Exercise 2.6

Question

2.6. Production Function III
As 2.4 (a), for: \( Z = 2 \log_e X + 4 \log_e Y \)

Solution from Manual

\[
\begin{align*}
MP_x &= \frac{2}{X} & MP_y &= \frac{4}{Y} \\
MRS &= -\frac{MP_x}{MP_y} = -\frac{2/X}{4/Y} = -\frac{Y}{2X}
\end{align*}
\]

RTS: As \( X \) and \( Y \) double, \( Z \) increases by \( 6 \log_e 2 \). Whether this represents increasing or decreasing RTS depends on the level of \( Z \).

Additional Notes

\[
\begin{align*}
MP_x &= \frac{\partial Z}{\partial X} = \frac{2}{X} \\
MP_y &= \frac{\partial Z}{\partial Y} = \frac{4}{Y}
\end{align*}
\]

\[
\begin{align*}
MRS &= \frac{\Delta Y}{\Delta X} = -\frac{MP_x}{MP_y} = -\frac{2/X}{4/Y} = -\frac{Y}{2X}
\end{align*}
\]

Note: it does not matter whether MRS is done as \( \Delta Y/\Delta X \) or \( \Delta X/\Delta Y \). The question is only to be consistent in the calculations.

In this case, we cannot use exponents to determine the RTS. Let’s see what happens to \( Z \) as inputs double:

\[
\begin{align*}
X = Y = 1 & \Rightarrow Z = 2 \ln(1) + 4 \ln(1) = 6 \ln(1) = 0 \\
X = Y = 2 & \Rightarrow Z = 6 \ln(2) = 4.16 \\
X = Y = 4 & \Rightarrow Z = 6 \ln(4) = 8.32 \\
X = Y = 8 & \Rightarrow Z = 6 \ln(8) = 12.48
\end{align*}
\]

As inputs changes double from 2 to 4, there is just about constant RTS since the output \( Z \) nearly doubles. As inputs double again from 4 to 8, \( Z \) does not double, and there is
decreasing RTS. Therefore, we see that RTS depends on the values chosen for X and Y, and does not always provide the same kind of RTS.