Exercise 2.6

Question

2.6. *Production Function III*

As 2.4 (*a*), for: $Z = 2 \log_e X + 4 \log_e Y$

Solution from Manual

2.6 Production Function III

$MP_X = 2/X$ $MP_Y = 4/Y$

MRS = - (2/X) / (4/Y) = - Y / 2X

RTS: As X and Y double, Z increases by 6 $\log_e 2$. Whether this represents increasing or decreasing RTS depends on the level of Z.

Additional Notes

$$MP_x = \frac{\partial Z}{\partial X} = \frac{2}{X}$$

$$MP_y = \frac{\partial Z}{\partial Y} = \frac{4}{Y}$$

$$MRS = \frac{\Delta Y}{\Delta X} = -\frac{MP_x}{MP_y} = -\frac{2/X}{4/Y} = -\frac{Y}{2X}$$

Note: it does not matter whether MRS is done as ΔY/ΔX or ΔX/ΔY. The question is only to be consistent in the calculations.

In this case, we cannot use exponents to determine the RTS. Let's see what happens to Z as inputs double:

X = Y = 1 ⇒ Z = 2 ln(1) + 4 ln(1) = 6 ln(1) = 0

X = Y = 2 ⇒ Z = 6 ln (2) = 4.16

X = Y = 4 ⇒ Z = 6 ln (4) = 8.32

X = Y = 8 ⇒ Z = 6 ln (8) = 12.48

As inputs changes double from 2 to 4, there is just about constant RTS since the output Z nearly doubles. As inputs double again from 4 to 8, Z does not double, and there is decreasing RTS. Therefore, we see that RTS depends on the values chosen for X and Y, and does not always provide the same kind of RTS.