

Knapsack Problem

Truck – 10t capacity

Optimum cargo combination:

- Item 1: \$5 (3t)
- Item 2: \$7 (4t)
- Item 3: \$8 (5t)

Knapsack Problem

Output function $f(i,w)$



Optimum output of a combination of items 1 to i with a cumulated weight of w or less.

- Item 1: $x_1=\$5$; $w_1=3t$
- Item 2: $x_2=\$7$; $w_2=4t$
- Item 3: $x_3=\$8$; $w_3=5t$

Knapsack Problem

Output function $f(i,w)$

$$f(i,w) = \text{Max} [x_i + f(i,w-w_i) ; f(i-1,w)]$$

ONE Item i + optimum combination of weight $w-w_i$

NO Item i + optimum combination items 1 to $i-1$

Knapsack Problem

Table

	1	2	3	4	5	6	7	8	9	10
1										
2										
3										

i (green arrow pointing up to row 3)


$f(i,w)$ (red arrow pointing to cell (3,6))


w (blue arrow pointing left to column 10)

Knapsack Problem

Table

	1	2	3	4	5	6	7	8	9	10	
1			Using only item 1								
2											
3											


 i


 W

Knapsack Problem

Table

	1	2	3	4	5	6	7	8	9	10
1										
2		Using only item 1 & 2								
3										

 i

 W

Knapsack Problem

Table

	1	2	3	4	5	6	7	8	9	10	
1											
2											
3			Using items 1, 2 & 3								

← W


↑
i

Knapsack Problem


Table

	1	2	3	4	5	6	7	8	9	10
1	0	0	5	5	5	10				
2										
3										

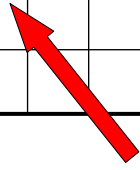
← W



0 items n°1



1 items n°1
w1 = 3



2 items n°1
2 w1 = 6

Knapsack Problem

Table

	1	2	3	4	5	6	7	8	9	10
1	0	0	5	5	5	10	10	10	15	15
2	0	0	5	7						
3										

$$w - w_2 = 5 - 4 = 1$$

+ x2 (= 7)

$$f(i,w) = \text{Max}[x_i + f(i,w-w_i); f(i-1,w)]$$

Knapsack Problem

Table

	1	2	3	4	5	6	7	8	9	10
1	0	0	5	5	5	10	10	10	15	15
2	0	0	5	7	7					
3										

+ x2 (= 7)

$$f(i,w) = \text{Max}[x_i + f(i,w-w_i); f(i-1,w)]$$

Knapsack Problem

Table

	1	2	3	4	5	6	7	8	9	10
1	0	0	5	5	5	10	10	10	15	15
2	0	0	5	7	7					
3										

$$w - w_2 = 6 - 4 = 2$$

+ x2 (= 7)

$$f(i,w) = \text{Max}[x_i + f(i,w-w_i); f(i-1,w)]$$

Knapsack Problem

Table

	1	2	3	4	5	6	7	8	9	10
1	0	0	5	5	5	10	10	10	15	15
2	0	0	5	7	7	10				
3										

+ x2 (= 7)

$$f(i,w) = \text{Max}[x_i + f(i,w-w_i); f(i-1,w)]$$

Knapsack Problem

COMPLETED TABLE

	1	2	3	4	5	6	7	8	9	10
1	0	0	5	5	5	10	10	10	15	15
2	0	0	5	7	7	10	12	14	15	17
3	0	0	5	7	8	10	12	14	15	17

Knapsack Problem

Path

	1	2	3	4	5	6	7	8	9	10
1	0	0	5	5	5	10	10	10	15	15
2	0	0	5	7	7	10	12	14	15	17
3	0	0	5	7	8	10	12	14	15	17



Optimal: 2 x Item 1 + 1 x Item 2