SOLUTION

Engineering Systems Analysis for Design

Mid-Semester Quiz **October 24, 2006**

This is a closed book exercise. Computers and other wireless enabled devices for communication with web and outside are not allowed.

You may use old-fashioned, non-communicating calculators that are not miniature computers with extensive memory (if you have them! And if so, show me first…)

Grade Table

There are 90 points possible for the regular test. Points associated with each question correspond to the estimated time it might take to answer them. .

Item	Score	
	Max	**Yours**
Your Name (provided we can read it)	1	
Concepts	22	
What's the best design?	17	
Static valuation of projects	11	
Effect of Uncertainty on Design	13	
Decision Analysis	26	
Total	90	

I have completed this test fairly, without copying from others, a book, or the web.

JOHN Q. SOLUTION

Please sign your name legibly _____ **(1 point)**

Feedback voluntary question (no credit):

Suggest a CD for classroom enjoyment in second half of semester:

SOLUTION

Concepts (22 points -- 2 points per part)

Note: Full marks only for conceptually precise responses

Write a short definition or description explaining the following:

Production Function

…is the functional relationship between a set of inputs **(1 pt.)** and the maximum or technically efficient **(1 pt.)** output. Commonly modeled for the one-output form via the Cobb-Douglas function **(0 pts.)**.

Criterion for Technical Efficiency

The maximum production level or amount of output **(1 pt.)** that can be achieved for a given **(1 pt.)** set of inputs.

Economic Efficiency

MPx/MCx = MPy/MCy **(2 pts.)** Answers that explain this condition also accepted: economic efficiency is defined by combination of technical efficiency (production function) **(1 pt.)** and economic (in general, value) **(1 pt.)** relationships. (Defining economic efficiency as the most economic output was not acceptable; you need to define economic!) Two points max.

Isoquant

The set of all input combinations that yield the same output **(1 pt.)** in a technically efficient **(1 pt.)** process.

Increasing Returns to Scale

Occur in a production function when the same change in scale in **all** input levels **(1 pt.)** leads to a greater corresponding change in the scale of the level of output **(1 pt.)**.

GRADING NOTE: If the element of efficiency is left off of both IRTS and EOS, only one point was deducted.

Economies of Scale

Occur in production when the production level increases faster than the cost of the optimal set of inputs **(1 pt)** required to produce that output level (in a technically efficient process.) **(1 pt.)**

SOLUTION

Expansion Path

The set of inputs that minimize cost **(1 pt.)** for all levels of production **(1 pt.)** or the locus of input combinations that define economically efficient design

(if you wrote "isoquant isocost tangencies along a set of isoquants"…. we accepted it)

Output Cost Function

Total cost as a function of the output **(1 pt.)**, assuming technical and economic efficiency **(1 pt.).**

Discount Rate

Risk-adjusted rate **(1 pt.)** used to evaluate the cash flows of a project over time **(1 pt.).**
The discount rate represents the opportunity cost of the resources used for the project **(1 pt.)**
Two points max.

WACC

Weighted Average Cost of Capital **(1 pt.)**
… the rates achieved by an organization on previous projects **(1 pt.)**
… or as rates set by project lenders external to the project. **(1 pt.)**
... or the historical lending rates received by the firm **(1 pt.)**
… often used as a measure of the discount rate that should be used for an average project **(1 pt.)**
Two points max.

CAPM

Capital Asset Pricing Model **(1 pt.)**
A rate of return set for a project that includes a factor to weight for project risk – internal to the project. **(1 pt.)**
Ri = Rf + Beta*(Rmarket premium) **(1 pt.)**
Two points max.

SOLUTION

What's the best design? (17 points)

You are given a production function: 8 R(exp 0.4) S (exp 0.8) $Z = 0.8R^{0.4}S^{0.8}$

And the cost of the resources as: 2 R (exp 0.6) + 6 S (exp 0.4) $C = 2R^{0.6} + 6S^{0.4}$

[Note: a (exp b) means "a" raised to the power of "b"]

Note: In calculating answers, you may leave exponents in fractional form rather than estimating numbers in decimal form. For example, (.4) exp (2/3) would be acceptable.

a) What can you say immediately, by inspection, about the returns to scale? The economies of scale? Explain answer (3 points)

RTS: $\alpha_R + \alpha_S = 1.2 > 1$ Therefore IRTS **(2 pts.)**
EOS: it is too early to tell **(1 pt.)**

b) What is the economically efficient relationship between the resources R and S? (6 points)

$$\frac{MP_R}{MP_S} = \frac{MC_R}{MC_S} \Rightarrow \frac{0.4S}{0.8R} = \frac{1.2S^{0.6}}{2.4R^{0.4}}$$ **(4 pts.)**

we can rewrite this as: $S^{*0.4} = R^{*0.6}$ or equivalent **(2 pts.)**
and conveniently plugs into the production function for later use: $Z^* = 8R^{*0.4}R^{*1.2} = 8R^{*1.6}$

c) What is the associated cost function? (6 points)

…and plug into the cost function

$$C^* = 2R^{*0.6} + 6R^{*0.6} = 8R^{*0.6}$$ **(2 pts.)**

Now we need to express C* in terms of Z*:

First, $R^* = f(Z^*) = \left(\frac{Z^*}{8}\right)^{5/8}$ **(2 pts.)**

So that: $C^* = f(Z^*) = 8\left(\frac{Z^*}{8}\right)^{3/8}$ **(2 pts.)**

d) What can you now say about the economies of scale? (2 points)

Expressing C* as a function of Z* shows that C* increases with a factor of (3/8) of the increase in Z*, or b=(3/8)<1. Economies of scale exist. **(2 pts.)**

SOLUTION

Static Valuation of Projects (11 points)

Consider the project with the following revenues and costs:

	Year				
	0	1	2	3	4
Revenues		145	500	600	550
Costs	800	90	137	200	260
Net Cash Flow	**(800)**	**55**	**363**	**400**	**290**
(1+r)expN	**1**	**1.10**	**1.21**	**1.331**	**1.4641**
Present value	**(800)**	**50**	**300**	**300.53**	**198.07**

Assume a discount rate of 10%. Use no more than 3 significant figures
Answers that used approximate values were accepted.

a) Define Net Present Value and calculate it for this case (5 Points)

Net present value is the sum total of net cash flows **(1 pt.)** appropriately discounted to the present time **(1pt.)** using appropriate discount factor.

Here, the total is 48.6 (answers between 48 and 50 were accepted). **(3 pts.)**

b) How would you calculate the benefit-cost ratio? What are the major advantages and disadvantages of the Benefit/Cost ratio as a criterion of evaluation? (3)

NOTE: THIS PROBLEM DOES NOT ASK YOU TO CALCULATE THE B-C RATIO

The benefit-cost ratio is the ratio of the present values of all benefits to all costs. **(1 pt.)** That is, while the NPV can simply discount the net cash flow for each year, the B-C ratio discounts each separately.

Advantages: simple to calculate; allows a rating of the alternatives; unit-less **(1 pt.)**

Disadvantages: biased against recurring costs **(1 pt.)**; does not give a definitive ranking; may disagree with NPV

c) Define and Calculate the Pay Back Period. What are the major advantages and disadvantages of this criterion of evaluation? (3 points)

Pay Back Period is the time period that will be required to recuperate the initial investment as based upon undiscounted **(1 pt.)** future cash flows

$$PBP = 2 years + \frac{800 - (55 + 363)}{400} = 2.995 years$$ (3 years also accepted) **(1 pt.)**

Advantages: useful when short turn-around is a priority; really simple; avoids the difficulty of picking the proper discount rate; avoids speculative long-term forecasts **(1 pt)**

Disadvantages: ignores cash flows after the initial pay-back period (that could turn negative); useless for ranking projects with lives beyond the shortest pay-back period; does not account for the time value of money **(1 pt.)**

Three points max, but must calculate the PBP correctly have partial answers for each part in order to receive full credit.

SOLUTION

Effect of Uncertainty on Design (13 points)

a) Under what circumstances is the value of a project, when calculated on the basis of the most likely forecasts, the same as its value when calculated for the range of possible scenarios that lead to those most likely forecasts? (3)

If and only if $E\left[f(x)\right] = f\left(E[x]\right)$ **(3 pts.)**
f(x) must be linear, convex, and continuous.

Partial credit given for something about Jensen's inequality

b) Therefore, how likely is it that the valuation based on the most likely forecasts are correct? (2)

Very unlikely.

c) What are the advantages of a staged approach that allows the designer to achieve a capacity through several increments, instead of one? (2)

Flexibility attained by delaying investment commitments until:
- Forecasts become more precise by incorporating more up-to-date data
- Uncertainty is mitigated with time – that is, investment decisions involve less guess work, which makes them more appropriate for the given conditions
- Ability to terminate investments into the project if the business plan does not pan out

(1 pt. each, max 2)

d) What are the disadvantages of the staged approach? (2)

Flexibility may have an upfront cost (the purchase cost of the option), which can increase the overall cost per unit.
Possible loss of economies of scale not related to the option
Additional design/modeling complexity
"Larger initial cost" was **not** accepted unless you fully explained what you meant

(1 pt. each, max 2)

e) Illustrate how a staged approach could affect the Value at Risk and Gain of the value of a project. (4)

Draw VAR chart	**(1 pt.)**
Labels and readability	**(1 pt.)**
Chart indicates benefits	**(1 pt.)**
Benefits are identified correctly	**(1 pt.)**

If the diagram is confusing or has incorrect statements, points may have been deducted.

For examples, please see next page

SOLUTION

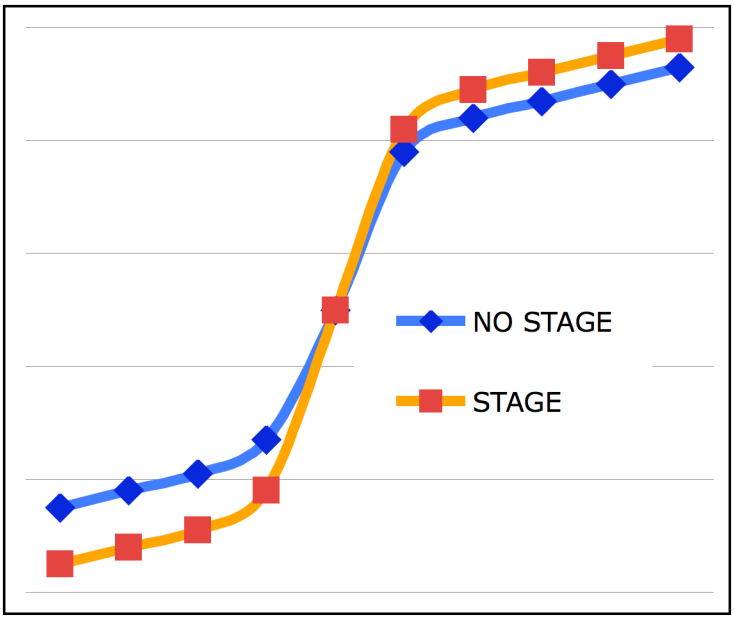


Diagram 1 shows a decrease in the downside potential (lower left) and a loss of upside (upper right), likely due to economies of scale

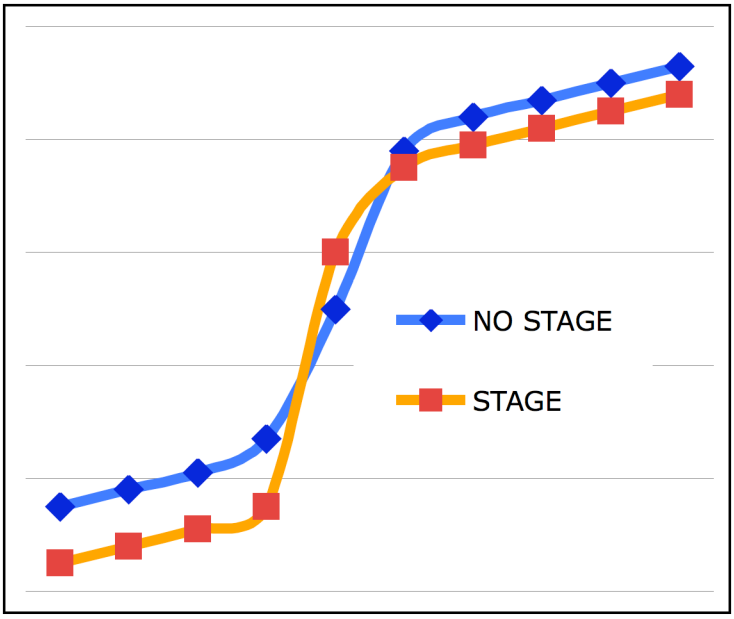


Diagram 2 shows a decrease in the downside potential (lower left) and an increase in the upside (upper right), but there is a point at which EOS losses occur (in the middle).

Either drawing is acceptable (or one in which the entire line is shifted to the right), but you MUST TELL ME WHAT YOUR PICTURE SHOWS AND IDENTIFY WHERE IT IS SHOWN!

SOLUTION

Decision Analysis (26 points)

You are advising the national arsenic company (NARCO) that wants to establish their production plan over the next 2 years. Specifically, they have to decide whether to hire staff either on a fixed contract for the full 2 years, or on a flexible contract that they can end after the first year. The fixed contract will be cheaper per year, but the flexible contract might save NARCO money if they decide to close operations. The fixed contract will cost 4M a year for 2 years, the flexible contract 6M per year, for either one or two years.

NARCO can decide not to produce in year 1 or year 2 in any case.

NARCO estimates they have a 3/4 chance that the price of arsenic will be "high" in the first year. If the price is "high" in the first year, there is a 40% chance it will also be "high" in the second year (and a corresponding 60% chance that it will be "not high" in the second year. NARCO will get $20M in any year in which the price is "high".

If the price is "not high" in the first year, NARCO estimates that it will stay that way in both years. NARCO will **lose** 5M in any year they produce when the price is "not high".

If NARCO closes in year 2 and has the flexibility to end its staff contract, it will neither make nor lose money.

a) Draw the decision tree for this choice, giving all information provided. (6)

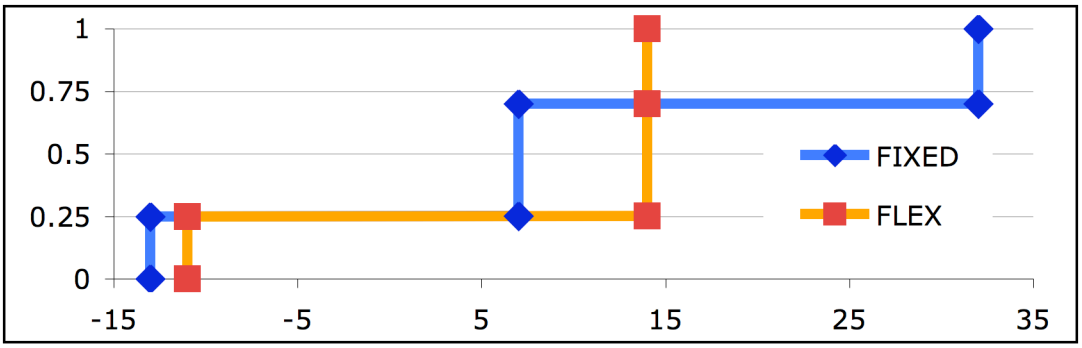
For full credit in this section, the two main braches of the decision tree **(3 pts. each)** need to be shown, including the value of each outcome and probabilities for each chance branch.

The third option (at bottom or tree) is not required, but it represents the hypothesis that production is not made in year one but *could be* made in year two with a flexible contract. If you completed the tree correctly and labeled everything…. and then got this branch completely right, a few points extra to you!

For the complete diagram, please see the last page

b) Which is the better production plan? (5)
The value of the current decision point is 9.5. **2pts.** each awarded for correctly calculating the expected value of the top and bottom half of the tree.
Choose FIXED labor contract. **(1 pt.)**

c) Graph the Value at Risk and Gain for the two production plans (4)



1 pt. each for correct shape (step function), **1 pt.** each for correct numbers

SOLUTION

d) Define the optimal strategy over the 2 years (2)

1: Choose FIXED labor contract.
2: Then, continue production in year 2 IFF year 1 has a high price. **(2 pts.)**
To get full points here (if there is an error in part a, I must be able to understand how you read your tree and reach the same result based on your tree. Read: label your trees!

e) Define Bayes' theorem, explaining the meaning of the terms (3)

$$P(A \mid B) = P(A)\frac{P(B \mid A)}{P(B)}$$ **(2 pts.)**

P(A|B) is the posterior probability, that is the probability of A once we know of B
P(A) is the prior probability, or the probability of A without any knowledge of B
P(B|A) is the probability or correlation of our conditioning event on A
P(B) is the probability of our conditioning event, or observation **(labels: 1 pt.)**

f) NARCO is concerned about the uncertainty associated with the price of arsenic, and is thinking about getting some extra information that would help you make your choice. Define the concept of the Expected Value of Perfect Information about the success in year 1, and calculate its value for this case. (6)

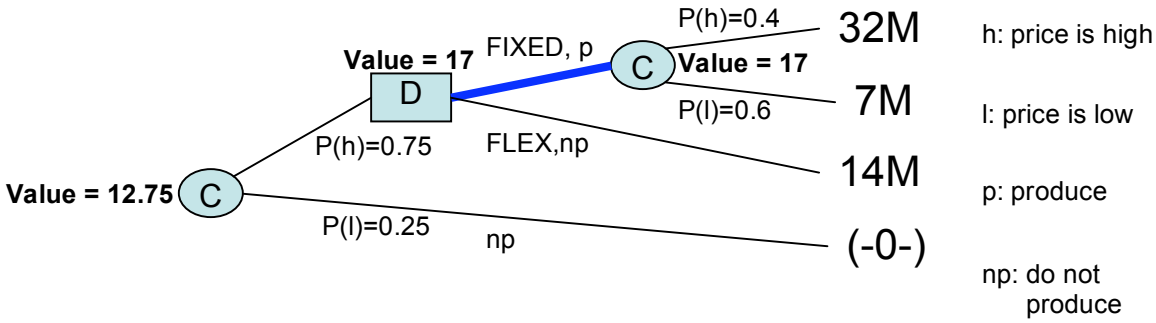
EVPI is the change in value of our situation (decision) if we could know a priori what will occur – we can think of it as the maximum willingness to pay for knowledge of the future before it happens. In this case, we are considering knowledge of the price in year 1.

To compute EVPI, simply draw another small decision tree with the initial point being the chance of receiving the high or low price in year 1.

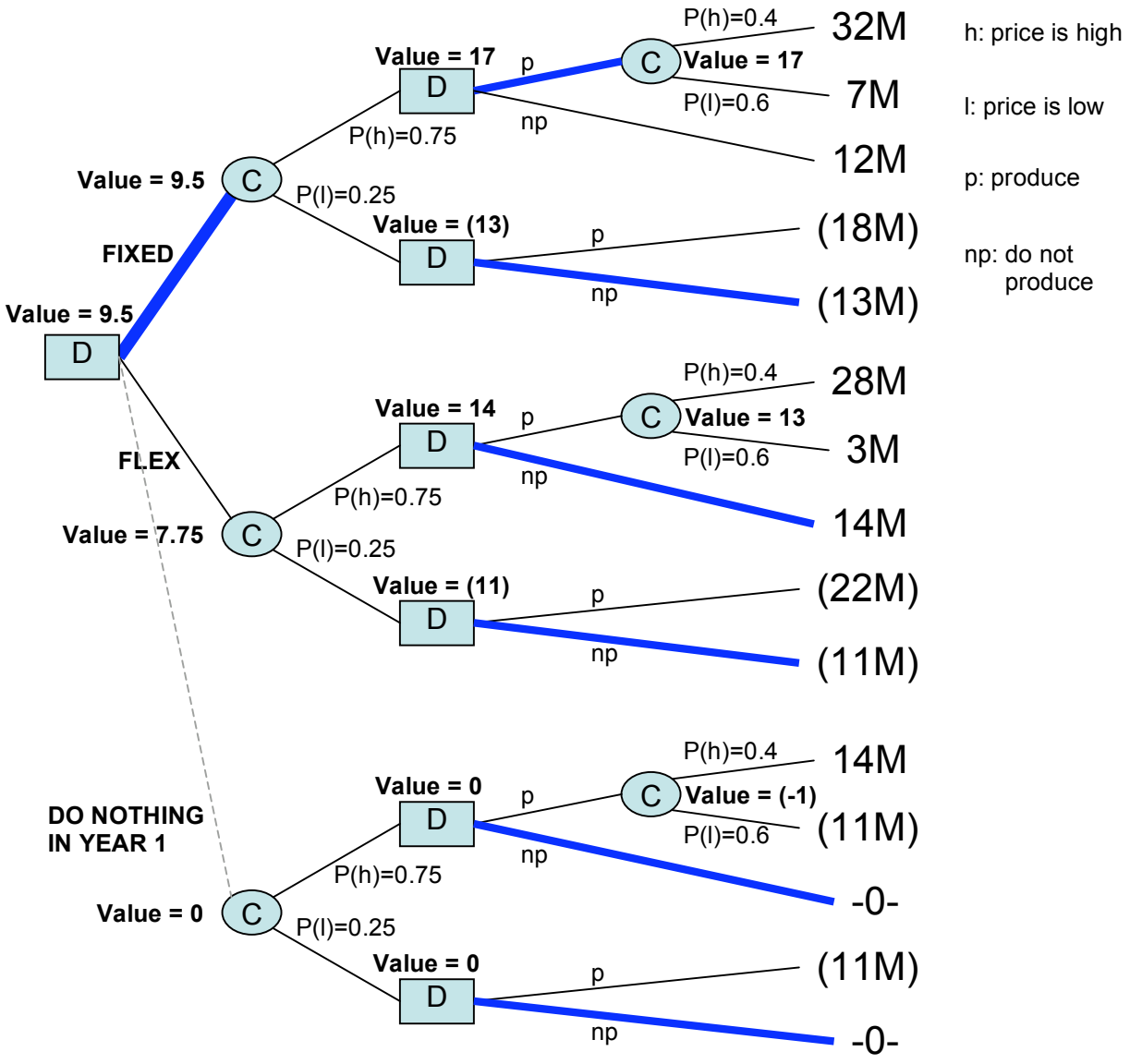
If we knew we were going to get a low outcome in year 1, we know from the diagram in (a) that (regardless of all other decisions, we do not produce in year 2). Moreover, if we knew we were also going to get a loss in year 1, we simply would not invest in the project at all.

Next, if we knew we were going to get the high outcome in year 1, we eliminate the possibility of losses in year 1 from the decision tree. Therefore, the tree is fairly simple: a ¾ chance of getting high in first year with its expected value, and a ¼ chance of getting zero (by not investing at all). The expected value given perfect information of year 1 is 12.75. **(2 pts.)**

The difference between the optimal expected value of this decision tree and the expected value of our original decision is EVPI **(2 pts.)**:
EVPI = 12.75 – 9.5 = 3.25. **(2 pts.)**



SOLUTION



h: price is high

l: price is low

p: produce

np: do not produce