Question from Session Two

Last time we used uniformly distributed random variables to model the uncertain demand. This implies identical probability of median as well as extreme high and low outcomes. It’s not too hard to imagine why this is not very realistic.

What alternative models for demand uncertainties should we try?
Modeling Uncertainty

- Generate random numbers from various distributions (Normal, LogNormal, etc)
- Random variables as time function (stochastic processes)
  - Geometric Brownian Motion
  - Mean Reversion
  - S-curve
- Statistical analysis to data-mine distribution and its descriptive stats from historical data

Random numbers generation redux

- Generate normally distributed random numbers:
  - Use norminv(rand(), μ, σ) (norminv stands for “the inverse of the normal cumulative distribution”)
  - μ is the mean
  - σ is the standard deviation
- In the data table output formula cell (B1 in “Sim” sheet of 1.xls) type in “=norminv(rand(), 5, 1)”. Press “F9”, see what happens

Link for Excel: http://ardent.mit.edu/real_options/ROcse_Excel_latest/Session3-1.xls
Random numbers from triangular distribution

- Triangular distribution could work as an approximation of other distribution (e.g. normal, Weibull, and Beta)
- Try “=rand()+rand()” in the data table output formula cell (B1 in “Sim” sheet of 1.xls), press “F9”, see what happens.

Random numbers from lognormal distribution

- A random variable X has a lognormal distribution if its natural logarithm has a normal distribution
- Using loginv(rand(), log_μ, log_σ)
  - log_μ is the log mean
  - log_σ is the log standard deviation
- In the data table output formula cell (B1 in “Simu” sheet of 1.xls) type in “=loginv(rand(), 2, 0.3)”. Press “F9”, see what happens
From probability to stochastic processes

- We can describe the probability density function (PDF) of random variable \(x\), or \(f(x)\).
- Apparently, the distribution of a random variable in the future is not independent from what happens now.
- Life is random in a non-random way…

From probability to stochastic processes

- We have to study the time function of distribution of random variable \(x\) across time, or \(f(x,t)\).
- That is a stochastic process, or in plain English language:
  
  TREND + UNCERTAINTY
Check the solution sheet.

Please ask questions now…

Three stochastic models

• Geometric Brownian Motion

• Mean-reversion

• S-Curve
Geometric Brownian Motion

• Brownian motion (aka random walk)
  – the motion of a pollen in water
  – a drunk walks in Boston Common
  – S&P500 return

• Rate of change of the geometric mean is Brownian, not the underlying observations
  – For example, the stock prices do not follow Brownian motion, but their returns do!

Simulate a stock price

• Google’s stock price is $378.49 per class A common share on 9/8/06 (see “GOOG” tab).
• Using historical data, we calculate monthly mean return and volatility of 6% and 14%
• These two values are key inputs into any forward-looking simulation models. We will be using them repeatedly, so let’s define their names…
Defining Excel variable names

1. Select sell with the historical mean value (6.16%) and go to: <Insert> → <Name> → <Define>
2. Enter field name “drift” and hit <OK>.
3. Repeat the same for historical standard deviation and call that variable “vol”.

Simulate a stock price (Cont)

Complete the following table for Google stock:

<table>
<thead>
<tr>
<th>Time</th>
<th>Stock Price</th>
<th>Random Draw from standardized normal distribution(^1)</th>
<th>Expected Return + random draw * volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>September</td>
<td>$378.49</td>
<td>=NORMINV(RAND(),0,1)</td>
<td>=drift+vol*C2</td>
</tr>
<tr>
<td>October</td>
<td>=B2*(1+D2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>November</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>December</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1). Standardized normal distribution with mean 0 and standard deviation 1
Simulating Google returns in Excel

1. Open a new worksheet, name it “GOOG forecast”
2. Copy or input forecasting time frame (i.e.: “Time” in cell A1, “September” in A2)
3. Type “=norminv(rand(),0,1)” in cell C2, and drag down to cell C13
4. Type “=drift+vol*C2” in cell D2, and drag down to cell D13
5. Type “=B2*(1+D2)” in cell B3, and drag down to cell B13
6. Click “Chart” under “Insert” menu

Simulating Google returns in Excel (cont)

7. “Standard types” select “Line”, “Chart sub-type” select whichever you like, click “Next”
9. “Chart options” select whatever pleases you, click “Next”
10. Choose “As object in” and click “Finish”
11. Press “F9” several times to see what happens.
Check the solution sheet.
Please ask questions now…

Brownian Motion Theory

• This is the standard model for modeling stock price behavior in finance theory, and lots of other uncertainties (enter the Central Limit Theorem)
• Mathematic form for Geometric Brownian Motion (you do not have to know)

\[ dS = \mu S dt + \sigma S dz \]

where \( S \) is the stock price, \( \mu \) is the expected return on the stock, \( \sigma \) is the volatility of the stock price, and \( dz \) is the basic Wiener process
Mean-reversion

• Unlike Geometric Brownian Motion that grows forever at the rate of drift, some processes have the tendency to
  – fluctuate around a mean
  – the farther away from the mean, the high the probability of reversion to the mean
  – the speed of mean reversion can be measured by a parameter $\eta$

Simulating interest rate

• In finance, people usually use mean reversion to model behavior of interest rates and asset volatilities
• Suppose the Fed rate $r$ is 4.25% today, the speed of mean reversion $\eta$ is 0.3, the long-term mean $\bar{r}$ is 7%, the volatility $\sigma$ is 1.5% per year
• Expected mean reversion is:

$$dr = \eta(\bar{r} - r)dt$$
Simulating interest rate (Cont)

Complete the following table for interest rate:

<table>
<thead>
<tr>
<th>Time</th>
<th>Interest rate</th>
<th>Random Draw from standardized normal distribution</th>
<th>Realized return (expected reversion + random draw * volatility)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>4.25%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2007</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2008</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2009</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Interest rate forecast in Excel

1. Open a new worksheet, name it “Interest Rates”
2. Copy or input the table in the previous slide into Excel, with “Time” as cell A1
3. Type “=norminv(rand(),0,1)” in cell C2, and drag down to cell C12
4. Type “=0.3*(0.07-B2)+C2*0.015” in cell D2, and drag down to cell D12
5. Type “=B2+D2” in cell B3, and drag down to cell B12
6. Click “Chart” under “Insert” menu
Interest rate forecast in Excel

7. “Standard types” select “XY(Scatter)”, “Chart sub-type” select any one with line, click “Next”
9. “Chart options” select whatever pleases you, click “Next”
10. Choose “As object in” and click “Finish”
11. Press “F9” several times to see what happens.

Check the solution sheet.

Please ask questions now…
Mean reversion Theory

• Mean reversion has many applications besides modeling interest rate behavior in finance theory

• Mathematic form (you do not have to know)  \( dr = \eta(r - \bar{r})dt + \sigma dz \)

where \( r \) is the interest rate, \( \eta \) is the speed of mean reversion, \( \bar{r} \) is the long-term mean, \( \sigma \) is the volatility, and \( dz \) is the basic Wiener process

S-curve

• Many interesting process follow the S-curve pattern

For example, demand for a new technology initially grows slowly, then the demand explodes exponentially and finally decays as it approaches a natural saturation limit
Modeling S-curve Deterministically

• Parameters:
  – Demand at year 0
  – Demand at year T
  – The limit of demand, or demand at time \( \infty \)

• Model:
  \[ \text{Demand}(t) = \text{Demand}(\infty) - \alpha e^{-\beta t} \]

• \( \alpha \) and \( \beta \) can be derived from demand at year 0 and year T
  \[ \alpha = \text{Demand}(\infty) - \text{Demand}(0) \]
  \[ \beta = -\ln\left(\frac{\text{Demand}(\infty) - \text{Demand}(10)}{\alpha}\right)/10 \]

Modeling S-curve dynamically

• We can estimate incorrectly the initial demand, demand at year T, and the limit of demand, so all of these are random variables

• The growth every year is subject to an additional annual volatility
S-curve example

- Demand(0) = 80 (may differ +/- 20%)
- Demand(10) = 1000 (may differ plus or minus 40%)
- Limit of demand = 1600 (May differ plus or minus 40%, not less than (Demand(10)+100))
- Annual volatility is 10%

Link for Excel: http://ardent.mit.edu/real_options/ROcse_Excel_latest/Session3-2.xls

Back to Big vs. small?

- We talked about the following models today
  - Normal
  - LogNormal
  - Geometric Brownian Motion
  - Mean Reversion
  - S-curve
- Which one is more appropriate for our demand modeling problem? Why?
Model calibration challenges

• Knowing the theoretical models is only a start. Properly calibrating them is critical
• Otherwise – GIGO
• In many cases, data is scarce for interesting decision modeling problems.
• A good everyday habit to contemplate plausible sources of data for your line of work.

Example

• We simulated the movement of Google stock price using the expected monthly return of 6% and quarterly volatility of 14%. Is it reasonable?
• When Google IPO-ed in 2004, there was no historical data to draw upon. Solution - use a comparable stock, like Yahoo, to estimate expected drift and volatility.
Issues in modeling

• Do not trust the model – this is the presumption for using any model.
  – Highly complicated models are prone (if not doomed) to be misleading
  – The more inputs are required – the more room for error
  – Always check sensitivity of inputs
• Dynamic models offer great insights, regardless of the output data errors
• In some sense, it is more a way of thinking, analysis and communication

Summary

• We have generated random numbers from various distributions
• Explored random variables as functions of time (stochastic processes)
  – Geometric Brownian Motion
  – Mean Reversion
  – S-curve
• Used statistical analysis to collect key model inputs
Next class…

The course has so far concentrated on ways to model the uncertainty.

Modeling is passive. As human being, we have the capacity to manage uncertainties proactively. This capacity is called flexibility and contingency planning.

The next class we’ll finally explore way to model and value the flexibility.