better information on the possibility that the local authorities might give you the tax benefit. You plan to do this using EVPI.

a) "What's this 'perfect information' stuff?" asks a fellow project manager. "Nobody's perfect!" You then explain the concept of EVPI using the following words: **(5 pts)**

The performance of a project depends on one or more uncertain factors over which we have no control. EVPI is difference in value of our project between the expected NPV and the NPV of the expected outcome if we are able to make all relevant decisions after finding out the values of these uncertainties.

b) To reinforce the above, you draw the diagram showing the decision tree associated with EVPI about the tax benefit **(5 pts)**



Note: a value of 35.4 was also accepted.

c) What is your estimate of EVPI in this case? **(4 pts)**

EVPI = 33.93M – 23.56M = 10.37M

d) What is the most money you might justify spending to improve your information? **(2 pts)**

Answers up to 10.37M (or your answer to part c) were accepted. For full credit, you must answer in context with the notion that the value of information increases with the quality.

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5 Lattice of Probabilities (27 pts)

After using decision analysis to reach some conclusions regarding the phasing of the plant in view of the possibility of tax benefits, you start looking into production uncertainty.

You model the evolution of production with a binomial lattice, based on these parameters:

- Initial production = 80,000 cars/year
- Annual Growth, v = 7%
- Std Deviation of change in demand over 1 year, σ = 35%

As previously, you are looking at 3 blocks of 5 year periods

Useful formulas:

p = 0.5 + 0.5 (v/σ) √ΔT u = e power[σ √ΔT] d = 1/u

a) Using the data table on next page, what are the values for p, u, d ? **(5 pts)**

p = 0.5 + 0.5*(.7/.35)*√5 = 0.7236
u = 2.19 (from table) d = 1/2.19 = 0.457

b) Fill in the Tables **(14 pts)**

Table of Probabilities at nodes

Year 0	Year 5	Year 10	Year 15
1	**0.7236**	**0.5236**	**0.3789**
	0.2764	**0.4000**	**0.4342**
		0.0764	**0.1658**
			0.0211

Table of Production at end of each 5-year period (in thousands)

80	**175.04**	**382.99**	**837.98**
	36.56	**80.00**	**175.04**
		16.71	**36.56**
			7.64

Answers limiting production to 160,000 were accepted.

c) Draw the distribution of production in year 15. Label axes and use grid to scale accurately **(8 pts)**

Answers showing production limited to capacity were accepted

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6. Value at Risk and Gain (VARG) (18 pts)

The distribution you have calculated sets off alarm bells in your head. Although the project could be a big winner, the data also indicate that there is a possibility that the production would not justify the capacity planned for the base case (160,000 cars/year).

To illustrate this possibility, you decide to draw up a Value at Risk and Gain Diagram for the revenues in the **third period (year 15)**, for these **two situations** that contrast the unrealistic assumption and the recognition of uncertainty:

- Production is as projected for the base case (Exercise 2)
- Production has the distribution you have just calculated

Note that the plant capacity is limited to 160,000 in both situations

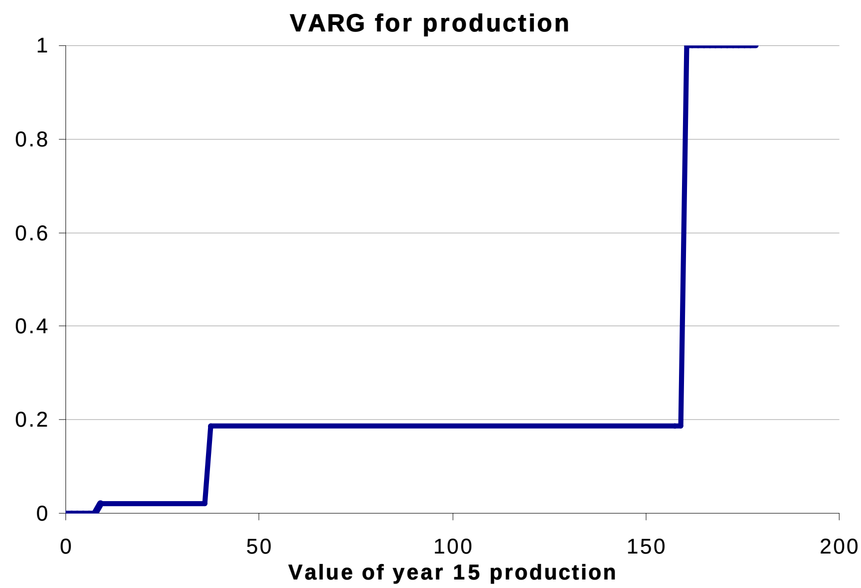
For your convenience, the assumptions used to calculate revenue in a period are repeated here:

- revenues in any period = 5 x (production at end of the period in thousands) x ($0.2 per 1000 cars)
- revenues are paid at the end of each period

a) Enter the revenues in the following table (the 2 left hand columns come from Exercise 5) **(8 pts)**

Probability	Production	Revenue
0.0211	**7.64**	**7.64**
0.1658	**36.56**	**36.56**
0.4342	**160**	**160**
0.3789	**160**	**160**
Sum = **1.0**	------	-----
Expected Value	------	**136.31**

b) Sketch the VARG diagram. Label axes and grid and draw accurately. **(6 pts)**



c) In telling your boss what you've been doing, you explain the VARG diagram as follows: **(4 pts)**

VARG tells us the final loss/gain that will not be exceeded with a given likelihood.

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7 Option Evaluation (60 points)

Using the VARG diagram, you have persuaded your boss that something ought to be done to protect the organization from the possibility of spending money on a lot of capacity that might not be used. You thus need to investigate the possibility of deferring investments in capacity.

So you revert to the possibility of building the assembly plant in two phases of 80,000 units of capacity, one at beginning and the other at the end of the 5^{th} year, as in Exercise 3.

Assume as before that if you expand in year 5, you immediately:

- incur the construction costs
- obtain the expanded project (so that the revenues for the expanded project are applied to the following 5-year period).

Do not consider any possible tax incentives in this analysis. The net revenues are as originally given:

- net revenues (in millions) in any 5-year period
  = 5 x (production at end of the period in thousands) x ($0.2 per 1000 cars) **[Eqn 2]**

Being pressed for time, you use the probability distribution you generated in Exercise 5

a) On your way to calculate the option value for the phased project, the Vice-President (Finance) challenges you: "how can you be doing options analysis? I bet you don't even know what 'arbitrage-enforced pricing' is!" But you show that you do understand this concept, explaining it as follows **(6 pts)**

"Arbitrage-enforced pricing" implies that an option can be replicated exactly **(1 pt)** as a portfolio of **assets and loans (2 pts).** This portfolio has a value that must prevail in the market or else traders could make **"risk-free" arbitrage profits (2 pts)** by buying or selling the option until its price matched the value of the replicating portfolio. **(1 pt).**

Also gave 1 pt for recognizing that market forces drive value of option/portfolio and allows pricing of option.

b) You reinforce your argument by showing him a version of the Black-Scholes formula:

$$\text{Option Price} = P * N(q_1) - A * e^{-rY} * N(q_2)$$

Where A = strike price; P = price of the Stock. You then explain the intuition behind its two terms. **(5 pts)**

Full Credit: (Given to students who describe intuition of assets and loans.) The first term relates to the notion of assets, showing the possible distribution of the asset value. The second term relates to the notion of loans, with the possible distribution of the amount borrowed discounted over time by the risk-free rate.

4 pts: (Given to students who describe equation without linking first term to assets and second term to loans.)

1 pt: (Given to students who only link Black-Scholes to European call options or non-dividend paying assets.)

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c) You also tell the VP (Finance) why you **do not** plan to base your analysis on "arbitrage-enforced pricing", citing the following reasons why using an analysis based on arbitrage enforced pricing would be inappropriate in this case **(5 pts)**

Arbitrage-enforced pricing requires (i) the ability to construct an accurate replicating portfolio to match the value of the asset **(2 pts)** and (ii) the existence of a functioning market to trade the option, the replicating portfolio, and the asset **(2 pts)**. Since neither of the two is true for the real option we are considering our analysis will not be based on arbitrage-enforced pricing. **(1 pt)**

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d) Use the table to calculate the Expected NPV of the base case (**capacity 160,000**). **(15 pts)**

Years	0	1-5	6-10	11 -15
Demand at period end (Exercise 5 b)	80	175.04	382.99	837.98
		36.56	80.00	175.04
			16.71	36.56
				7.64
Actual Production to meet demand at period end	80	160	160	160
Remember! plant size limits production		36.56	80.00	160
			16.71	36.56
				7.64
Net Revenue at each node (millions)	80	160	160	160
		36.56	80.00	160
			16.71	36.56
				7.64
Probability of each node (Exercise 5b)	1	0.7236	0.5236	0.3789
		0.2764	0.4000	0.4342
			0.0764	0.1658
				0.0211
Cost of 160,000 capacity plant (Exercise 2)	96			
Expected Value in each period (millions)	(96)	125.88	117.05	136.31
Discount factor at 15% (Exercise 2)	1	0.497	0.247	0.123
Discounted Value in each period (millions)	(96)	62.59	28.93	16.75
Net Present Value	12.27			

e) Similarly you calculate the Expected NPV of having a **capacity = 80,000** cars/year **(10 pts)**

Actual Production at period end	80	80	80	80
Remember! plant size limits production		36.56	80.00	80
			16.71	36.56
				7.64
Net Revenue at each node (millions)	80	80	80	80
		36.56	80.00	80
			16.71	36.56
				7.64
Probability of each node (Exercise 5b)	1	0.7236	0.5236	0.3789
		0.2764	0.4000	0.4342
			0.0764	0.1658
				0.0211
Cost of 80,000 Capacity plant (Exercise 3)	63.57			
Expected Value in each period (millions)	(63.57)	67.99	75.17	71.27
Discount factor at 15% (Exercise 2)	1	0.497	0.247	0.123
Discounted Value in each period (millions)	(63.57)	33.81	18.58	8.76
Net Present Value	(2.43)			

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f) Now calculate the value of starting with the smaller capacity and expanding it optimally – **which you assume can only be done in year 5. (15 pts)**

You need to choose which plant size you will select for end of year 5. To do this, you need to calculate the value of being in the 2 possible nodes at year 5, for each of two capacities.
Reminder: The value of being in a node at period N = Expected value of following 2 nodes, properly discounted over period.

As a reminder in your calculations, from Exercise 5: p = 0.72 (1-p) = 0.28

Expected Value of being in node (millions)	----	**185.77**	**239.55**	**160**
160,000 cars/year case		**92.11**	**142.59**	**160**
			30.91	**36.56**
				7.64
Cost of Expansion	----	**63.57**	----	----
Expected Value of being in node (millions)	----	**138.73**	**119.77**	**80**
80,000 cars/year case		**81.75**	**113.81**	**80**
			30.91	**36.56**
				7.64
Expected Value of being in node (millions)	----	**138.73**	**119.77**	**80**
Based on optimal choice, remember to Deduct cost of expansion if used		**81.75**	**113.81**	**80**
			30.91	**36.56**
				7.64
Probability of each node (Exercise 5b)	1	**0.6**	**0.36**	**0.216**
		0.4	**0.48**	**0.432**
			0.16	**0.288**
				0.064
Cost of 80,000 Capacity plant (Exercise 3)	**63.57**			
Expected Value in each period (millions)				
Discount factor at 15% (Exercise 2)				
Discounted Value in each period (millions)				
Net Present Value	**20.41**			

Please see more detailed solution on the next page.

g) Under what circumstances, if ever, do you expand capacity in year 5? **(4 pts)**

Actually, as it turns out, we don't expand. The cost of expansion ($63.57M) in year 5 is more than the expected increase future revenues when these revenues are scaled back in time.

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THANK YOU FOR YOUR PARTICIPATION IN THE CLASS. ALL BEST WISHES!!!!

Lattice valuation example -- 2006 Final Prob. #7

u	d	p	i	cost
2.19	0.46	0.72	101%	63.57

Step 1: Define Lattice of Demand values

	80.00	175.04	382.99	837.98
		36.56	80.00	175.04
			16.71	36.56
				7.64
EXP	80.00	136.77	233.81	399.72

1.00	0.7236	0.5236	0.3789
	0.2764	0.4000	0.4342
		0.0764	0.1658
			0.0211

Step 2: Define Lattice of production

Max of: 160

		160.00	160.00	160.00
		36.56	80.00	160.00
			16.71	36.56
				7.64
EXP		125.88	117.05	136.31
PV		62.59	28.93	16.75
NPV	12.27			

Max of: 80

	80.00	80.00	80.00
	36.56	80.00	80.00
		16.71	36.56
			7.64
	67.99	75.17	71.27
	33.81	18.58	8.76
-2.43			

Step 3: Calculate the value of being in a state (NO OPTION)

Production Capacity = 160

108.27	265.77	239.55	160.00
	92.11	142.59	160.00
		30.91	36.56
-63.57	-63.57		7.64

13.096 = NPV if we force expansion

Production Capacity = 80

61.14	138.73	119.77	80.00
	81.75	113.81	80.00
		30.91	36.56
-63.57			7.64

-2.427 = NPV without expansion

Step 4: Calculate the vlaue of being in a state (WITH OPTION)

83.98	**202.20**	119.77	80.00
	81.75	113.81	80.00
		30.91	36.56
			7.64

Execute Option

202.20 =max(265.77-63.57, 138.73)

Step 5: Calculate the NPV of the approach with option

Note: the above calculation did not include the cost of the original plant

20.41 = Step 4 NPV - Cost of 80k Cap
= 83.98 - 63.57