Chapter 4

4.1 Marginal Analysis I

a) $X = Y^2$

b) The optimum design is on the expansion path, if contraints on the inputs do not prevent it.

c) $C = (5 \times 10^{-4})Z^4$

d) $Z = 20$; $Y = 4$; $X = 16$; $C = 80$

e) $Z = 20$. This is the same as in (d), but the cost is much higher ($C = 196$) since (2, 8) is not along the expansion path.

4.2 Marginal Analysis II

a) $X = Y^3$

b) as above

c) $C = (10^{-2})\ Z^3$

4.3 Marginal Analysis III

a) $MP_X = 3X^{-0.7}X^{0.4} = .3Z/X$ $\qquad MP_Y = 4X^{0.3}Y^{-0.6} = .4Z/Y$
$MC_X = 3 \qquad MC_Y = 4Y$

b) At optimum: $MP_X/MC_X = MP_Y/MC_Y$

c) $MRS_{XY} = \Delta X/\Delta Y = -MP_Y/MP_X = -(.4Z/Y)/(.3Z/X) = -4X/3Y$

d) At optimum: $(.3Z/X)/3 = (.4Z/Y)/4Y$, or $Z/X = Z/Y^2$
So, $X^* = (Y^*)^2$.

e) We wish to find $C = f(Z)$ along the optimum path $X^* = (Y^*)^2$ substituting $X = Y^2$ into $C(X,Y)$ and $Z(X,Y)$, we get $C = Z^2/20$.

4.4 Marginal Analysis IV

a) $MP_X = 2(.5/X)Z$ decreasing over entire range.
$MP_Y = 2(2.5/Y)Z$ increasing over entire range.

b) $RTS = \Sigma a_i > 1$, therefore increasing RTS.

c) $MC_X = 64X^2 \qquad MC_Y = 5Y^2$
Optimality: $MP_X/MP_Y = MC_X/MC_Y$, therefore expansion path $Y = 4\ X$

d) $Q^* = 64X^3$ so $X^3 = Q^*/64$
$C^* = 128X^3 + 17 = 2Q^* + 17$

4.5 Marginal Analysis V

a) Decreasing MP for X. Increasing MP for Y.

b) $\Sigma a_i = 2.1 > 1$. Increasing RTS.

c) At optimum, $MP_X\ MC_X = MP_Y\ MC_Y$.
$[(.7/X)Q]/X^2 = [(1.4/Y)Q]/16Y^2$; So, $X = 2Y$

d) Substitute equation for expansion path into the production function to get $Y = 2.77Q^{0.476}$. Also substitute the equation for the expansion path into the cost equation to get $C = 8Y^3$. Substitute for Y into the simplified cost equation to get $C = 170Q^{1.43}$.

4.6 Marginal Analysis VI

a) $\Sigma a_i = 2$. Increasing returns to scale.

b) $MP_X = 2Y$ and $MP_Y = 2X$

c) $MC_X = 12X$; $MC_Y = 12Y^3$; $MP_X/MC_X = MP_Y/MC_Y$; $2Y/12X = 2X/12Y^3$
$Y = X^{0.5}$

(d) Substitute for Y in the cost function to get: $C = 9X^2 + 5$
Substitute for Y in the production function to get: $Q = 2X^{1.5}$
Solving the production function for X yields: $X = (.5Q)^{0.67}$
Substituting for X in the cost function gives: $C = 3.57Q^{1.3} + 5$

4.7 Marginal Analysis VII

Optimality criterion: $Y / 4X = 1/2$
Expansion path: $Y = X$
Along expansion path: $Z = 6 \log_e X$; $C = 6X$; so: $C = 6e^{Z/6}$

4.8 Marginal Analysis VIII

a) Optimality criterion: $4Y / 3X = 4X$
Expansion path: $Y = 3X^2$
Along expansion path: $Z = X^2$; $C = 5X^2$; so: $C = 5Z$

b) The significant returns to scale are counterbalanced by non-linear costs to define a cost function with constant economies of scale.

4.9 Vi-Tall Again

It's always better to use straight rye (in this case).

4.10 Economies of Scale?

a) $MP_X = .3X^{-0.7}Y^{0.8}$; $MP_Y = .8X^{0.3}Y^{-0.2}$

b) RTS increasing since $\Sigma a_i > 1$

c) At optimum: $MP_X/MC_X = MP_Y/MC_Y$
This condition yields: $Y^2 = X^3$ or $Y^* = (X^*)^{1.5}$

d) Substituting the expansion path into the cost function yields: $C = 5Y^2$. Again, substitute $X = Y^{2/3}$ into $Z = X^{0.3}Y^{0.8}$ to obtain $Z = Y^{0.2}Y^{0.8} = Y$. Then, $C = 5\ Z^2$

e) The cost-effectiveness function shows diseconomies of scale since a doubling of output leads to a quadrupling of cost.

4.11 More Road Work

a) $MP_L\ /\ C_L = (0.2H/L)\ /\ 5 = H\ /\ 25L$
$MP_M\ /\ C_M = (0.8H/M)\ /\ 80 = H\ /\ 100M$
So: $L = 4M$ as before

b) The Lagrangean for Prob. 3.5 can be written as:
$L = \text{Cost} - \lambda\ (\text{Production function})$
So the optimality criterion is: $MC - \lambda\ (MP)$
and: $\lambda = MC\ /\ MP$

4.12 Tim Burr, III

About a 90 percent learning curve.

4.13 Efficient Design

a) Since all a_i are less than one, the MP's are decreasing. The RTS are also decreasing because $\Sigma a_i < 1$. Therefore, the feasible region is convex.

b) $\Sigma\ a_i = .8$ Decreasing RTS.

c) The MRS is the rate at which marginal increases in an input must be substituted for marginal decreases in another input so output remains constant:
$MRS_{XY} = -(.5/Y)/(.3/X) = -5X/3Y$

d) $MP_X/MC_X = MP_Y/MC_Y$ $[(.3/X)Q]/3 = [(.5/Y)Q]/20Y$
$20Y^2 = 5X$; $Y = 0.5X^{0.5}$

e) The cost-effectiveness function is: $C = 38 + 22(Z/4)^{(2/1.1)}$
Setting $C = 60$, $Z = 4$
Since $Z = 4Y^{1.1}$ and $X = 4Y^2$: $Y = 1$, $X = 4$

4.14 Chemical Plant

a) $n = 4$; $V = (3/5)Q*$
b) $C = 0.128(Q*)^{0.6}$

4.15 Potamia

a) $MP_L/MC_L = MP_B/MC_B$
$[(.2/L)M]/500 = [(.8/B)M]/500$ $B/L = 4$

b) The expansion path is kinked. After the point where L = 100 and B = 400, output can be increased only by using more bulldozers.

c) Industrialized countries typically have a higher B/L ratio because the marginal cost of labor is higher. As Potamia develops, the B/L ratio will probably increase, but its decisions on road construction should be based upon the current MP/MC ratios for Potamia and not on the ratios of other countries.

Increasing M/L is simply increasing the average product of labor. Optimal decisions should be based upon MP's and MC's, not average products.

The criteria of both ministers lead to solutions that are not optimal.

4.16 Electric Company

a) $MP_F = 4L^{0.1}F^{-0.6}$; $MC_F = 24$; $MP_L = L^{-0.9}F^{0.4}$; $MC_L = 6$

b) $L = F$

c) Since L = F, then: $C = 30L$ $Z = 10L^{0.5} \Rightarrow (Z/10)^2 = L$
Thus: $C = .3Z^2$
See Figure S4.1

Use marginal costs. Thus, MC for plant 1 is $MC = 0.6Z$

$Z \le 10$	Use (1), it's cheaper	
$10 \le Z \le 15$	Use (2)	in addition to (1)
$15 \le Z \le 20$	Use (3)	exclusively
$20 \le Z \le 30$	Use (1)	in addition to (3)
$30 \le Z$	Use (2)	in addition to (1) and (3)

4.17 Railroad Crossing

a) As gates and grade separations are placed at the intersections, accidents are avoided. Thus, accidents avoided or lives saved is the measure of effectiveness of gates and grade separations.

b) Gates and grade separations. Money is never considered when discussing resources or the production function.

c) Inputs: gates and grade separations
Outputs: accidents avoided

d) i) An isoquant for this problem is combinations of gates and grade separations that avoid a certain number of accidents. See Figure S4.2
8 gates avoid 4 accidents
5 grade separations avoid 4 accidents
(.5)(gates) + (.8)(grade separations) = 4

ii) Looking at gate axis:
20 gates group A save 10 lives
50 gates group B save 10 lives
16.667 gates group C save 2 lives
Thus: 86.667 gates save 22 lives
Similarly: 30 grade separations save 22 lives
On Figure S4.2, the change in slope at different points occurs because you are moving to another group.

iii) See Figure S4.2

e) Moving the budget line outward, it crosses (0,10) and (15,20) before the other points on the individual lines.

f) The marginal benefits are, in lives saved / \$100K:
gates at B => .2
gates at C => .12
GS at A => .2
GS at B => .15
GS at C => .075
By spending \$300K on gates at 3 of group B crossings, you can save 0.6 lives vs. saving 0.3 additional lives by upgrading a group A crossing with a grade separation.
The marginal benefits are, in lives saved / \$100K:
gates at A => .5
gates at B => .2
gates at C => .12
GS at A => .1
GS at B => .133
GS at C => .06

g) See Figure S4.3. One should build gates at A first then:
gates at B => GS at B => gates at C => GS at A => GS at C