Exercise 12.3

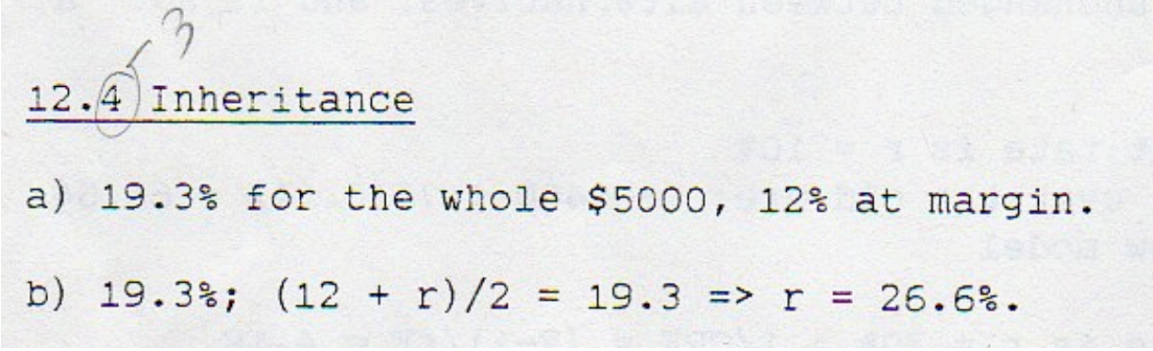
Question

12.3. *Inheritance*
Assume you receive $5000 to invest for your future. Your available alternatives for this money are:

- Use up to $2500 of it to pay a tuition loan, on which you pay 12% interest on unpaid balances.
- Put as much as you want of the $5000 in a savings bank to earn 5% per year.
- Buy a discounted bond for $5000 (the smallest denomination), that is redeemable only after 3 years for $8500. You can then reinvest this money in more of the same type of bonds.

(*a*) What is the opportunity cost, in percent, of the $5000?
(*b*) What discount rate should you use for evaluating an investment opportunity for the entire $5000 in some other business? for only $2500 of the $5000?

Solution from Manual


12.4 Inheritance

a) 19.3% for the whole $5000, 12% at margin.

b) 19.3%; (12 + r)/2 = 19.3 => r = 26.6%.

Additional Notes

a) Ranked in order, the investment opportunities go as follows:

$5,000 to buy a bond that will give $8,500 exactly three years from now. However, we do not know yet the implicit return on this, so we need to calculate (see below)
$2,500 invested at 12% to pay for tuition loan
$5,000 invested in a savings account at 5% return

The return on the bond investment is obtained as follows:

$$F = P(1+r)^N$$

where F is the future value of money, P it's present value, r the discount rate, and N the number of time periods we are considering for the investment. Here, N = 3 years, P =

$5,000, F = $8,500. We are looking for the value of r that satisfies these constraints in the above relationship.

$$r = \left(\frac{F}{P}\right)^{\frac{1}{N}} - 1 \Rightarrow r = 19.3\%$$

Since 19.3% is the highest of all returns, it is also the opportunity cost in percent of our next $5,000. This is the value of the next best alternative investment at our disposal. In this case, this is also our discount rate because we may decide to invest in the bond only.

b) For the first part of the question, we have the entire $5,000 at our disposal, and since we already have the opportunity of investing in the bond at 19.3% return, this should be the minimal return will we accept for taking on any other investment opportunity.

For the second part, the question is about investing $5,000, of which $2,500 go to another business. For a total investment of $5,000, our discount rate is still 19.3% as found in the first part because we still have the possibility to choose to invest the entire amount in the bond. If we split the $5,000 into two, we can't however get the bond because the smallest denomination is $5,000. Therefore, we can invest $2,500 in the loan at 12%, and the remaining $2,500 in some other business. Since we are splitting our original amount into two investments, our discount rate should be a weighted average of two returns:

$$r = \frac{12\% \times \$2{,}500 + r_{New\ \ Business} \times \$2{,}500}{\$5{,}000} = 19.3\% \Rightarrow r_{New\ \ Business} = 26.6\%$$

where $r_{\text{New Business}}$ is the minimal return (or discount rate) we should use to evaluate whether we want to invest in this other business or not.