Knapsack Problem

Truck – 10t capacity

Optimum cargo combination:

- Item 1: $5 (3t)
- Item 2: $7 (4t)
- Item 3: $8 (5t)

Knapsack Problem

Output function f(i,w)

⇔

Optimum output of a combination of items 1 to i with a cumulated weight of w or less.

- Item 1: x1=$5 ; w1=3t
- Item 2: x2=$7 ; w2=4t
- Item 3: x3=$8 ; w3=5t

Knapsack Problem

Output function f(i,w)

f(i,w)=Max[xi + f(i,w-wi) ; f(i-1,w)]

ONE Item i + optimum combination of weight w-wi

NO Item i + optimum combination items 1 to i-1

Knapsack Problem

Table

	1	2	3	4	5	6	7	8	9	10
1										
2										
3										

W

f(i,w)

i

Knapsack Problem

Table

	1	2	3	4	5	6	7	8	9	10
1			**Using only item 1**							
2										
3										

W

i

Knapsack Problem

Table

	1	2	3	4	5	6	7	8	9	10
1										
2			**Using only item 1 & 2**							
3										

W

i

Knapsack Problem

Table

	1	2	3	4	5	6	7	8	9	10
1										
2										
3			**Using items 1, 2 & 3**							

W

i

Knapsack Problem

Table

	1	2	3	4	5	6	7	8	9	10
1	0	0	5	5	5	10				
2										
3										

W

0 items n°1

1 items n°1
w1 = 3

2 items n°1
2 w1 = 6

Knapsack Problem

Table

	1	2	3	4	5	6	7	8	9	10
1	0	0	5	5	5	10	10	10	15	15
2	0	0	5	7						
3										

w – w2 =
5 – 4 = 1

+ x2 (= 7)

f(i,w)=Max[xi + f(i,w-wi) ; f(i-1,w)]

Knapsack Problem

Table

	1	2	3	4	5	6	7	8	9	10
1	0	0	5	5	5	10	10	10	15	15
2	0	0	5	7	7					
3										

+ x2 (= 7)

f(i,w)=Max[xi + f(i,w-wi) ; f(i-1,w)]

Knapsack Problem

Table

	1	2	3	4	5	6	7	8	9	10
1	0	0	5	5	5	10	10	10	15	15
2	0	0	5	7	7					
3										

w – w2 =
6 – 4 = 2

+ x2 (= 7)

f(i,w)=Max[xi + f(i,w-wi) ; f(i-1,w)]

Knapsack Problem

Table

	1	2	3	4	5	6	7	8	9	10
1	0	0	5	5	5	10	10	10	15	15
2	0	0	5	7	7	10				
3										

+ x2 (= 7)

f(i,w)=Max[xi + f(i,w-wi) ; f(i-1,w)]

Knapsack Problem

COMPLETED TABLE

	1	2	3	4	5	6	7	8	9	10
1	0	0	5	5	5	10	10	10	15	15
2	0	0	5	7	7	10	12	14	15	17
3	0	0	5	7	8	10	12	14	15	17

Knapsack Problem

Path

	1	2	3	4	5	6	7	8	9	10
1	0	0	5	5	5	10	10	10	15	15
2	0	0	5	7	7	10	12	14	15	17
3	0	0	5	7	8	10	12	14	15	17

Optimal: 2 x Item 1 + 1 x Item 2