

Chapter 5

5.1 Feasible Regions

- a) Yes. $2XY \leq 3$ is not a linear constraint.
- b Z c) Yes. A feasible regions exists.
- d, e Z f) No. A feasible region does not exist.

5.2 Feasible Region I

- a) See Figure S5.1
Maximum X_1 at segment between $(6, 5/2)$ and $(6, 1)$
Maximum $X_1 + X_2$ at $(3.2, 6)$
Maximum $X_1 - 2X_2$ at $(6, 1)$
Maximum $10X_1 + 8X_2$ at segment of line $5X_1 + 4X_2 = 40$
between $(3.2, 6)$ and $(6, 5/2)$ since $10X_1 + 8X_2$ and $5X_1 + 4X_2$
have the same slope.
- b) Minimum X_2 at $(48/13, 3/13)$
Minimum $X_1 + X_2$ at $(0, 3)$
Minimum $X_1 - 2X_2$ at $(0, 6)$

5.3 Feasible Region II

See Figure S5.2. The feasible region is convex. $(2, 2)$ is in the interior so it is never an optimum for any linear objective function.

5.4 Graphic Interpretation

- a) See Figure S5.3
- b) The extreme points are: $(8, 0)$; $(20/3, 20/3)$; $(10/3, 40/3)$; $(0, 15)$ and $(0, 0)$
- c) Z will reach a maximum at $(10/3, 40/3)$ giving $Z^* = 53 \frac{1}{3}$
- d) The primal is of the form $\text{Max } \underline{Z} = \underline{C}\underline{X}$
S.t.: $\underline{A}\underline{X} \leq \underline{B}$; $X_1 \geq 0$.
Here, $\underline{C} = [4, 3]$; $\underline{B}^T = [40, 20, 30]$

$$A = \begin{vmatrix} 5 & 1 \\ 2 & 1 \\ 1 & 2 \end{vmatrix}$$

The dual takes the form: $\min \underline{Y} = \underline{B}^T \underline{W}$

S.t.: $\underline{A}^T \underline{W} \geq \underline{C}^T$; $W_i \geq 0$.

So, the dual becomes: $\min Y = 40W_1 + 20W_2 + 30W_3$

S.t.: $5W_1 + 2W_2 + W_3 \geq 4$

$W_1 + W_2 + 2W_3 \geq 3$

$W_i \geq 0$

5.5 Meat Market

Let X_1 = amount of low priced (c_1) beef purchased
 X_2 = amount of high priced (c_2) beef purchased
 X_3 = amount of oatmeal purchased
 X_D = amount of deluxe hamburger output
 X_R = amount of regular hamburger output

Then: $c_1X_1 + c_2X_2 + c_3X_3 \leq 100D$

$.20X_R + .30X_D \leq X_1 + X_2$

$.80X_R + .70X_D \leq X_3$

$X_1 \leq L$

$X_1, X_2, X_3, X_R, X_D \geq 0$

Where the objective is to maximize net profit:

$$Z = p_1X_R + p_2X_D - c_1X_1 - c_2X_2 - c_3X_3$$

5.6 STOMP Manufacturing

a) Let H = cases of hinges produced per week
 N = cases of nameplates produced per week
 M = pounds of metal used per week
 L_1 = operator hours per week on regular pay
 L_2 = operator hours per week at overtime pay

b) Stomp, Inc. wishes to maximize profit:

$$P = \text{Rev} - \text{Cost} = 150H + 300N - 20M - 25(L_1 + L_2) - 12L_2$$

c) We have constraints on:

$H/4 + N/3 - (L_1 + L_2) \leq 0$ hours

$5H + 10N - M \leq 0$ metal needed

$L_1 \leq 40$ regular pay hours

$N \leq 75$ maximum weekly sales

$20M + 25(L_1 + L_2) + 12L_2 \leq 30,000$ budget

These completely specify the problem for an LP solution. Note: The program will choose $L_2 = 0$ until L_1 has reached 40. This is a piecewise linear approximation.

5.7 Mountain Movers

- a) Decision variables: N_1 - number of type A trucks.
 N_2 - number of type B trucks.
 N_3 - number of type C trucks.

Objective function:

$$\max C = 7200N_1 + 12600N_2 + 13230N_3 \quad (\text{carrying capacity}).$$

Constraints:

$$\text{Price: } 24,000 N_1 + 39,000 N_2 + 45,000 N_3 \leq 1,200,000$$

$$\text{Drivers: } 3 N_1 + 6 N_2 + 6 N_3 \leq 150$$

$$\text{Parking: } N_1 + N_2 + N_3 \leq 50$$

$$N_1, N_2, N_3 \geq 0$$

- b) Assumptions: every driver works one shift a day. All trucks park at the same time.

5.8 Alloy Optimization

- a) Maximize profit $P = 30X_1 + 23X_2 + 29X_3$

Where X_1 , X_2 , and X_3 are units of outputs by process 1, 2, and 3.

$$\text{Subject to: } 6X_1 + 5X_2 + 3X_3 \leq 52,000 \quad \# \text{ chrome}$$

$$4X_1 + 2X_2 + 5X_3 \leq 14,000 \quad \# \text{ carbon}$$

$$X_1, X_2, X_3 \geq 0$$

- b) Activities are means of combining resources in a fixed ratio to produce output. Activities are intermediate processes. Here, the activities are each process specifying a fixed ratio of chrome to carbon yielding X_1 , X_2 , and X_3 . Hence X_1 , X_2 , and X_3 are activities intermediate between resources (Cr and C) and profit P.

5.9 Paint it easy

Decision variables:

Let F_i be the parts of each component (by volume) in the mixture.

Let $F_1 + F_2 + F_3 + F_4 = 100$ so we get F in percent.

Objective function: $\min C = 1.0 F_1 + 2.4F_2 + 1.5F_3 + .8F_4$.

Constraints:

$$\text{Boiling point: } 60F_1 + 99F_2 + 15F_3 + 30F_4 \geq 70 \times 100$$

$$\text{Hardening point: } F_1 + 4F_2 + 3F_3 + 2F_4 \leq 4 \times 100$$

$$\% \text{ plastic: } 80F_1 + 50F_2 + 2F_3 + 14F_4 \leq 70 \times 100$$

$$\% \text{ acid: } 20F_1 + 0F_2 + 0F_3 + 10F_4 \geq 15 \times 100$$

$$\text{Total quantity: } F_1 + F_2 + F_3 + F_4 = 100$$

$$F_1, F_2, F_3, F_4 \geq 0$$

5.10 Commodity Shipment

a) Let X_{ij} = amount shipped from warehouse i to store j . Then

$$\begin{aligned} \text{Minimize} \quad & \sum \sum c_{ij} X_{ij} \\ \text{Subject to:} \quad & \sum_i X_{ij} \geq D_j \\ & \sum_j X_{ij} \leq S_i \quad X_{ij} \geq 0 \end{aligned}$$

The minimum condition necessary for a solution to exist is that total supply must exceed (or at worst be equal to) total demand. That is:
 $S_1 + S_2 \geq D_1 + D_2 + D_3$

b) Let X_{oi} = amount shipped from factory to warehouse i .

$$\begin{aligned} \text{Minimize } Z = & a_1 X_{o1} + a_2 X_{o2} + \sum \sum X_{ij} c_{ij} \\ \text{Subject to:} \quad & \sum_j X_{ij} = X_{oi} \\ & \sum_i X_{ij} \geq D_j \\ & X_{oi} \leq S_i \\ & X_{o1} + X_{o2} \geq D \\ & X_{ij} \geq 0 \end{aligned}$$

5.11 Wheat Shipment

a) Decision Variables: X_{ij} = shipments from source i to warehouse j .

Objective Function:

$$\text{Minimize costs} = 145 X_{A1} + 130 X_{A2} + 105 X_{B1} + 90 X_{B2}$$

Constraints:

$$\text{Capacity: } X_{A1} + X_{A2} \leq 400 ; \quad X_{B1} + X_{B2} \leq 50$$

$$\text{Contract: } X_{A1} + X_{A2} \geq 100$$

$$\text{Demand: } X_{A1} + X_{B1} \geq 200 ; \quad X_{A2} + X_{B2} \geq 150$$

$$\text{Nonnegativity: } X_{ij} \geq 0$$

b) Add two decision variables X'_{B1} , X'_{B2} , the amounts shipped from B to W1 and W2 over 50 tons.

$$\text{Add to objective function: } 205X'_{B1} + 190X'_{B2}$$

$$\text{Add the capacity constraint: } X'_{B1} + X'_{B2} \leq 350$$

Alter the demand constraints to:

$$X_{A1} + X_{B1} + X'_{B1} \geq 200$$

$$X_{A2} + X_{B2} + X'_{B2} \geq 150$$

5.12 Heavy Metals Inc.

A_1 = amount of 60/40 alloy produced

A_2 = amount of 20/80 alloy produced

R_1 = ounces of rhodium secured from cheap source

R_2 = ounces of rhodium secured from expensive source

I = ounces of iridium secured

$$\text{Max profits} = P_1A_1 + P_2A_2 - (C_1R_1 + C_2R_2 + C_3I)$$

Subject to:

$$R_1 + R_2 \geq .6A_1 + .2A_2 \quad \text{purchased} > \text{used}$$

$$I \geq .4A_1 + .8A_2 \quad \text{purchased} > \text{used}$$

$$R_1 \leq L \quad \text{only L available}$$

$$C_1R_1 + C_2R_2 + C_3I \leq D \quad \text{budget constraint}$$

$$A_1, A_2, R_1, R_2, I \geq 0$$

5.13 Red Cross Relief

a) $S = \text{number of semis}$; $V = \text{number of vans}$

$$\text{max} = 40S + 15V$$

Subject to:

$$S \leq 100 \quad V \leq 200$$

$$50S + 12V \leq 4500 \quad S + V \leq 250$$

$$S, V \geq 0$$

b) Assuming only whole numbers of vans and semis can be used, the isoquant for 120 tons is only two points. The Red Cross can use either all semis (3) or all vans (8) to transport 120 tons.

c) The problem can be solved in a single run because semis have a decreasing marginal product.

d) The problem cannot be solved directly as an LP because vans have an increasing marginal product.

5.14 SMC Factory

Define:

SMC = Pounds of sheet molding compound

P = Pounds of Polyvester

V = Pounds of Vinylester

E = Pounds of E-glass fiber

C = Pounds of carbon fiber

K = Pounds of Kevlar fiber

Objective function:

$$\text{Min. cost} = .55P + 1.05V + 1.00E + 25C + 21K$$

Subject to:

$$SMC = P + V + E + C + K \quad \text{conservation of material}$$

$$100 \leq 8P^* + 10V^* + 500E^* + 300C^* + 400K^* \quad \text{tensile strength}$$

$$200 \geq 8P^* + 10V^* + 500E^* + 300C^* + 400K^* \quad \text{tensile strength}$$

$$10 \leq .4P^* + .5V^* + 10E^* + 30C^* + 18K^* \quad \text{tensile modulus}$$

$$1.9 \geq 1.10P^* + 1.10V^* + 2.54E^* + 1.7C^* + 1.44K^* \quad \text{specific gravity}$$

$$0 \geq .215(E^* + C^* + K^*) - .785(P^* + E^*) \quad \text{volume of fibers}$$

$$SMC, P, V, E, C, K \geq 0 \quad \text{non-negativity}$$

Where $X_i^* = (X_i / \text{sp. gravity } i) / \sum_i (X_i / \text{Sp.G. } i)$

To reflect the volume proportion of material X_i in the mixture.

