

## Exercise 2.4

## Question

## 2.4. Production Function I

(a) Describe the marginal products, marginal rates of substitution, and the returns to scale for the production function:

$$Z = 10X^{0.1}Y^{0.3}$$

(b) Is the feasible region convex? Explain.

## Solution from Manual

2.4 Production Function I

a)  $MP_X = (0.1/X)Z$        $MP_Y = (0.3/Y)Z$   
 $= X^{-0.9}Y^{0.3}$        $= 3X^{0.1}Y^{-0.7}$   
 Both are decreasing (both  $a_i < 1.0$ )

$MRS = -MP_Y/MP_X = -(0.3/Y)/(0.1/X) = -3X/Y$

$\Sigma a_i = 0.4$ , therefore decreasing.

b) Convex. Both MP and RTS decreasing.

## Additional Notes

$$MP_x = \frac{\partial Z}{\partial X} = 0.1 \cdot 10 \cdot X^{0.1-1}Y^{0.3} = X^{-0.9}Y^{0.3}$$

$$MP_y = \frac{\partial Z}{\partial Y} = 0.3 \cdot 10 \cdot X^{0.1}Y^{-0.7} = 3X^{0.1}Y^{-0.7}$$

$$MRS = \frac{\Delta X}{\Delta Y} = -\frac{MP_y}{MP_x} = \frac{-3X^{0.1}Y^{-0.7}}{X^{-0.9}Y^{0.3}} = -\frac{3X}{Y}$$

Now, RTS is increasing if  $\Sigma a_i > 0$ , decreasing if  $\Sigma a_i < 0$ , and constant if  $\Sigma a_i = 0$ .

Given that  $RTS = \frac{Y''/Y'}{s} = \frac{g(sX)/g(X)}{s}$

Thus, if  $Y' = a_0 \prod_i X_i^{a_i}$ ,  $Y'' = g(sX) = a_0 \prod_i (sX_i)^{a_i}$   
 $= a_0 \prod_i s^{a_i} X_i^{a_i} = s^{\sum_i a_i} a_0 \prod_i X_i^{a_i} = s^{\sum_i a_i} Y'$

so that  $RTS = \frac{Y''/Y'}{s} = \frac{s^{\sum_i a_i} Y'/Y'}{s} = \frac{s^{\sum_i a_i}}{s}$

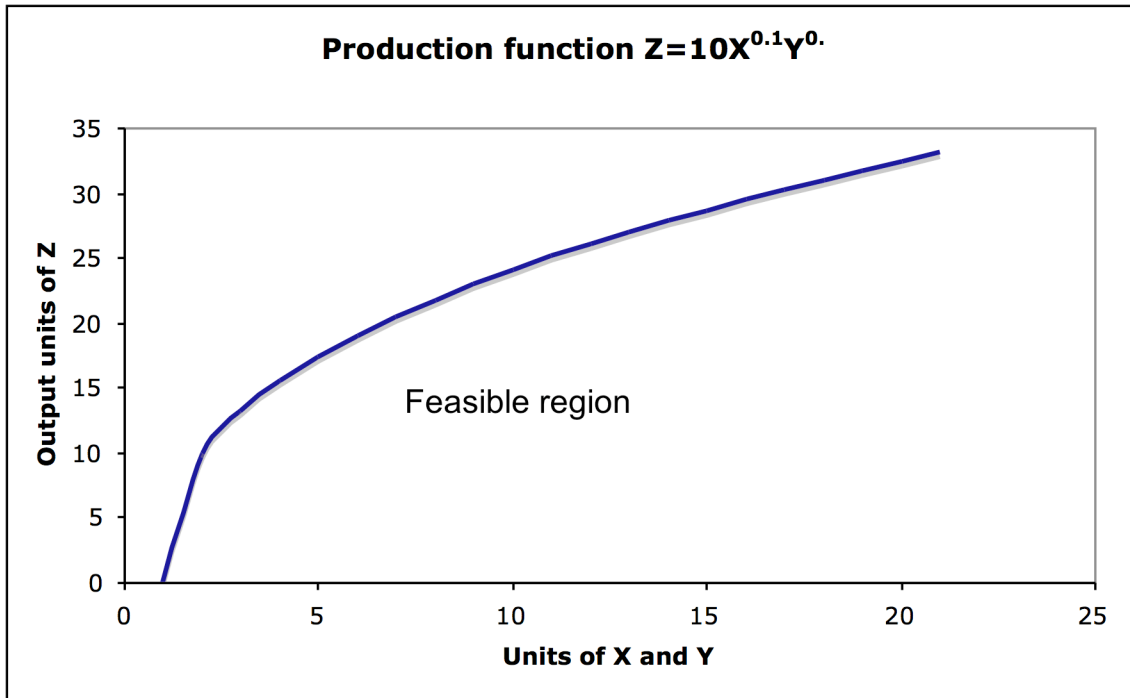
and  $RTS > 1 \Leftrightarrow s^{\sum_i a_i} > s \Leftrightarrow \sum_i a_i > 1$

Similarly,  $RTS = 1 \Leftrightarrow \sum_i a_i = 1$ , and  $RTS < 1 \Leftrightarrow \sum_i a_i < 1$

So here,  $\sum a_i = 0.1 + 0.3 = 0.4 < 1 \Rightarrow$  we have decreasing RTS.

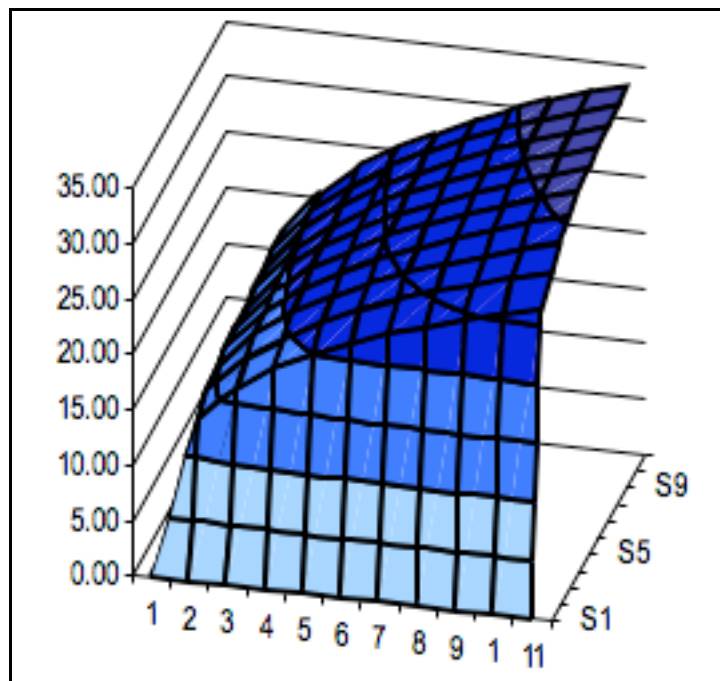
b) Draw the function in Excel:

X	Y	Z=10X <sup>0.1</sup> Y <sup>0.3</sup>	a <sub>0</sub>	10
0	0	0	a <sub>x</sub>	0.1
1	1	10	a <sub>y</sub>	0.3
2	2	13.19507911		
3	3	15.51845574		
4	4	17.41101127		
5	5	19.03653939		
6	6	20.47672511		
7	7	21.77906424		
8	8	22.9739671		
9	9	24.08224685		
10	10	25.11886432		
11	11	26.09498635		
12	12	27.01920077		
13	13	27.89827436		
14	14	28.73764756		
15	15	29.54176939		
16	16	30.31433133		
17	17	31.05843502		
18	18	31.77671523		
19	19	32.47143191		
20	20	33.14454017		



We see here that the production function bounding the feasible region exhibits convexity.

Otherwise one can use the spreadsheet ProdFun+MargAnal.xls and see this convexity in 3D:



Accompanying Data Table

		Y										
2-WAY		10.000	0	2	4	6	8	10	12	14	16	18
X		0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	2	0.00	13.20	16.25	18.35	20.00	21.38	22.59	23.66	24.62	25.51	
	4	0.00	14.14	17.41	19.66	21.44	22.92	24.21	25.35	26.39	27.34	
	6	0.00	14.73	18.13	20.48	22.32	23.87	25.21	26.40	27.48	28.47	
	8	0.00	15.16	18.66	21.07	22.97	24.56	25.95	27.17	28.28	29.30	
	10	0.00	15.50	19.08	21.55	23.49	25.12	26.53	27.79	28.92	29.96	
	12	0.00	15.78	19.43	21.95	23.92	25.58	27.02	28.30	29.45	30.51	
	14	0.00	16.03	19.73	22.29	24.30	25.98	27.44	28.74	29.91	30.99	
	16	0.00	16.25	20.00	22.59	24.62	26.33	27.81	29.12	30.31	31.40	
	18	0.00	16.44	20.24	22.85	24.91	26.64	28.14	29.47	30.67	31.78	
	20	0.00	16.61	20.45	23.10	25.18	26.92	28.44	29.78	31.00	32.11	