

Engineering Systems Analysis for Design

Quiz on Deterministic part of course

October 22, 2002

This is a closed book exercise. You may use calculators

Grade Tables

There are 90 points possible for the regular test, or 1 point per minute. Points associated with each question correspond to the estimated time it might take to answer them. There are also a possible 10 points of extra credit. Final scores will be based on 90 possible points.

Item	Score	
	Max	Yours
Your Name (provided we can read it)	1	
Concepts	36	
What's the best design?	20	
Money, Money, Money	13	
Ship it out!	20	
Extra Credit	10	
Total	100	
Percentage grade on basis of 90 maximum:		

I have completed this test fairly, without copying from others or from a textbook

Please sign your name legibly _____

Concepts (36 points -- 2 points per part)

Write a short definition or description explaining the following:

Production Function

A physical function represents the technically efficient transformation of physical resources into products.

Technical Efficiency

Technical efficiency represents the maximum product that can be obtained from any given set of resources.

Economic Efficiency

Economic efficiency represents the best design economically. Not the same as Technical Efficiency.

Marginal Product

A marginal product is the change in output due to a unit change in a specific input.

Isoquant

An isoquant is a locus on the production function of all equal specific levels of product.

Returns to Scale

Returns to scale is the ratio of the rate of change in output due to a proportional change in all inputs simultaneously.

Economies of Scale

Economies of scale exist for a production process when it is cheaper to produce in quantity.

How are the previous two related (or not)?

Both refer to the idea of somehow getting proportionately more as the scale of production increases. Their principal difference arises from the fact that the notion of economies of scale incorporates information about the input cost function.

Shadow Prices

A shadow price is the rate of change of the objective function with respect to a particular constraint.

Opportunity Costs

An opportunity cost is the rate of degradation of the optimum per unit use of a non-optimal variable in the design.

Lagrangean Multipliers

For the problem: optimize $g(\vec{X})$ subject to $h_j(\vec{X}) = b_j$ and $\vec{X} \geq 0$, Lagrangean is

$L = g(\vec{X}) - \sum_j \lambda_j [h_j(\vec{X}) - b_j]$. The parameters of λ_j are known as Lagrangean multipliers, and actually they are the shadow prices.

Optimality Criteria in Marginal Analysis

$$\frac{MP_i}{MC_i} = \frac{1}{\lambda} \text{ for all } i$$

Complementary Slackness

$$\lambda_j s_j = 0 \text{ for all } j \text{ so that either } \lambda \text{ or } s \text{ is equal to zero}$$

Expansion Path

The expansion path is the locus of all the optimum designs for every level of output Y.

Cost Function

A cost function describes the optimal, the least cost of producing any level of product Y.

Activities

An activity is a specific way of combining basic materials or resources to achieve some objective or output.

Fixed Charge Problem

A fixed charge problem is a specific amount, typically a cost, associated with any level of a decision variable. For example, cost of $X_i = c_0 + c_i X_i$. It cannot ordinarily be handled by LP software.

Data Tables

An Excel tool to execute sensitivity analysis automatically by examining the consequences of varying one, two, and sometimes three variables at once.

What's the best design? (20 points)

You are given a production function: $3R^{0.6}S^{0.3}$

And the cost of the resources as: $6R^{0.8} + 10S^{1.2}$

- a) What can you say by immediately, by inspection, about the returns to scale? About the economies of scale? Justify your answer. (4 points)

Returns to scale are decreasing because $0.6 + 0.3 < 1$.
Economies of scale can not be told by inspection.

- b) What is the optimal relationship between the resources R and S? (10 points)

$$MP_R = 1.8R^{-0.4}S^{0.3} \quad MP_S = 0.9R^{0.6}S^{-0.7}$$

(Solution using Y in the expression is good also $= (0.6/R) Y$; $= (0.3/S) Y$)

$$MC_R = 4.8R^{-0.2} \quad MC_S = 12S^{0.2}$$

Applying Optimality Criteria in Marginal Analysis:

$$\frac{MP_R}{MC_R} = \frac{MP_S}{MC_S}$$
$$\frac{1.8R^{-0.4}S^{0.3}}{4.8R^{-0.2}} = \frac{0.9R^{0.6}S^{-0.7}}{12S^{0.2}}$$

$$5S^{1.2} = R^{0.8}$$

- c) What is the associated cost function? (6 points)

$$Z = 3R^{0.6}S^{0.3} = 3 \cdot (5^{\frac{3}{4}} \cdot S^{0.9}) \cdot S^{0.3} = 3 \cdot 5^{\frac{3}{4}} \cdot S^{1.2}$$

$$C = 6 \cdot 5 \cdot S^{1.2} + 10 \cdot S^{1.2} = 40S^{1.2}$$

so the cost function is

$$C = \frac{40}{3 \cdot (5^{\frac{3}{4}})} Z \approx 3.99Z \quad \text{and has no economies of scale.}$$

Money, Money, Money (13 points)

What is Net present value? (3 Points)

Net Present Value = Present Value Revenues – Present Value Costs

Present value is the discounted value of future sums of money using the appropriate discount rate.

What are the major advantages and disadvantages of the Benefit/Cost ratio as a criterion of evaluation? (6 Points)

Advantages:

- It compares projects on a common scale
- It provides a direct indication of whether a project is worthwhile (the ratio exceeds 1)
- It provides an easy means to rank projects in order of relative merit

Disadvantages:

- It requires all benefits to be assigned a monetary value
- Ambiguity of the treatment of recurring costs
- Bias in favor of capital-intensive projects
- Relative rank of projects can depend on discount rate used

Why might the rank order of projects change when you calculate their benefit-cost ratios using different discount rates? (4 Points)

Lower discount rates favor projects with longer-term benefits. Thus, projects with longer-term benefits can appear better than projects with benefits accruing sooner if a lower rate is used in the analysis.

Ship it out! (20 points)

A plant manager wants to minimize the cost of shipments from plants A and B (capacity of 1000 and 500) to markets K, L, M (requirements of 300, 800, 200, respectively).

The shipping costs are as in the table

From	To		
	K	L	M
A	30	40	20
B	10	50	80

a) Set up the Linear Program. You may use a vector notation if convenient. (7 points)

Let X_{ij} be the shipment from plant i to market j .

Minimize $30X_{AK} + 40X_{AL} + 20X_{AM} + 10X_{BK} + 50X_{BL} + 80X_{BM}$

Subject to $X_{AK} + X_{AL} + X_{AM} \leq 1000$

$X_{BK} + X_{BL} + X_{BM} \leq 500$

$X_{AK} + X_{BK} = 300$

$X_{AL} + X_{BL} = 800$

$X_{AM} + X_{BM} = 200$

$X_{AK}, X_{AL}, X_{AM}, X_{BK}, X_{BL}, X_{BM} \geq 0$

b) Suppose the results gave the following results: (9 points)

Requirement at	Shadow Price	Range
A	0	(0, 800)
B	30	(300, 600)
K	20	(200, 400)
L	a	...
M	40	(250, 450)

What is the meaning of the shadow price on production at B?

The rate of cost reduction if capacity of B increases.

What can you say about the shadow price on requirements at K if these rise to 450 units?

Greater than 20 because as a constraint tightened, the shadow price increases beyond the range

What can you say about the shadow price at L?

$a > 0$ because if I change requirement that will change amount shipped and this has a cost

c) The manager thinks it might be a good idea to set up a facility between B and M at a cost of

$$20 + 0.02 \times \textit{throughput} .$$

The object would be to reduce the shipping costs on this route. How does the LP handle such an extension to the basic formulation? (4 points)

This is a fixed charge problem.

We can solve two problems:

- one without the new facility
- one with the new facility, but drop off the fixed charge (20) off when solving the LP, and add the fixed charge (20) back to get the final result

Comparing the optimal solutions of the above two problems, whichever lower is the final solution. The above is valid only for trivial problems. The better answer is that standard LP does not know how to deal with this problem.

For 10 points extra credit:

d) Suppose that the actual cost of shipping between B and M is not

$$80 \times (\textit{volume_on_route}) ,$$

but

$$(\textit{volume_on_route})^a .$$

Under what conditions could you incorporate this feature into the LP? (4 points)

If $a \geq 1$, then the feasible region is convex (the feasible region is above the curve because we can always spend more) and we can incorporate it into the LP. If $a=1$, we can solve LP directly; if $a>1$, we need piecewise approximation of the curve. If $a<1$, we cannot deal with this problem using LP.

How would including this feature change the LP? Show specific equations. (6 points)

This is for the case that $a > 1$. If $a = 1$, we can incorporate the feature directly into the LP.

Let X_{BM} denote *volume_on_route*.

We have the formulation related to this feature:

1, Define breakpoints on X_{BM} that define the new variables X_{BM}' , X_{BM}'' , X_{BM}''' , etc. and then $X_{BM} < V_1, X_{BM}'' < V_2$, etc for as many segments as relevant, also $X_{BM} > 0$.

2, Define $(\textit{volume_on_route})^a$ as $c_1 X_{BM}' + c_2 X_{BM}'' + c_3 X_{BM}'''$ with c_i determined by the breakpoints selected, and change the objective function to

$$30X_{AK} + 40X_{AL} + 20X_{AM} + 10X_{BK} + 50X_{BL} + 80(X_{BM}' + X_{BM}'' + X_{BM}''')$$

3, Insert the new definitions of X_{BM} in the constraints for shipments out of B and into M.