Chapter 2

2.1 Feasible Regions

a) $\Sigma a_i = 0.7 < 1$. Decreasing RTS. Decreasing MP for X and Y.

b) $\Sigma a_i = 2.1 > 1$. Increasing RTS. Increasing MP for X. Decreasing MP for Y.

c) $\Sigma a_i = 1.2 > 1$. Increasing RTS. Decreasing MP for X and Y.

d) Developing f(KX,KY) and K.f(X,Y) leads to the comparison of $K(3XY - 2Y^2)$ and $(3XY - 2Y^2)$, yielding: increasing RTS for $3X > 2Y$; decreasing RTS for $3X < 2Y$; constant RTS for $3X = 2Y$. Constant MP for X. Decreasing MP for Y.

e) Varying RTS over the domain of the function. Constant MP for X. Varying MP for Y. Decreasing MP for Z.

f) $\Sigma a_i = .8 < 1$. Decreasing RTS, Decreasing MP for X and Y.

g) $\Sigma a_i = 1.6 > 1$. Increasing RTS. Increasing MP for X. Decreasing MP for Y.

h) RTS = $X + k(-4Y^2 + 2XY)/X + (-4^2 + 2XY)$. For any scale increase ($k>1$), RTS will be increasung only if $4Y^2 + 2XY > 0$. Hence, increasing RTS for $-4Y^2 > -2XY$ or $2Y > X$; Constant RTS for $2Y = X$; and decreasing RTS for $2Y > X$. MP_X constant. MP_Y Decreasing.

i) $\Sigma a_i = .4$. Decreasing RTS. Decreasing MP for X and Y.

j) $\Sigma a_i = 1.1$. Increasing RTS. Decreasing MP for X,Y, and Z.

k) $\Sigma a_i = 1.6$. Increasing RTS. Increasing MP for X. Decreasing MP for Y.

l) RTS variable. Increasing if $Y > \ln(1/K)$. Constant if $Y = \ln(1/K)$. Decreasing if $Y < \ln(1/K)$. Constant MP for X. Increasing MP for Y.

m) RTS variable. Increasing if $KY^2 > 1$. Constant if $KY^2 = 1$. Decreasing if $KY^2 < 1$. Constant MP for X. Increasing MP for Y.

n) Constant MP for X and Y, Constant RTS.

Only (a), (f), (i), and (n) define a convex Feasible Region over their entire range. To satisfy this condition all factors must have constant or decreasing MP and RTS must not be increasing.

2.2 Thought Problems

a) For chemical reactions, input ratios, pressure and temperature are usually fairly narrowly fixed, so changing any of the resources will not yield much, if any, additional output and may actually decrease the output. *Therefore, the MP are at least decreasing and may soon become zero or even negative.* However, since the chamber size increases faster than the chamber cost, increasing all the inputs generally gives a more efficient plant.

b) Each additional farm worker yields less additional output, given a fixed amount of land. For example, adding a 51st worker on a Soviet farm will increase production less than adding the same person to a Minnesota farm, where this person would be, perhaps, the fifth worker. Similarly, the same number of people, farming twice as much land, would not do as good a job, and would therefore not get twice the output. Hence, the MP for both labor and land are decreasing. The size of farms has been increasing as greater amounts of machinery make it easier for 5 workers to plant, care for, and harvest 500 acres, than for each farmer to farm 100 acres by himself. Great corporations now own farms of many thousands of acres, taking advantage of the increasing returns to scale. These apply only when the quality of land is consistent. If we are farming in a small valley, doubling the size of our farm may require us to use less fertile hillside land, so the returns to scale decrease.

c) Ships have been getting larger over the last couple decades -- there are economies of scale. Since fluid friction is proportional to r^2 doubling horsepower will not double speed or T-mi/yr: MR (horsepower) decreases. (Also, turn around time in port is essentially independent of travel speed, so even doubling speed does not double productivity.) As ship weight (and size) increase, so does drag, so doubling ship weight probably won't double productivity: MR (weight) decreases.

d) There is some optimal employee/truck ratio, which most firms operate at or near, so marginal returns for both trucks and employees should be decreasing. Returns to scale should be nearly constant, but there are factors which differentially affect various size firms, primarily managerial. There is evidence to suggest that small family trucking firms are quite efficiently run, as are large firms which can afford a corporate superstructure; but medium size firms are too large to be run by Ma and Pa, too small to have personnel specialists and accountants. Therefore, RTS decrease from small- to medium-scale *firms, then increase into the corporate-giant range.*

e) If you increase your advertising effort, you reach some new people, but you also repeat your message to many who have already heard it. Since the advertiser is interested only in new people exposed to his product, the MR & RTS are decreasing.

2.3 Translations

a) $Y = X^{0.5}$ $\quad 2500 < X < 90000$

b) $Y = C/5 + S/3$ $\quad Y \leq 10$; $Y = (2/3)(C/5 + S/3)$ $\quad Y > 10$
The function is, realistically, limited to integer values of housing units. See Figure S2.1.

2.4 Production Function I

a) $MP_X = (0.1/X)Z$ $\qquad MP_Y = (0.3/Y)Z$

$= X^{-0.9}Y^{0.3}$ $\qquad = 3X^{0.1}Y^{-0.7}$

Both are decreasing (both $a_i < 1.0$)

$MRS = - MP_Y/MP_Y = - (0.3/Y)/(0.1/X) = - 3X/Y$

$\Sigma a_i = 0.4$, therefore decreasing.

b) Convex. Both MP and RTS decreasing.

2.5 Production Function II

a) Both decreasing, $\Sigma a_i < 1.0$

$MRS = MP_Y/MP_X = (0.4/Y)/(0.2/X) = 2X/Y$

Decreasing, $\Sigma a_i < 1.0$

b) Convex

2.6 Production Function III

$MP_X = 2/X$ $\qquad MP_Y = 4/Y$

$MRS = - (2/X) / (4/Y) = - Y / 2X$

RTS: As X and Y double, Z increases by $6 \log_e 2$. Whether this represents increasing or decreasing RTS depends on the level of Z.

2.7 Production Function IV

$MP_X = (0.8/X) Z$ $\qquad MP_Y = (0.6/Y) Z$

$MRS = - 4Y / 3X$

RTS : $\Sigma a_i = 1.4 > 1$ $\qquad$ Strong RTS

2.8 Vi-Tall Cereal

a) No, production function is piece-wise linear and discontinuous

$Z = (2/3)R + B \qquad Z < 10$

$Z = (4/9)R + (2/3)B \qquad Z \geq 10$

b) Straight lines across (R,B) plane.
For Z = 4, goes from (6,0) to (0,4)
For Z = 12, goes from (27,0) to (0,18)

c) Both above and below Z = 10, ratio of substitution is the same: -2/3

d) $MC_R = 2 \qquad MC_B = 4$

$MP_R = 2/3 \qquad MP_B = 1 \qquad Z < 10$

$MP_R = 4/9 \qquad MP_B = 2/3 \qquad Z \geq 10$

2.9 Plywood Factory

a) $MP_S = 0$ This is because Process 3 already uses the highest possible ratio of soft to hard wood; there is no other process available that can use the soft wood effectively.

b) It is easy to calculate that multiples of each Process can produce 10,000 bd.ft. of plywood:

Process 1: 7.5 units ; (7,500 ; 3,750)
Process 2: 4 units ; (4,000 ; 7,400)
Process 3: 2.5 units ; (2,500 ; 8,000)

b) The total amount used would be (1,000 ; 1,850). But notice that the total amount produced is (1,333 + 4,000) / 2 = 2,667. This is more than Process 2 could do with the same inputs; Process 2 is therefore technically inefficient.

c) The isoquant is a straight line through the points for Process 1 and 3, for the reason given in (b). Outward of these two points, the isoquant is parallel to the axes, for the reason suggested in (a).

d) The MRS is the slope of the isoquant. Between the rays defining the use of woods by Process 1 and Process 3, it is the slope taken from (c): MRS = - 4,250 / 5,000 = - 0.85

2.10 Timothy Burr

MRS = - (MP_Y) / (MP_X) Since MP_X is proportional to $X^{-0.5}$, it is also to (0.5 / X) Z --- Assuming the production function is Cobb-Douglas. Likewise, MP_Y is proportional to (0.75 / Y) Z. Thus MRS = - 3X / 2Y.

Assuming a Cobb-Douglas form: $Z = A X^{0.5} Y^{0.75} + C$
There are thus increasing returns to scale if the C term is insignificant or zero.