ENGINEERING SYSTEMS ANALYSIS FOR DESIGN

Final Examination, 2004

Item	Points	
	Possible	Achieved
Your Name	**2**	
1 Cost Function	**18**	
2 Engrg Economy Valuation	**26**	
3 Decision Analysis	**18**	
4 Value of Information	**15**	
5 Lattice Development	**27**	
6 Value at Risk	**18**	
7 Option Evaluation	**56**	
TOTAL POSSIBLE	**180**	
TOTAL ACHIEVED		
GRADE ON 100% (TOTAL/1.80)		

Structure of the Exam
The questions all tie to the design and justification of a proposed project. They take you through the material of the course from the beginning to the end.

The technological part is deliberately simple, to reduce the complexity of the calculations and focus on the concepts of the course.

Grading
The concepts are indeed the focus of the exam. You will earn most of your points by demonstrating that you know what the correct procedures are, and how to use them. You will do this by "running the numbers'. However, in this case the numbers themselves are not too important – points will be deducted for mistakes, but neither the course nor the exam is focused on arithmetic.

Because the focus is on the concepts, you should clearly indicate how you going about each part of the exam so that we can give you the credit you deserve.

Project
You are designing a fiber-optic communications line. As project manager, you need to be able to justify the design to the decision-making board that can authorize the project.

I have completed this test fairly, without copying from others, a book, or the web.

Please sign your name legibly _______________________________________ (2 points)

1. Cost Function (18 points)

Your design engineers have come up with a plan for the fiber optic cable project. They have sized it in the traditional way, that is, to provide the maximum capacity projected over its life.

This maximum capacity has been projected at 8 billion packets/year, based on an estimated initial load of 2 billion packets/year, that would double every 8 years (that is, would grow at about 9% / year compounded).

As you wonder whether if it might be better to roll out the capacity in a couple of stages, your staff provides you with their formula for estimating project costs:

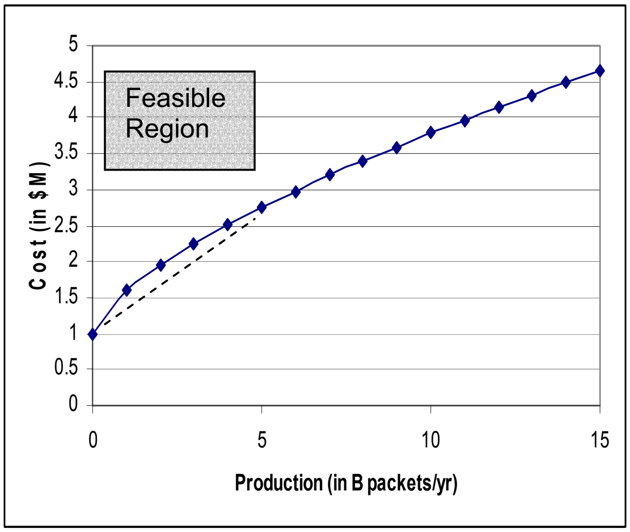
Cost of building line = Cost of building trenches + Cost of Cable
$= 1 + 0.6\ (\text{Capacity})^{2/3}$
The cost is expressed in terms of millions of dollars; the capacity in terms of billions of packets per year.

a) Define the concept of the "cost function" and its connection to the production function. **(4 pts)**

The cost function provides us with the least cost (1pt) for any given level of production (1pt). It can be derived from the production function by substituting the inputs with their relevant costs (2pts).

b) Does this cost function define a convex feasible region? Explain your answer. **(3 pts)**

This cost function defines a **non-convex** feasible region (1pt) as shown in the graph below.



The dashed line connects two points of the line but it lies outside (2pts) the feasible region.

The formal definition of monotonically decreasing marginal products **and** returns to scale could be used but it required more calculations to prove.

c) Define "economies of scale". How is this different from "increasing returns to scale" ? **(4 pts)**

Economies of scale (EOS) indicate that average cost per unit of output decreases (1pt) when output (scale of production) increases (1pt). EOS differs from returns to scale (RTS) because the latter describes rate of change in a specific direction on the production function and has nothing to do with values (2pts)

Note: It is possible to have EOS without RTS if the cost of inputs falls rapidly with scale. RTS indicate an increase in output from an increase to all inputs that is greater than the proportional increase of inputs.

d) It you knew that the cost function did NOT have economies of scale, how would it affect your thoughts about developing the project in stages? **(4 pts)**

If a project does NOT offer EOS then early development of high capacity offers no advantage over staged development. Hence staged development should be the preferred method for the project.

e) Does this cost function exhibit economies of scale? Explain your answer. **(3 pts)**

Yes, this cost function exhibits EOS since the exponent on capacity is 2/3 < 1 which means that is cheaper to produce in quantity.

2 Engineering Economy Valuation (26 pts)

Being pressed for time, you decide to see if you can justify the "base case" project proposed by your design staff. Therefore you do some traditional calculations.
USE ONLY TWO-DECIMAL PLACE ACCURACY FOR THIS AND OTHER CALCULATIONS

As previously, you assume that the

- design is for Capacity = 8 billion packets/year = maximum projected demand
- cost formula is: Cost = 1 + 0.6 (Capacity)$^{2/3}$ -- in millions of dollars
- initial demand = 2 billion packets/year
- growth rate = doubling every 8 years ~ 9 % /year
- life of project = 16 years

Assume that

- revenues in any 4-year period = 4 x (demand at the end of the period) x ($0.3 per 1000 packets)
- revenues are paid in at the end of each period
- the company normally uses a discount rate = 15%, which is its estimated WACC

Not having a spreadsheet available, you will do your analysis by filling in the table below.

Base Case Project

Years	0	1-4	5-8	9-12	13-16
Demand at period end (billions)	**2.00**	**2.82**	**4.00**	**5.63**	**8.00**
Revenue for 4 years (millions)	**0.00**	**3.38**	**4.80**	**6.76**	**9.60**
Cost	**-3.40**	**0.00**	**0.00**	**0.00**	**0.00**
Net Cash Flow	**-3.40**	**3.38**	**4.80**	**6.76**	**9.60**
Discounting Factor at 15%	**1.00**	**0.57**	**0.33**	**0.19**	**0.11**
Discounted Cash Flow	**-3.40**	**1.93**	**1.57**	**1.26**	**1.03**
Net Present Value	**2.39**				

a) What is your estimate of the project NPV? **(15 pts)**

As calculated above project NPV is $2.39 M.

b) Define pay-back period. Estimate it for the project. **(4 pts)**

Payback period is the time needed of net **undiscounted** revenues (1pt) to equal initial investment (1 pt). In this case payback period can be estimated as **4 years** since 3.38 ~= 3.4 (2pts).

c) Define internal rate of return. By looking at the table, what would you estimate it to be? **(3 pts)**

Internal rate of return (IRR) is the discount rate for which the project has NPV = 0. (1pt). In this case it MUST be >15% (1pt) in fact substantially so (1 pt) [it is in fact 29%].

d) One of your engineers asks you to justify the apparently high discount rate. You therefore explain that the WACC is….. **(4 pts)**

The Weighted average cost of capital (WACC) represents the opportunity costs (average cost of money over all projects) for a firm (2pts) and is a weighted proportion of this company's source of capital in the form of stock and bond opportunities (2pts).

3 Decision Analysis (18 pts)

Wondering about the desirability of an alternate design that would roll out the project in 2 phases you decide to do a simple decision analysis to test the following alternatives:

- Base Case: build full 8 billion capacity now
- Phased Case: build 4 billion at start, add 4 billion at end of the second period (year 8)

The cost of construction is as before: $1 + 0.6\ (\text{Capacity})^{2/3}$

As an experienced professional, you estimate that there is a 10% probability that your company will not be able to obtain the permits to build the second phase in year 8, so that revenues for the "phased case" would be capped for the last 2 periods (years 9-16)

The tables below indicate where new data needs to be inserted (compared to previous table).

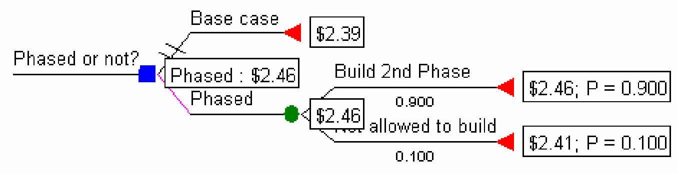
Phased Project (with second phase built)

Years	0	1-4	5-8	9-12	13-16
Demand at period end (billions)	**2.00**	**2.82**	**4.00**	**5.63**	**8.00**
Revenue for 4 years (millions)	**0.00**	**3.38**	**4.80**	**6.76**	**9.60**
Cost	**-2.51**	**0.00**	**-2.51**	**0.00**	**0.00**
Net Cash Flow	**-2.51**	**3.38**	**2.29**	**6.76**	**9.60**
Discounting Factor at 15%	**1.00**	**0.57**	**0.33**	**0.19**	**0.11**
Discounted Cash Flow	**-2.51**	**1.93**	**0.75**	**1.26**	**1.03**
Net Present Value	**2.46**				

Phased Project (with second phase NOT built, revenues capped)

Years	0	1-4	5-8	9-12	13-16
Sales at period end (billions)	**2.00**	**2.82**	**4.00**	**4.00**	**4.00**
Revenue for 4 years (millions)	**0.00**	**3.38**	**4.80**	**4.80**	**4.80**
Cost	**-2.51**	**0.00**	**0.00**	**0.00**	**0.00**
Net Cash Flow	**-2.51**	**3.38**	**4.80**	**4.80**	**4.80**
Discounting Factor at 15%	**1.00**	**0.57**	**0.33**	**0.19**	**0.11**
Discounted Cash Flow	**-2.51**	**1.93**	**1.56**	**0.90**	**0.51**
Net Present Value	**2.41**				

a) Draw the decision tree. Fill in all relevant data. **(15 pts)**



b) Which is the better choice? The full project from start or the phased project? **(3 pts)**

As shown above, the better choice (by a thin margin) is to go with the **phased project**.

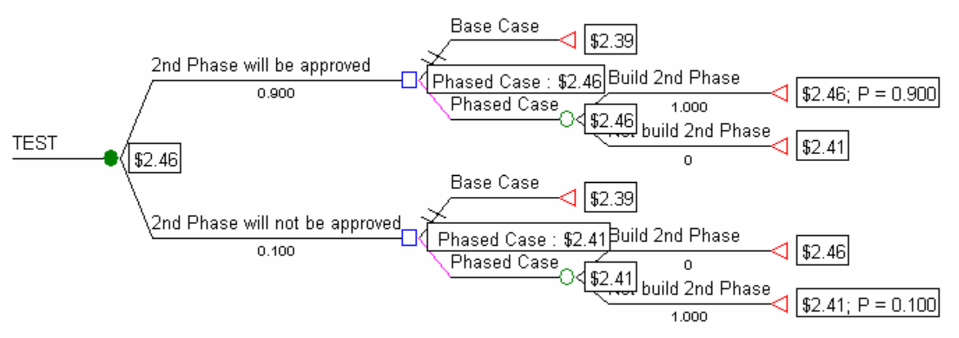
4 Value of Information (15 pts)

Thinking about the possibility that the phased alternative is attractive, you decide to estimate whether if would be worthwhile to gather better information on the possibility that a second phase might not be constructed. You plan to do this using EVPI.

a) “What’s this ‘perfect information’ stuff?” asks a fellow project manager. “Nobody’s perfect!” You then explain the concept of EVPI using the following words: **(5 pts)**

EVPI is the increase in expected value from hypothetically knowing the exact outcome of a chance situation without regarding the cost of obtaining it. It helps us estimate an upper bound of testing costs we are willing to accept.

b) To reinforce the above, you draw the diagram showing the decision tree with EVPI **(5 pts)**



c) What is your estimate of EVPI in this case? **(3 pts)**

As calculated above the EV of the project given a perfect test would still be around $2.46M. In fact, since the additional information does not change the decision, your results are identically equal in this case. Thus: **EVPI = 0**.

d) For any EVPI, what guidance does it give you about the money you might justify spending to improve your information? **(2 pts)**

As discussed in (4.a) it provides an upper bound or maximum for the amount of money one is willing to spend for testing a hypothesis. In reality any test should cost less than the EVPI. As a rule of thumb, if the cost of a test is less than half the EVPI then one would proceed in calculating the EVSI (if enough is at stake) and then decide if an actual test is worthwhile.

5 Lattice of Probabilities (27 pts)

After reaching your conclusions regarding the optimal staging of the plant in view of the uncertainty in building the second plant using decision analysis, you start looking into another uncertainty that you had not previously considered: demand uncertainty.

You model the evolution of demand with a binomial lattice, based on these parameters:

- Initial demand = 2 billion packets/year
- Annual Growth, v = 9%
- Std Deviation of change in demand over 1 year, σ = 30%

To reduce your calculations, you decide to examine a time horizon of 12 years (and not 16).

Useful formulas:

$$p = \frac{1}{2} + \frac{1}{2}\frac{growth}{s}\sqrt{\mathrm{D}t}$$

$$u = e^{s\sqrt{\mathrm{D}t}}, \quad d = 1/u$$

a) Give the values you have calculated for p, u, d **(5 pts)**

p = 0.5 + 0.5 (0.09/0.3) $4^{(0.5)}$ = **0.80** (2 pts)

u = e $^{0.3\ 4\text{^}(0.5)}$ = **1.82** (2 pts)

d = 1 / u = **0.55** (1 pt)

b) Fill in the Tables **(14 pts)**

Table of Probabilities at nodes

Year 0	Years 1 - 4	Years 5 - 8	Years 9 - 12
1	**0.80**	**0.64**	**0.51**
	0.20	**0.32**	**0.38**
		0.04	**0.10**
			0.01

Table of Demand at end of period

2	**3.64**	**6.64**	**12.10**
	1.10	**2.00**	**3.64**
		0.60	**1.10**
			0.33

c) Use the grid to sketch the resulting distribution at the end of the 3rd period (year 12) **(8 pts)**

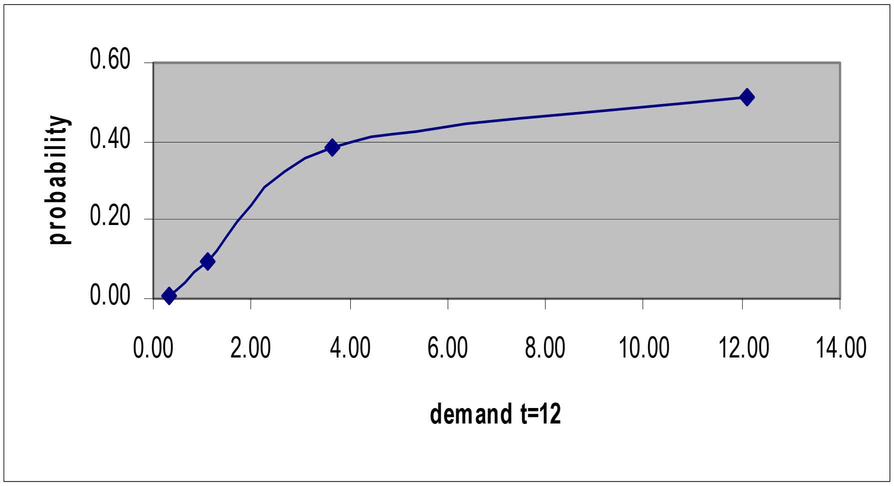


Image of Data Table defining formula

sigma, σ	0.1									
Δ T	0.5									
					Sigma, σ					
e power[σ $\sqrt{\Delta T}$]	1.073271	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5
	0.5	1.07	1.11	1.15	1.19	1.24	1.28	1.33	1.37	1.42
	1	1.11	1.16	1.22	1.28	1.35	1.42	1.49	1.57	1.65
	1.5	1.13	1.20	1.28	1.36	1.44	1.54	1.63	1.74	1.84
	2	1.15	1.24	1.33	1.42	1.53	1.64	1.76	1.89	2.03
	2.5	1.17	1.27	1.37	1.48	1.61	1.74	1.88	2.04	2.20
	3	1.19	1.30	1.41	1.54	1.68	1.83	2.00	2.18	2.38
	3.5	1.21	1.32	1.45	1.60	1.75	1.92	2.11	2.32	2.55
	4	1.22	1.35	1.49	1.65	1.82	2.01	2.23	2.46	2.72
	4.5	1.24	1.37	1.53	1.70	1.89	2.10	2.34	2.60	2.89
	5	1.25	1.40	1.56	1.75	1.96	2.19	2.45	2.74	3.06
	5.5	1.26	1.42	1.60	1.80	2.02	2.27	2.56	2.87	3.23
	6	1.28	1.44	1.63	1.84	2.09	2.36	2.66	3.01	3.40
	6.5	1.29	1.47	1.67	1.89	2.15	2.44	2.77	3.15	3.58
	7	1.30	1.49	1.70	1.94	2.21	2.52	2.88	3.29	3.75
	7.5	1.32	1.51	1.73	1.98	2.27	2.61	2.99	3.43	3.93
	8	1.33	1.53	1.76	2.03	2.34	2.69	3.10	3.57	4.11
	8.5	1.34	1.55	1.79	2.07	2.40	2.77	3.21	3.71	4.30
	9	1.35	1.57	1.82	2.12	2.46	2.86	3.32	3.86	4.48
	9.5	1.36	1.59	1.85	2.16	2.52	2.94	3.43	4.00	4.67
	10	1.37	1.61	1.88	2.20	2.58	3.02	3.54	4.15	4.86

6. Value at Risk (18 pts)

The distribution you have calculated sets off some alarm bells in your head. Although the project could be a big winner, the data also indicate that there is a distinct possibility that the traffic would not justify the capacity planned for the base case design (8 billion packets/year).

To illustrate this possibility, you decide to draw up a Value at Risk Diagram for the revenues obtained in the **third period (year 12)**, for these **two situations**:

- Traffic and revenues are as projected for the base case (Exercise 2)
- Traffic has the distribution you have just calculated

For your convenience, the assumptions used to calculate revenue in a period are repeated here:

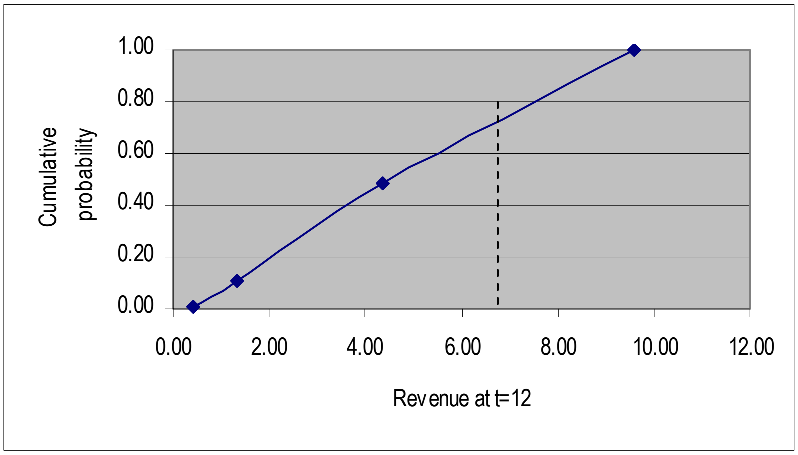
- revenues in any period = 4 (demand at the end of the period) x ($0.3 per 1000 packets)
- revenues are paid in at the end of each period

a) Enter the revenues in the following table (the 2 left hand columns come from Exercise 5) **(8 pts)**

Probability	**Cumulative Probability***	Demand	Revenue**
0.51	**1.00**	12.10 **(8**)**	**9.60**
0.38	**0.49**	3.64	**4.37**
0.10	**0.10**	1.10	**1.32**
0.01	**0.01**	0.33	**0.40**

* We need the cumulative probabilities in order to sketch the VaR.
** Total demand cannot exceed 8 B packets / year so our revenues are bounded by capacity.

b) Sketch the VAR diagram **(6 pts)**



c) In telling your boss what you've been doing, you explain the VAR diagram as follows: **(4 pts)**

VaR is the cumulative distribution of the possible levels of value. (1pt) For uncertain outcomes it slopes up to the right (1pt) as shown. For a deterministic outcome it is a vertical line (1pt)) It gives the level of confidence for any outcome. (1pt) For example, it indicates that there is a ~25% to exceed the $6.8M of revenue (dashed line) obtained from the NPV calculation from 2a.

Note: Strictly speaking, the VaR for the project would discount the revenues and deduct the cost of the project. This was not required for the exam.

7 Option Evaluation (56 points)

You have persuaded your boss that something ought to be done to protect the organization from the possibility of spending money on a lot of capacity that won't be used. You thus need to investigate the possibility of deferring investments in capacity until uncertainty in demand resolves in the years ahead.

However, it's become plain that the idea of building a parallel project 8 years from now is not acceptable. Your boss reports that the decision-making board does not feel it could get public approval for digging another trench – the politics are too difficult.

Thus, you plan to redesign the way you will install the fiber optic line, so that you can actually expand it in the future. Since the problem is a new trench, you decide to investigate how much more you can afford to spend *today* in building a deeper trench, so that you are able to install more capacity in the future without having to dig a new trench then. You realize you are thus building a real option "in" the system.

The extra cost of the deeper trench will be **X**, so that:

- the initially installed system will cost: $(1 + \mathbf{X}) + 0.6\,(\text{Capacity})^{2/3}$
- But then, increasing capacity will cost less: $0.6\,(\text{Capacity})^{2/3}$

Your task is to estimate what the value of having this flexibility would be, that is, to estimate how much increase in expected value would be available to justify the expenditure of **X.**

You decide to compare two cases of building in year 0 the:

- "full" project, with total capacity = 8.
- "half" project, with capacity = 4, but with a deeper trench that allows you to expand to a total capacity of 8 at the end of the second period (in year 8).

Being pressed for time, you:

- simplify your analysis by neglecting what happens in the 4th period (years 13-16) because the impact of anything occurring in year 16 when discounting at 15% is less than 10%
- use the data on probability distribution you have generated in Exercise 5

Assume that if you decide to exercise the option to expand at end of second period (in year 8), you immediately:

- incur the construction costs
- obtain the expanded project (so that the revenues for the expanded project are applied to the following 4-year period).

a) On your way to calculate the option value for the project, the Vice-President (Finance) challenges you: "how can you be doing options analysis? I bet you don't even know what "arbitrage-enforced pricing" is!. But you carefully show that you do understand this concept, explaining it as follows **(6 pts)**
"Arbitrage-enforced pricing" implies that an option can be replicated *exactly* through a portfolio of assets and loans. (3pts) This portfolio has a value that must prevail in the market or else traders could make "risk-free" arbitrage profits by buying or selling the option until its price matched the value of the replicating portfolio. (3pts).

b) You also tell him why you **do not** plan to base your analysis on "arbitrage-enforced pricing" **(4 pts)**
Arbitrage-enforced pricing requires (i) the ability to construct an accurate replicating portfolio to match the value of the asset (2pts) and (ii) the existence of a functioning market to trade the option, the replicating portfolio, and the asset (2pts). Since neither of the two is true for the real option we are considering our analysis will not be based on arbitrage-enforced pricing.

c) First, you calculate the net present value of the base case. **(15 pts)**

Specifically, you use the following table to do this in 2 phases:

1. You construct a lattice of the value of having a capacity = 8. Remember that the value V_i at every node *i* will be the revenue at that node, plus the expected value from the two succeeding nodes, appropriately discounted according to the formulas below:

$$V_i = \text{Revenue}(\text{demand}_i) + \frac{1}{(1+r)^{\Delta t}} \left[pV^u_{t+1} + (1-p)V^d_{t+1} \right]$$

$$\text{Revenue}(\text{demand}_i) = (0.3)(4)\min\left[\mathit{capacity}, \text{demand}_i\right]$$

2. You deduct the cost of the initial construction, and thus obtain the NPV of the base case.

Years	0	1-4	5-8	9-12
Demand at end of period (Exercise 5 b)	2	**3.64**	**6.64**	**12.10**
		1.10	**2.00**	**3.64**
			0.60	**1.10**
				0.33
Revenue = (0.3) (4) Demand (in billions)	0	**4.37**	**7.97**	**9.60**
Remember! Sales capped at 8 billion		**1.32**	**2.40**	**4.37**
			0.72	**1.32**
				0.40
Value of Having Capacity = 8 in each node	**5.34**	**10.78**	**12.86**	**9.60**
		3.56	**4.55**	**4.37**
			1.38	**1.32**
				0.40
Cost	**-3.40**			
Net Present Value	**$1.94M**			

d) You now consider building the "half" project, with deeper trenches: construct the lattice for the value of the having a capacity = 4. **(10 pts)**

Revenue = (0.3) (4) Demand (in billions)	0	**4.37**	**4.80**	**4.80**
Remember! Sales capped at 4 billion		**1.32**	**2.40**	**4.37**
			0.72	**1.32**
				0.40
Value of Having Capacity = 4 in each node	**4.21**	**8.32**	**7.50**	**4.80**
		3.56	**4.55**	**4.37**
			1.38	**1.32**
				0.40

e) Now calculate the value of starting with the smaller capacity and expanding it optimally – **which can only be done in year 8**. **(15 pts)**

Again, you do this in 2 phases

1. You construct a lattice of the value of the having a flexible system
2. You deduct the cost of the initial construction of smaller system, and thus obtain the NPV of the flexible system

Remember that owning the smaller, "half" project, gives you the right but not the obligation to expand it at the end of the second period (at the additional construction cost = 0.6 $(\text{Capacity})^{2/3}$), thus obtaining the 8 billion capacity facility.

Therefore, the value of the small capacity (4 billion) at each node will be the sum of the current revenue (calculated in part d), plus the expected value of next period, if you exercise your expansion option optimally. Do not forget that expansion has a cost.

Value of Flexible System	**4.35**	**8.63**	**8.18**	**9.60 (not 4.80)**
		3.56	**4.55**	**4.37**
			1.37	**1.32**
				0.40
Cost of initial construction of smaller system	**2.51 + X**			
Net Present Value	**4.35 – 2.51 - X**			

The additional construction cost if we choose to add capacity will be = 0.6 * $4^{2/3}$ = \$1.51M and is subtracted from the value of expanding case giving a

net = 4.80 + MAX{ [1.15^{-4} * (0.8*4.80 + 0.2*4.37)], [(1.15^{-4} * (0.8*9.60 + 0.2*4.37) – 1.51]} = *8.18*.

In this case the value of the option is 1.84 – X which is less than 1.94 (the expected value without the option) so that the value of the option is zero (0)

f) Give the optimal exercise decision rules at end of second period (in year 8) **(6 pts)**

Year 0	Year 4	Year 8	Year 12
	No change	**Expand**	No change
	No change	**Stay**	No change
		Stay	No change
			No change

Apparently it would be better to expand at year 4 but this is not allowed by the restrictions of the problem.

THANK YOU FOR YOUR PARTICIPATION. ALL BEST WISHES!!!!