ESD.70J Engineering Economy

Fall 2008
Session Three

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Question from Session Two

Yesterday we used uniformly distributed random variables to model uncertain demand. This implies identical probability of median as well as extreme high and low outcomes. This is not very realistic…

⇒ What alternative probability distribution models should we use to sample demand?

Session three – Modeling Uncertainty

- Objectives:
  - Generate random numbers from various distributions (Normal, Lognormal, etc)
    - So you can incorporate in your model as you wish
  - Generate and understand random variables that evolve through time (stochastic processes)
    - Geometric Brownian Motion, Mean Reversion, S-curve

About random number generation

Open ESD70session3-1Part1.xls

(Two parts because RAND() calls and graphs take long to compute and update for every Data Table iteration…)

http://ardent.mit.edu/real_options/ROcse_Excel_latest/ESD 70 2008/ESD70session3-1Part1.xls

About random number generation

- Generate normally distributed random numbers:
  - Use NORMINV(RAND(), μ, σ) (NORMINV stands for "the inverse of the normal cumulative distribution")
  - μ is the mean
  - σ is the standard deviation
- In cell B1 in "Sim" sheet, type in "=NORMINV(RAND(), 5, 1)"
- Create the Data Table for 2000 samples
- Press "F9", see what happens

Random numbers from triangular distribution

- Triangular distribution could work as an approximation of other distribution (e.g. normal, Weibull, and Beta)
- Try "=RAND()+RAND()" in the Data Table output formula cell B1
- Press "F9", see what happens

Random numbers from lognormal distribution

- A random variable X has a lognormal distribution if its natural logarithm has a normal distribution
- Using LOGINV(RAND(), ln_μ, ln_σ)
  - ln_μ is the mean of ln(X)
  - ln_σ is the standard deviation of ln(X)
- In the Data Table output formula cell B1, type "=LOGINV(RAND(), 2, 0.3)"
- Press "F9", see what happens

Give it a try!

Check with your neighbors…

Check the solution sheet…

Ask me questions…

From probability to stochastic processes

- We have just described the probability density function (PDF) of random variable x, or f(x)
- We can now study the time function of distribution of random variable x across time, or f(x,t)
- That is a stochastic process, or in plain English language:

TREND + UNCERTAINTY

Three stochastic models

- Geometric Brownian Motion
- Mean-reversion
- S-Curve

Geometric Brownian Motion

- Brownian motion (also called random walk)
  - The motion of a pollen in water
  - A drunk walk in Boston Common
  - S&P500 return
- Rate of change of the geometric mean is Brownian, not the underlying observations
  - For example, the stock prices do not necessarily follow Brownian motion, but their returns do!

Brownian motion theory

- This is the standard model for modeling stock price behavior in finance theory, and lots of other uncertainties
- Mathematic form for Geometric Brownian Motion (you do not have to know):

$$dS = \underbrace{\mu S dt}_{\text{trend}} + \underbrace{\sigma S dz}_{\text{uncertainty}}$$

where S is the stock price, μ is the expected return on the stock, σ is the volatility of the stock price, and dz is the basic Wiener process

Simulate a stock price

Open ESD70session3-1Part2.xls

http://ardent.mit.edu/real_options/ROcse_Excel_latest/ESD 70 2008/ESD70session3-1Part2.xls

Simulate a stock price

- Google's common stock price as of 8/21/08 is $473.75 (see "GOOG" tab)
- Using regression analysis on historical price data, we calculate monthly growth rate (drift) of 2.79% and volatility of 23.46%
- These two values are key inputs into any forward-looking simulation models. We will be using them repeatedly, so lets define their names…

Defining Excel variable names

1. Select cell with the historical mean value (2.79%) and go to: "Insert" → "Name" → "Define"
2. Enter field name "drift" and hit "OK"
3. Repeat the same for historical standard deviation and call that variable "vol"

Simulate a stock price (Cont)

Complete the following table for Google stock in tab "GOOG forecast":

Time	Stock Price	Random Draw from standardized normal distribution[1]	Expected Return + random draw * volatility
September	$473.75	=NORMINV(RAND(),0,1)	=drift+vol*C2
October	=B2*(1+D2)		
November			
December			

1) Standardized normal distribution with mean 0 and standard deviation 1

Simulating Google returns in Excel

1. In worksheet "GOOG forecast", type "=NORMINV(RAND(),0,1)" in cell C2, and drag down to cell C13
2. Type "=drift+vol*C2" in cell D2, and drag down to cell D13
3. Type "=B2*(1+D2)" in cell B3, and drag down to cell B13
4. Click "Chart" under "Insert" menu

Simulating Google returns in Excel

7. "Standard Types" select "Line", "Chart sub-type" select whichever you like, click "Next"
8. "Data Range" select "= 'GOOG forecast'!$A$2:$B$13", click "Next"
9. "Chart options" select whatever pleases you, click "Next"
10. Choose "As object in" and click "Finish"
11. Press "F9" several times, see what happens

Give it a try!

Check with your neighbors…

Check the solution sheet…

Ask me questions…

Mean reversion

- Unlike Geometric Brownian Motion that grows forever at the rate of drift, some processes have the tendency to
  - Fluctuate around a mean
  - The farther away from the mean, the high the probability of reversion to the mean
  - The speed of mean reversion can be measured by a parameter η

Mean reversion theory

- Mean reversion has many applications besides modeling interest rate behavior in finance theory
- Mathematic form (you do not have to know): $dr = \eta(\mu - r)dt + \sigma dz$

where r is the interest rate, η is the speed of mean reversion, μ is the long-term mean, σ is the volatility, and dz is the basic Wiener process

Simulating interest rate

- In finance, people usually use mean reversion to model behavior of interest rates and asset volatilities
- Suppose the Fed rate r is 4.25% today, the speed of mean reversion η is 0.3, the long-term mean μ is 7%, the volatility σ is 1.5% per year
- Expected mean reversion is: $dr = \eta(\mu - r)dt$

Simulating interest rate

Complete the following table for interest rate:

Time	Interest rate	Random Draw from standardized normal distribution	Realized return $dr = \eta(\mu - r)dt + \sigma dz$
2006	4.25%	=NORMINV(RAND(),0,1)	=$H$2*($H$3-B2)+C2*$H$4
2007	=B2+D2		
2008			
2009			
2010			

Interest rate forecast in Excel

1. In worksheet "Interest Rates", type "=NORMINV(RAND(),0,1)" in cell C2, and drag down to cell C12
2. Type "=$H$2*($H$3-B2)+C2*$H$4" in cell D2 to represent the model, and drag down to cell D12
3. Type "=B2+D2" in cell B3, and drag down to cell B12. NOTE: the two values are added since the model expresses a change in return compared to initial return, not a change in stock price as for the GBM model
4. Click "Chart" under "Insert" menu

Interest rate forecast in Excel

7. "Standard Types" select "XY(Scatter)", "Chart sub-type" select any one with line, click "Next"
8. "Data range" select "='Interest Rates'!$A$2:$B$12", click "Next"
9. "Chart options" select whatever pleases you, click "Next"
10. Choose "As object in" and click "Finish"
11. Press "F9" several times, see what happens

Give it a try!

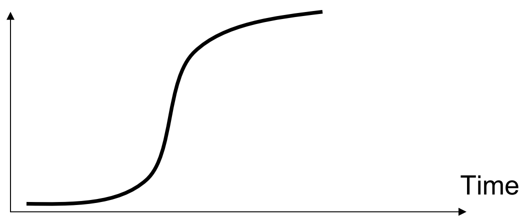
Check with your neighbors…

Check the solution sheet…

Ask me questions…

S-curve

- Many interesting process follow the S-curve pattern



For example, demand for a new technology initially grows slowly, then the demand explodes exponentially and finally decays as it approaches a natural saturation limit

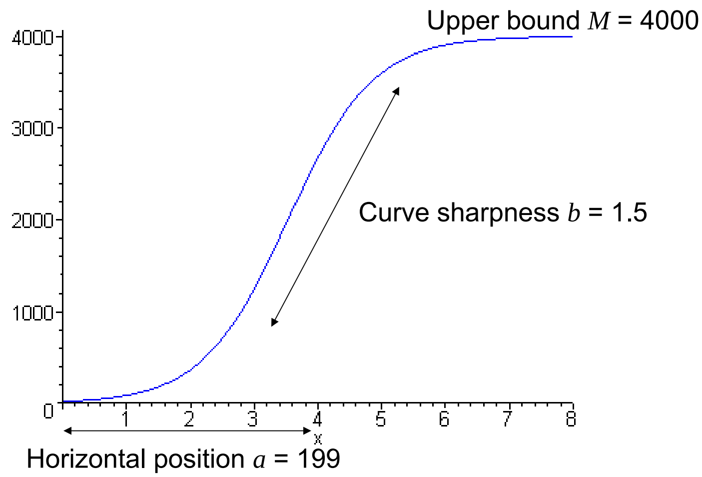
S-curve

- Overall form of S-curve

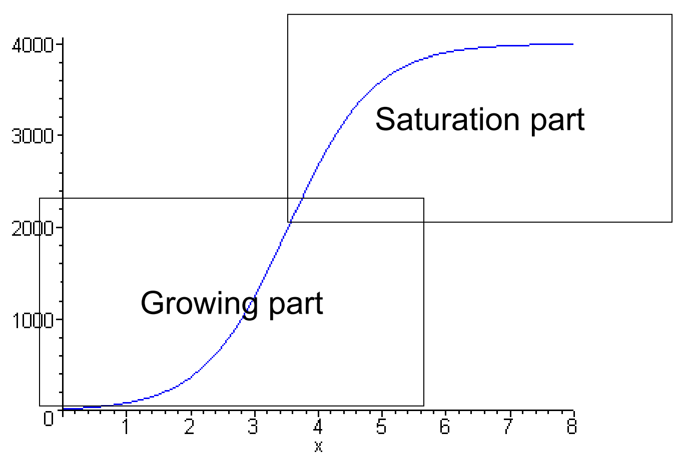
$$y(x) = \frac{M}{1 + ae^{(-bx)}}$$

  - M is upper bound on maximum value
  - b determines how fast we go through the temporal range to reach the upper bound
  - a interacts with b, but translates the curve horizontally

S-curve



S-curve



Modeling S-curve deterministically

- Parameters:
  - Demand at year 0
  - The limit of demand (M), or demand at time ∞
  - Sharpness parameter b
- Model:

$$D(t) = \frac{M}{1 + ae^{(-bt)}}$$

  - Translation parameter a can be approximated from demand at year 0 and the upper bound M at ∞:

$$a = \frac{M}{Demand(0)} - 1$$

Modeling S-curve dynamically

- We can estimate incorrectly the initial demand, the limit of demand, and the sharpness parameter, so all of these are random variables
- The growth every year is subject to an additional annual volatility

S-curve example

- In tab "S-curve"
  - Demand(0) = 80 (may differ ± 20%)
  - Limit of demand M = 1600 (± 40%)
  - Sharpness parameter b = 1 (± 40%)
  - Annual volatility is 10%

Give it a try!

Check how the model is built…

Ask me questions…

Back to Big vs. small?

- We talked about the following models today
  - Normal, Triangular, Lognormal
  - Geometric Brownian Motion
  - Mean Reversion
  - S-curve
- Which one is more appropriate for our demand modeling problem? Why?

Model calibration challenges

- Knowing the theoretical models is only a start. Properly calibrating them is critical
- Otherwise – GIGO
- In many cases, data is scarce for interesting decision modeling problems
- It is good habit to study plausible sources of data for your line of work
  - So you have a model that is representative of reality!

Example

- We simulated the movement of Google stock price using the expected monthly growth rate (return) of 2.79% and volatility of 23.46%. Is it reasonable?
- In the Google IPO of 2004, there was no historical data to draw upon

⇒ Solution - use a comparable stock, like Yahoo, to estimate expected drift and volatility

Issues in modeling

- Do not trust the model – this should be the basic assumption for using any model
  - Highly complicated models are prone (if not doomed) to be misleading
  - The more inputs are required – the more room for error
  - Always check sensitivity of inputs
- Dynamic models offer great insights, regardless of the output data errors
- In some sense, it is more a way of thinking, analysis and communication

Summary

- We have generated random numbers from various distributions
- Explored random variables as functions of time (stochastic processes)
  - Geometric Brownian Motion
  - Mean Reversion
  - S-curve

Next class…

The course has so far concentrated on ways to model uncertainty

Modeling is passive. As human being, we have the capacity to manage uncertainties proactively. This capacity is called flexibility and contingency planning

⇒ Next class we'll finally explore ways to extract additional value from uncertainty and assess the value of flexibility!