Chapter 16

16.1 Election Campaign

a) Go to city B for an expected number of votes equal to 1020. See Figure S16.1

b) The votes have value only if they are sufficient to win. Taking winning to have a value of plus 1 and losing equal to zero, going to a maximized expected value. Chances of winning are maximal in city A (50% vs. only 30% in city B).

16.2 Money Bags, Take 2

a) See Figure S16.2

Prior probabilities: $P(1000) = P(320) = 0.5$

$P(20/320) = .25$ $\quad P(20/1000) = 0$

$P(100/320) = .75$ $\quad P(100/1000) = 1.0$

$P(20) = P(20/320)\ P(320) + P(20/1000)\ P(1000) = 0.125$

$P(100) = 1 - P(20) = 1 - .125 = .875$

$P(1000/100) = [P(100/1000)\ P(1000)]/\ P(100) = 1.0\ (0.5)/.875 = .57$

$P(320/100) = [P(100/320)\ P(320)]/\ P(100) = 0.75\ (0.5)\ /\ 0.875 = 0.43$

b) Take the wallet.

16.3 Assembly Robot

a) See Figure S16.3

b) Notation: A = Aligned; $\quad$ $\underline{A}$ = Not aligned

OK = Test results okay $\quad$ $\underline{OK}$ = Test results not okay

Given: $P(OK/A) = .8 \rightarrow P(\underline{OK}/A) = .2$

$P(\underline{OK}/A) = .9 \rightarrow P(OK/\underline{A}) = .1$

$P_o(A) = .7 \rightarrow P_o(\underline{A}) = .3 \rightarrow P(OK) = .7(.8)+.3(.1) = .59$

$P(\underline{OK}) = .41$

Revised probabilities after test:

$P(A/OK) = P_o(A)\,[P(OK/A)/P(OK)] = .95 \rightarrow P(\underline{A}/OK) = 0.5$

$P(A/\underline{OK}) = P_o(A)\,[P(\underline{OK}/A)/\ P(\underline{OK})] = .34 \rightarrow P(\underline{A}/\underline{OK}) = .66$

c) From these revised probabilites, least expected time actions can be selected (pound if test okay; nudge if not) to arrive at an expected time of 0.36 seconds <u>after</u> the test. the total expected time for the branch starting with a test is 0.41 sec., which is less than the expectation for either pound (.044 sec.) or nudge (0.46 sec).

16.4 Mars Probe

a) Notation: S_X = Success of experimental method X
$\underline{S}_X$ = Failure of experimental method X
$\underline{S}_Y$ = Success of experimental method Y
Sim = Simulation of method X experiment succeeds
$\underline{\text{Sim}}$ = Simulation of method X experiment fails

Since the initial probabilities that theories A or B is true are 50/50:

$P(S_X) = P(S_X/A)P_o(A) + P(S_X/B)P_o(B) = .8(.5) + .4(.5) = 0.6$
$P(S_Y) = P(S_Y/A)P_o(A) + P(S_Y/B)P_o(B) = .4(.5) + .7(.5) = 0.55$

b) See Figure S16.4.
EV(method X) = EV_X = .6(\$10M) + .4(\$0M) = \$6M
EV_Y = .55(\$10M) + .45(\$0M) = \$5.5M

c) Since $EV_X > EV_Y$, Method X should be used

16.5 Software Development

a) See Figure S16.5

b) Buy license for A, which has expected value \$4.5M > \$3.75M for B.

16.6 Traffic Department

(a), (b), (c) See Figure S16.6.

d) Notation:
A means theory A is true
0 or 1 means payoff in first period is 0 or 1.

P(A) = P(B) = 1/2
P(1/A) = .8   P(0/A) = .2
P(1/B) = .1   P(0/B) = .9

Then: P(1) = P(1/A)P(A) + P(1/A)P(B) = .45
P(0) = 1-P(1) = .55

Now revise original 50 : 50 probabilities of theory A and B given that doing X yields 0 or 1:
P(A/1) = P(A)[P(1/A)/P(1)] = 8/9
P(B/1) = 1/9
P(A/0) = 2/11   P(B/0) = 9/11

Hence, the new probability of getting 1 are, if we got 0 the first time:

$$P(1) = P(1/A)P(A/0) + P(1/B)P(B/0) = .8(2/11) + .1(9/11) = 5/22$$
$$P(0) = 17/22$$

If we got 1 on first time: $P(1) = 13/18$ $P(0) = 5/18$
The expected values can then be computed, as in Figure S16.6.

e) The best strategy is to choose X in the first period, and then choose Y or X in the second, depending on whether the result in the first period is 0 or 1.

16.7 Oil Exploration

See Figure S16.7

$$EV(A) = .05(0) + .4(20k) + .5(100k) + .05(1000k) = 108$$
$$EV(B) = .1(0) + .75(20k) + .1(100k) + .05(1000k) = 75$$
$$EV(C) = .7(0) + .05(20k) + .05(100k) + .2(1000k) = 206$$

Choose site C.

16.8 Earthquake Protection

a) See Figure S16.8

b) Choose option (2). Design the house to survive a moderate earthquake.

16.9 Sonny's PV's

a) See Figure S16.9.

b) Pick new method, EV = 8.3.

16.10 PY-RIC Enterprises

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a)                                    |---- P = 0.9 ---      20 M
                |---- Launch  ---- C -----|
                |                         |---- P = 0.1 ---    -120 M
                |
    D ------    |                         |---- P = 0.95 --      10 M
                |---- Delay   ----- C -----|
                |                         |---- P = 0.05 --    -120 M
                |
                |---- Abort   ----- C ---------- P = 1.0 ---    - 10 M
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b) EV (Launch) = 0.9 (20) + 0.1 (-120) = 18 - 12 = 6
EV (Delay) = 0.95 (10) + 0.05 (-120) = 3.5
EV (Abort) = - 10
Therefore launch.

16.11 Marian Haste

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                               ____ No Peel ____        0
         ___ Go Ahead___|
        |                |____ Repaint ____        - 150k
        |    ___|
        |                   ____ No Peel ____        - 30k
        |___ Defer _____|
                         |____ Repaint ____        - 30 - 150k
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P(Repaint / Go Ahead) = 0.4(0.1) + 0.6(1/3) = 0.24
P(Repaint / Defer) = 1.0(0.1) + 0 (1/3) = 0.1

EV(Go Ahead) = 0.24 (-150) = - 36k best choice
EV(Defer) = 0.10 (-150) - 30 = - 45k

16.12 P. O'Toole

See Figure S16.10

	Low	Med	High
Level of Production :	Low	Med	High
Cost of Production :	14,000	28,000	42,000

EV(High) = 0.1(72) + 0.4(66) + 0.3(58.5) + 0.2(51) - 42
= 7.2 + 26.84 + 17.55 + 10.4 - 42 = 19.15
EV(Med) = 0.5(54) + 0.3(49.5) + 0.2(42) - 28
= 27 + 14.85 + 8.4 - 28 = 22.25 Best
EV(Low) = 0.8(36) + 0.2(33) - 14 = 21.4