Chapter 15

15.1 Money Bags

a) Use Bayes' Theorem.
P(draw \$10 bill) = (.5)(.6)+(.5)(.2) = .4
P(bag contains \$640/draw \$10 bill) = .5(.6/.4) = .75
Choose the bag you sample.

b) Use Likelihood Ratios.
Prior probabilities: P(\$640) = .5 P(\$280) = .5
P(\$10/\$640) = .6 P(\$1/\$640) = .4
P(\$1/\$280) = .8 P(\$10/\$280) = .2

$CLR_{\$10}$ = P(\$10/\$640)/P(\$10/\$280) = .6/.2 = 3
$CLR_{\$1}$ = .4/.8 = 1/2

LR_O = P(\$640)/P(\$280) = 1
$LR_N = (1)(3)(1/2)^2 = 3/4$

P(\$640/1 \$10,2 \$1) = $LR_N/(1+LR_N)$ = 3/7 = 0.429
Choose the other bag.

c) $LR_N = (1)(3^2)(1/2)^2 = 9/4$
P(\$640/2 \$10,2 \$1) = (9/4)/(1+9/4) = 0.692

A \$10 bill is more likely to come from the bag containing \$640, so drawing a \$10 bill from a bag increases the probability that that bag is the one you want. Pulling out the \$10 bill in this case changes your choice from part (b) above.

15.2 Diskette Drives

Notation: DP = Diskette problem; $\underline{DP}$ = No diskette problem
S = Error signal; $\underline{S}$ = No error signal

We know: P(S/DP) = 1.0
P(S/$\underline{DP}$) = 3 , therefore P($\underline{S}$/$\underline{DP}$) = 7
P(DP) = 0.2 , therefore P($\underline{DP}$) = 0.8

At what n is $P(DP/S^n) \geq .99$? (S^n means S observed n times) This requires $LR_n/(1+LR_n) \geq .99$ or $LR_n \geq 99$.
$LR_o = P(DP)/P(\underline{DP}) = .2/.8 = 1/4$.
$CLR_S = P(S/DP)/P(S/\underline{DP}) = 1/.3 = 10/3$.
$LR_n = LR_o(CLR_s)^n = (1/4)(10/3)^n$.
What minimum n satisfies: $(1/4)(10/3)^n \geq 99$?
Since $\ln(1/4) + n \bullet \ln(10/3) \geq \ln 99$, $n \geq \ln(99/.25)/\ln(10/3) = \underline{4.97}$
As n must be an integer, the minimum number of unsuccessful attempts to read diskette data is n = 5.

15.3 Lie Detector

a) Notation: l , t = person lies or is truthful
L = machine says "LIE"

P (L/l) = 0.8 P (L/t) = 0.5 P (l) = 0.2
P (L) = P(L/l) P(l) + P(L/t) P(t) = 0.8 (0.2) + 0.5 (0.8) = 0.56
The factor revising the prior is: P(L/l) / P(L) = 0.8 / 0.56 = 1.43
The posterior probability is thus: P(l/L) = 0.2 (1.43) = 0.286

b) Same formula, new numbers: P(L) = 0.99(0.2) + 0.5 (0.8) = 0.598
The factor revising the prior is: P(L/l) / P(L) = 0.99/ 0.56 = 1.65
The posterior probability is thus: P(l/L) = 0.2 (1.65) = 0.33

This is a noticeable improvement. But to get really good results one has to work also on the false positives.

c) P (L) = 0.8 (0.8) + 0.5 (0.2) = 0.74
The factor revising the prior is: P(L/l) / P(L) = 0.8 / 0.74 = 1.08
The posterior probability is thus: P(l/L) = 0.8 (1.08) = 0.864
In this case the machine is virtually useless.

15.4 VLSI Chips

a) Use Likelihood Ratios

P(meets specs)	= .75	P(not meet specs)	= .25
P(fail/meets specs)	= .4	P(pass/meet specs)	= .6
P(fail/not meet specs)	= .8	P(pass/not meet specs)	= .2

LR_O = P(meets specs)/P(does not meet specs) = .75/.25 = 3
CLR_P = P(pass/meets specs)/P(pass/not meet) = .6/.2 = 3
CLR_F = .4/.8 = 1/2
$LR_N = (3)(3^3)(1/2) = 81/2$

P(meets specs/3 pass,1 fail) = $LR_N/(1+LR_N)$ = 0.976

b) Test failures indicate a likelihood of not meeting specifications. The more failures, the greater the probability that a chip does not meet specs. The minimum number of tests to achieve 90% probability that a chip does not meet specs is five, in which case there would be 5 fails and no passes as shown below.

P(not meet specs/tests) > .9, so P(meet specs/tests) < .1

$LR_N/(1+LR_N) < .1$ implies $LR_N < 1/9 = .111$
$LR_N = (3)(3^0)(1/2)^5 = 0.0938 < .111$

P(not meet specs/0 pass,5 fail) = .914

Note: If any tests are passed, then additional fails could still give a probability of .9 for not meeting specs. For example, 1 pass and 7 fails yields P(not meet spec) = .934.

15.5 Oil Drilling

a) P(profit)=2/3 P(no profit)=1/3
P(pos/profit)=.75 P(neg/profit)=.25
P(neg/no profit)=.5 P(pos/no profit)=.5

Find P(profit/1 pos, 1 neg)

Use Bayes Theorem, apply once for each observation.

First stage: P(pos) =(2/3)(.75) + (1/3)(.5) = 2/3
P(profit/1 pos) = (2/3)[(.75/.67)] = .75

Second stage: P(neg) = (.75)(.25) + (.25)(.5) = .3125
P(profit/1 pos,1 neg) = .75[(.25/.3125)] = 0.6

Using Likelihood Ratios:

$LR_o = (2/3)/(1/3) = 2$ $CLR_{pos} = 3/2$ $CLR_{neg} = 1/2$

$LR_N = (2)(3/2)(1/2) = 3/2$

P(profit/1 pos,1 neg) = $LR_N/(1+LR_N)$ = .6

b) $LR_N = (2)(3/2)^3 (1/2)^2 = 27/16$

P(profit/3 pos,2neg) = 27/43 = .628

c) Use Likelihood Ratios whenever there are a series of observations. This requires fewer calculations than Bayes' Theorem.

15.6 Sonny Reyes

a) $LR_o = .8/.2 = 4$

b) $CLR_p = .8/.2 = 4$ $CLR_f = .2/.8 = .25$

c) $LR_2 = LR_o(CLR_p)(CLR_f) = 4(4)(.25) = 4$

d) P (S/1P, 1F) = 4/5 = .8

15.7 SIDA

a) 1 % (two decimal place accuracy)

b) As often with other rare events, the false positives dominate the results. That is, most of the positive results stem from errors in the diagnosis of healthy people.

c) 99% after three positive results; 10^{-6} after 1 positive, 2 negative.

15.8 Weather Expert

Prior: P(Frost) = 0.6

P(says Frost/Frost) = P(says NF/NF) = 0.8

P(says Frost) = 0.6(0.8) + 0.4(0.2) = 0.56

P(says NF) = 0.4(0.8) + 0.6(0.2) = 0.44

a) P(Frost / says F) = P(Frost)[P(says F/Frost) / P(says F)]
= 0.6[0.8 / 0.56] = 6 / 7

b) P(NF / says NF) = 0.4[0.8 / 0.44] = 8 / 11
P(Frost / says NF) = 3 / 11

15.9 Summer Goods

Prior: P(Def) = 0.02

P(Visual says D / D) = P(Visual says OK / OK) = 0.5

P(Detail says D / D) = P(Detail says OK / OK) = 0.9

P(Visual says OK) = 0.02 (0.5) + 0.98 (0.5) = 0.5

a) P(Def/Says OK) = P(Def)[P(says OK/ OK) / P(says OK)]
= 0.02 [0.5 / 0.5] = 0.02

b) P(Def / Visuals say D) = 0.02 [0.5 / 0.5] = 0.02

P(Detail says OK) = 0.02 (0.1) + 0.98 (0.9) = 0.884

P(Def/Detail says OK) = P(Def) [P(says OK/Def) / P(says OK)]
= 0.02 [0.1 / 0.884] = 0.002 / 0.884 ~ 0.002

Probability of passing both tests = 0.5 (0.02) + 0.5 (0.002)
= 0.011 = 1.1%

Percent of defectives not found = 1.1 / 2 = 55%

c) P(Def/ Details says OK) = 0.02 (0.1 / 0.884) ~ 0.002

By comparing (b) and (c) we see that applying detailed examination to half the goods reduces the percent defectives from 2 to 1.1%, a reduction of 0.9%. Applying this examto all goods we get a reduction of 1.8%. Therefore, if the detailed examination is cost-effective at all it should be applied to all goods, especially since this plicy eliminates the cost of the visual examination which is worthless.

15.10 Championship Playoff

Given : $P(TC) = 0.5$ $P(W/TC) = 0.7$

a) To apply Bayes' theorem, we need to know P(W). This depends on the current estimate of P(TC). For the first game:

$P(W) = P(TC)\ P(W/TC) + P(Loser)\ P(W/Loser)$
$= 0.5\ (0.7) + 0.5\ (0.3) = 0.5$
$P(TC/W) = P(TC)\ [P(W/TC)\ /\ P(W)] = 0.5\ [0.7\ /\ 0.5] = 0.7$

b) For two wins in a row:

$P(W/W) = 0.7\ (0.7) + 0.3\ (0.3) = 0.58$
$P(TC/WW) = 0.7\ [0.7\ /\ 0.58] = 0.84$

For three wins in a row:

$P(W/WW) = 0.84\ (0.7) + 0.16\ (0.3) = 0.64$
$P(TC/WWW) = 0.84\ [0.7\ /\ 0.64] = 0.92$

By likelihood ratios:

$L_O = 0.5\ /0.5 = 1$; $CLR_W = 0.7\ /0.3 = 7/3$; $CLR_L = 0.3\ /0.7 = 3/7$

$LR_3 = (7/3)^3 = 343/27$
$P(TC/WWW) = LR_3/\ (1 + LR_3) = 0.92$

c) $LR_{N,(N-1)} = (7/3)^N\ (3/7)^{N-1} = 7/3$
$P(TC) = 0.7$ is the same in all cases.