ESD.70J Engineering Economy

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Session Two

Michel-Alexandre Cardin – macardin@mit.edu
Prof. Richard de Neufville – ardent@mit.edu
Session two – Simulation

• Objectives:
  – Generate random numbers
  – Get familiar with Monte Carlo simulation
  – Set up simulation using Data Table
  – Generate statistics from simulation
  – Draw histogram and cumulative distribution function (CDF)
    • Also called “target curve”
  – Repeat exercises using @Risk software
    • Time permitting
Questions for “Big vs. Small”

From the base case spreadsheet, we’ve calculated NPVs

However, we assumed deterministic demand forecasts for years 1, 2, and 3. This assumption is over-simplifying since actual demand will vary

⇒ Since life is uncertain, we want to simulate a range of possible NPV outcomes, the Min, Max, distributions, and the $E[\text{NPV}]!$
Set up random generator

Open ESD70session2-1.xls
Excel’s RAND() function

• Returns random number greater than or equal to 0 and less than 1, sampled from a uniform distribution
• To generate a random real number between a and b, use: =RAND()*(b-a)+a
  – Returns a uniformly distributed random demand for year 1 centered around 300, which may differ by plus or minus 50%
• Same logic applies for cell C4 and C5
Random number generator

Follow the instructions, step by step

1. Go to tab “RAND”
5. Press “F9” several times to see what happens
Random number generator

6. Click “Chart” under “Insert” menu
7. “Chart Type” select “XY(Scatter)”, “Chart sub-type” select any one with lines, click “Next”
9. “Chart options” select whatever pleases you, click “Next”
10. Choose “As object in” and click “Finish”
11. Press “F9” several times to see what happens

We have built a random demand generator for the 3 years that assumes independent demand (0 correlation) from year to year
Give it a try!

Check with your neighbors…

Check the solution sheet…

Ask me questions…
How Monte Carlo Simulation works

Calculate two NPV_As corresponding to the two random demand simulations

<table>
<thead>
<tr>
<th>Demand in Year 1</th>
<th>Demand in Year 2</th>
<th>Demand in Year 3</th>
<th>NPV_A</th>
</tr>
</thead>
<tbody>
<tr>
<td>345</td>
<td>678</td>
<td>1001</td>
<td>?</td>
</tr>
<tr>
<td>189</td>
<td>579</td>
<td>690</td>
<td>?</td>
</tr>
</tbody>
</table>

How about generating many sets of random demands, and get the corresponding NPV_As automatically?
Monte Carlo Simulation

1. Generate many sets of random demands for the three-year span
2. Calculate corresponding NPVs
3. Generate Distribution of NPVs
4. Statistical Analysis
Setup simulation by Data Table

Follow these instructions, step by step:

1. Link demand in sheet for Plan A to the random demand generator, specifically, Plan A!E5 = Rand!C3; Plan A!G5 = Rand!D3; Plan A!I5 = Rand!E5

2. In “Simulation” sheet, type “=‘Plan A’!C16” in cell B8 (“=‘Plan A’!C16” is the output of result for $NPV_A$)

3. Create the Data Table. Select “A8:B2008”, click “Table” under “Data” menu, in “column input cell” put “A7”, leave “row input cell” blank.

4. Same thing already done for Plan B

NOTE: there is no input in the value column of the Data Table; an empty cell is selected as the “column input cell”. Why?
Explanation

• For the One-Way Data Table, there is no need to set up the input values in a list, since each row of the Data Table calls RAND() and generates an $NPV_A$ projection

• We have 2,000 rows in the Data Table, so we have simulated 2,000 times

• Click “command =” or “F9” to try another simulation run
Excel crashing note

If Excel crashes during simulation runs, input some numbers (0’s or whatever) into the input value column to the left of the data series. Do not leave the area of input values blank in the Data Table.

You can hide the dummy values by setting their font value to “white” color.
Give it a try!

Check with your neighbors…

Check the solution sheet…

Ask me questions…
Calculating descriptive statistics

- Useful to know E[NPV], maximum, and minimum values for the simulated results

Follow step by step:

1. In Cell D1 type “=AVERAGE(B$9:B$2008)”
2. In Cell D2 type “=MAX(B$9:B$2008)”
3. In Cell D3 type “=MIN(B$9:B$2008)”
Give it a try!

Check with your neighbors...

Check the solution sheet...

Ask me questions...
Deterministic vs. dynamic results

- From the base case spreadsheet, we learn $\text{NPV}_A = $162.1M and $\text{NPV}_B = $156.5M
- What is your result for the $E[\text{NPV}_A]$ and $E[\text{NPV}_B]$ when considering demand uncertainty?
- Jensen’s inequality and the Flaw of Averages:
  
  $$ f[E(x)] \neq E[f(x)] $$
Target curve

- The target curve is another name for cumulative distribution function (CDF)

- In our case, a target curve aims at making a representation to managers that
  - “There is a probability X that NPV will be lower (higher) than a targeted Y dollars for this project”

- Value At Risk is a common language on Wall Street. It stresses downside risk, though we should also look at CDF for upside potential of a project, or Value At Gain!
Target curve

Follow the instructions, step by step:

1. In sheet “Simulation”, set Cell G7 “=D$3+(D$2-D$3)/20*F7”, and drag the formula down to G27
2. Set Cell H7 “=COUNTIF(B$9:B$2008,"<"&G7)”, and drag the formula down to H27
3. Set Cell I7 “=H7/2000”, and drag down to cell I27
4. Same is already done for Plan B
Target curve

6. Right-click the chart on the right, select “Source Data”
7. Select “Series”, and press “Add”. This adds a new data series to the graph. Call it “NPV_A”
8. Select the range =Simulation!$G$7:$G$27 for X values, and the range =Simulation!$I$7:$I$27 for Y values. Click “OK”
9. Right-click the curve and change “Weight” to 3
10. Hit “command =” or “F9” and watch the target curve move!
Explanation

• We set up 20 data buckets and count how many data points fall into each interval.
• “=COUNTIF()” function counts the number of cells within a range that meet the criteria.
• The Excel file demonstrates how you can:
  – Add $E[NPV_A]$ and $E[NPV_B]$ as vertical lines.
  – Add histograms for two NPV distributions using the information created earlier.
• Can also use the Histogram analysis tool in “Data Analysis” package, but it won’t refresh.
Values At Risk and Gain

• Use your cursor on the graph to find different Values At Risk and Values At Gain
• Alternatively, use the percentile function
  – In cell N5, type 10%
  – In cell R5, type
    “=PERCENTILE(B9:B2008,N5)”
• What does this tell you?
• That’s interesting information for managers and decision-makers!
Question

- Why are high NPV values more cut off for Plan B on the target curve and histogram than for Plan A?
  - A matter of constraints…
Give it a try!

Check with your neighbors…

Check the solution sheet…

Ask me questions…
Repeat exercise in @Risk

Open ESD70session2-1_@Risk.xls
@Risk RAND generator

Follow the instructions, step by step

1. Go to tab “RAND”

2. Type “=RiskUniform(Entries!C9*(1-Entries!C25),Entries!C9*(1+Entries!C25))” in cell C3


Run simulations in @Risk

1. Go to tab “Plan A”
2. Type “=RiskOutput(“NPV A (Million dollar)”) + SUM(C15:I15)” in cell C16
3. Click on menu “@Risk”
4. Change “Iterations” to 2,000
5. Click on “Start Simulation” and see what happens
6. Right-click on figures, select “Distribution Format” ⇒ “Relative Frequency”
Simulation outputs

NPV A (Million dollar)

NPV B (Million dollar)
Target curves

- Click on icon at the bottom of figure for NPV A
- Select “Cumulative Ascending”
- Click on icon at the bottom of figure
- Click on cell C16 under tab “Plan B”, click OK
Target curve outputs

@RISK Student Version
For Academic Use Only
Comparison

- How do statistical results compare between Data Tables and @Risk approaches?
- What about histograms and target curves?
Summary

• Random number generation is fairly straightforward in Excel and @Risk
• At least two ways to run Monte Carlo simulation:
  – Direct RAND() calls - too long…
  – Using Data Table or @Risk - the way to go!
• Descriptive statistics from simulations
  – E[NPV], Max, Min, target curve
Next class...

- Today’s session modeled demand uncertainty based on a uniformly distributed random variable
- This is not necessarily realistic, though it is simple and sufficient for today’s purposes
- Next session explores alternative probability distributions from which to sample and stochastic models
- STAY TUNED!