

Exercise 4.10

Question

4.10. Economies of Scale?Given: $Z = X^{0.3}Y^{0.8}$

(a) What are the marginal products?

(b) Are the returns to scale increasing? Why or why not?

For the input cost function $C = X^3 + 4Y^2$

(c) Write an equation defining the expansion path.

(d) Write an equation defining the cost-effectiveness function.

(e) Does the cost-effectiveness function show economies of scale?

Solution from Manual

4.10 Economies of Scale?

a) $MP_X = .3X^{-0.7}Y^{0.8}$; $MP_Y = .8X^{0.3}Y^{-0.2}$

b) RTS increasing since $\sum a_i > 1$ c) At optimum: $MP_X/MC_X = MP_Y/MC_Y$ This condition yields: $Y^2 = X^3$ or $Y^* = (X^*)^{1.5}$ d) Substituting the expansion path into the cost function yields:
 $C = 5Y^2$. Again, substitute $X = Y^{2/3}$ into $Z = X^{0.3}Y^{0.8}$
to obtain $Z = Y^{0.2}Y^{0.8} = Y$. Then, $C = 5Z^2$

e) The cost-effectiveness function shows diseconomies of scale since a doubling of output leads to a quadrupling of cost.

Additional Notes

- d) Recall that the cost-effectiveness is an alternate view of the cost function (Figure 4.5, p. 49) which depicts the product as a function of cost. This is the inverse of the cost function.