Valuation of Financial Options

Outline

- Examined options basics
  - Rights, not obligations
  - Asymmetric payoffs (limited losses)
  - Value different from immediate payoff
  - Value increases with volatility of underlying asset, time to expiration
  - Current stock price and option strike price also affect value
- What is exact option value?
  - Why NPV does not work for options
  - Boundaries on option value
  - Replicating option returns
  - The Black-Scholes model
  - A general binomial model
- Goal: build background for understanding real options

The Problem of Pricing Financial Options

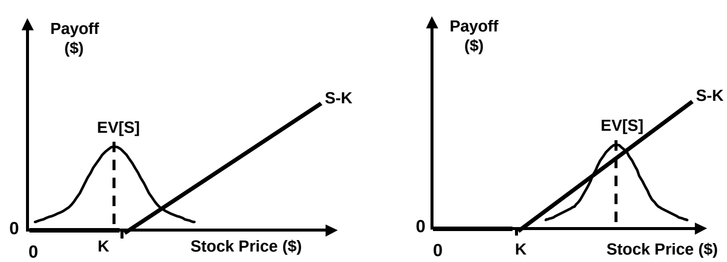
- Traditional NPV not applicable
  - NPV requires two steps
    - Estimation of cash flows
    - Discounting to present using risk-adjusted rate from CAPM

Step 1: option cash-flows (payoffs) depend on stock price
- Future stock price is uncertain
- Could describe with probability distribution

Step 2: adjusted discount rate depends on risk of option
- Option risk changes with stock price
- Stock prices change continually and unpredictably
- Cannot predict option risk over time
- No single, risk-adjusted, discount rate applies

Why Call Option Risk Changes Unpredictably

- Payout becomes more certain with increased S
  - Possibility of losing entire investment decreases
  - Decreases volatility (risk)



Risk of option changes every time stock price changes
Stock price changes continually and unpredictably

Moving Toward Options Pricing

- Need a framework other than NPV for valuing options

  Still want to account for time value of money and risk

  Begin by identifying logical constraints on price

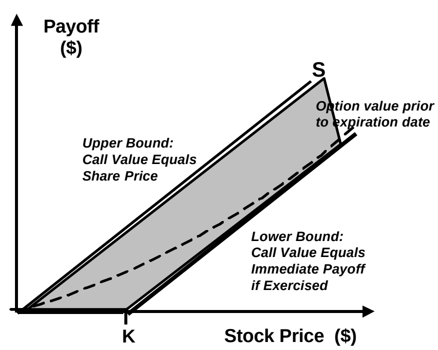
Narrowing the Scope: Boundaries on Price

- Some logical boundaries on the price of an American call

  Price ≥ 0
  Otherwise buy option immediately

  Price ≤ S
  Stock yields S*
  Option yields S*- K
  Option worth less than stock

  Price ≥ S - K
  Or buy and exercise immediately



Valuation By Comparison

- Identified several influences and boundaries of options value
- Still do not have concrete option valuation method
- One idea is to replicate options payoffs using other assets
  - If end payoffs are the same, then
  - The initial value of these assets and the option should be equal
- Key is to find replicating assets that can be valued directly

Breaking a Call Option into Separate Components

- If exercised, call option results in stock ownership
  - Option owner effectively controls shares of stock
- Payment for stock delayed until option is exercised
  - Delayed payments are essentially loans
- Call options are like buying stock with borrowed money
- Use this analogy to develop estimate of option value

A One-Period Example

- Stock
  - Current price = $100
  - Price at end of period either $80 or $125
- One-period call option
  - Strike price = $110
- Assume funds can be borrowed at risk-free rate
  - One-period risk-free rate = 10%
- Identify conditions where end-of-period payoffs are equal
  - Buying stock and borrowing money
  - Buying call options
- Then, initial values should be equal

Call Option Cost and Payoffs

- Pay C dollars to acquire option
- If S>K, call payoff = S - K
- If S<K, call payoff = 0

	Start (Stock = 100)	End (Stock = 80)	End (Stock = 125)
Buy Call Strike = 110	- C	0	(125 - 110) = 15

Stock and Loan Cost and Payoffs

- Buy stock and borrow to have payoffs look like option
- If S>K, want stock and loan payment to net to positive return
  - Can develop ratio to equalize stock and loan payments to option returns
- If S<K, want stock and loan payment to net to zero

Stock and Loan Cost and Payoffs (2)

	Start (Stock = 100)	End (Stock = 80)	End (Stock = 125)
Buy Stock	-100	80	125
Borrow Money	80/(1+r)	- 80	- 80
Net	-100 + 80/(1+r)	0	45

Comparing Costs and Payoffs

- **If S>K, stock and borrowing returns more than call**
  Ratio of returns in this case is 3:1

  If S<K, returns are equal
  Buying 3 calls should equalize payoffs

	Start (Stock = 100)	End (Stock = 80)	End (Stock = 125)
Buy Call (Strike = 110)	- C	0	(125-110) = 15

	Start (Stock = 100)	End (Stock = 80)	End (Stock = 125)
Buy Stock and Borrow	-100 + 80/(1+r)	0	45

Equalizing Costs and Payoffs

- **Equal payoffs suggest initial costs should be equal**
  - **Otherwise could buy cheaper alternative and sell more expensive result would be instant profit**

	Start (Stock = 100)	Start (Stock = 80)	End (Stock = 125)
Buy 3 Calls Strike = 110	-3C	3*0 = 0	3*(125 - 110) = 45

	Start (Stock = 100)	Start (Stock = 80)	End (Stock = 125)
Buy Stock and Borrow	100+80/(1.1)	0	45

- **3C = -100 + 80/(1.1), therefore C = $9.09**

One-Period Example Summary

- Call option payoff replicated using stock and borrowing
  - Cost of loan and price of stock are known
  - Allows value of option to be assessed
- Information needed to determine call value
  - Stock price
  - Strike price
  - Time (one-period)
  - Volatility of stock (range of final prices)
  - Interest rate also influenced option value

Options Pricing Models

- Concept of example important, must extend to be practical
  - Multiple periods
  - Dividends
- Present two options valuation frameworks
- Black-Scholes
  - Reasonably compact formula
  - Prices European calls only (assumes exercise can occur only at expiration)
  - Can be modified to include dividends
- A more general binomial model
  - Less limited in scope, possibly more difficult to apply
  - Considers exercise at any time and dividends

The Black-Scholes Options Pricing Formula

- **The value of a European call on a *non-dividend paying stock***

$$C = S * N(d_1) - K * e^{-rt} * N(d_2)$$

S = current stock price
K = striking price
r = risk-free rate of interest
t = time to expiration
σ = standard deviation of returns on stock
N(x) = standard cumulative normal distribution
$d_1 = LN [S/(K * e^{-rt})] / (\sigma * \sqrt{t}) + (\sigma * \sqrt{t})$
$d_2 = d_1 - (\sigma * \sqrt{t})$

The Black-Scholes Options Pricing Formula (2)

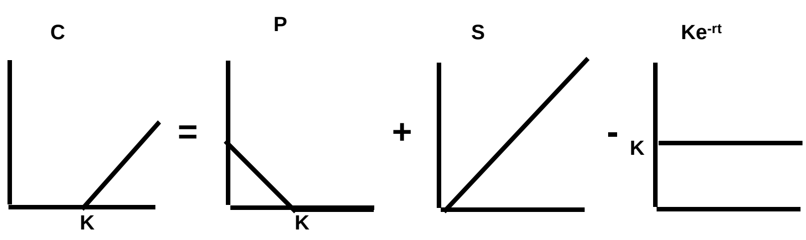
- **Note similarities to previous examples**
  - Same factors required
  - Volatility replaces stock outcomes from one-period example
  - Resembles replicating portfolio (buy stock and borrow)
- **Derivation complicated, not the focus here**

Using Black-Scholes Model

- Essentially a substitution and solve formula
  - Programmed into most financial calculators
  - Ubiquitous to Wall Street community
- S, K, t are directly stated terms of option
- r is risk-free rate of currency named in strike price
- Volatility of stock must be estimated from historical data

A Relationship Between Calls and Puts

- Put-Call parity
  - Put option value can be determined indirectly using Black-Scholes
  - For European options, on non-dividend paying stocks $C = P + S - Ke^{-rt}$



Including Dividends in Black-Scholes

- Two adjustment methods

- Assumption of constant dividend yield
  Replace S in formula with $S*(1-d)^n$
  d = constant dividend yield
  n = number of dividend periods

- Estimation of present value of dividends
  Replace S in formula with S-D
  D = present value of dividends

- Put-call parity becomes either

$$C = P + S*(1-d)^n - Ke^{-rt}$$

$$C = P + S - D - Ke^{-rt}$$

Limitations to Black-Scholes

- Black-Scholes values European options

  Most traded options are American type...
  As are most real options

  American options can be exercised at any time
  In general, early exercise is not optimal (option more valuable than payoff)
  Sometimes can be an extremely valuable feature

  Overall, a more general approach is needed

Example of Early Exercise Being Valuable

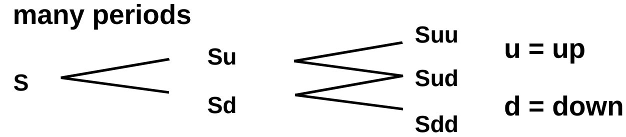
- Company unexpectedly decides to pay large dividend
  - Option lasts well beyond payment date
  - Stock will be worth much less after dividend payment, so will option
  - If option is in the money, would make sense to exercise just prior to pay-out
- Looking ahead to real options
  - Opportunity cost acts like dividends
  - Early exercise possible and likely

Origin of Black-Scholes Model

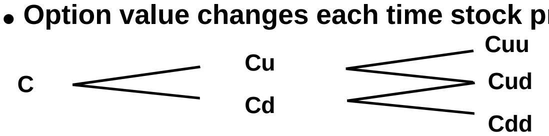
- One-period example
  - Compared end-of-period option value to stock and borrowing portfolio value
  - Equated beginning-of-period option value to initial portfolio value
- Black-Scholes model
  - Assumes many small periods
  - Represents limit as time period approaches zero
  - Calculates call option value based on statistically described stock movements
  - Assumes early exercise is not possible
- Needed for general model
  - Ability to decide to hold or exercise, at beginning of each period

A General Binomial Model for Options

- **One-period call option example**
  - Compared option value to portfolio of stock and borrowing
  - If stock price increased, call option had positive value
  - If stock price decreased, call option was worthless
- **In reality, stock price continues to change over many periods**



- **Option value changes each time stock price changes**



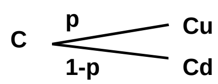
General Binomial Model Procedure

- **Assumes many periods**
- **Works backward from date of expiration**
- **For each period, applies one-period valuation methodology**
- **At each node, compares**
  - Value of option
  - Immediate exercise payoff
- **Optimal policy determined for each period and stock price**
  - Hold option for another period
  - Exercise immediately

General Binomial Model Results (Single Period)

- **Value of call if held for single period**

C = [p*Cu + (1-p)*Cd]/(1+r)



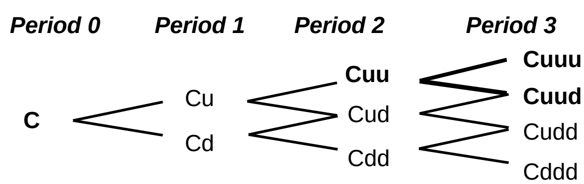
- **where, p acts as a probability**
  Cu and Cd determined by stock volatility

- **Value of option is maximum of**
  - **Immediate exercise**
  - **Holding for another period**
  - **Zero**

C = Max{S-K, [p*Cu + (1-p)*Cd]/(1+r), 0}

General Binomial Model Results (Multi-period)

- **Many periods are treated like a decision tree**



- **Work backward from last to first period to value C**

- **Apply one-period methodology at each node**
  example:
  Cuu = Max{Suu-K, [p*Cuuu + (1-p)*Cuud]/(1+r), 0}

Comments on Binomial Model

- Binomial model is a recursive technique
  - Start with end-period values and work backward to present
  - Tedious for anything other than short examples
  - Can be automated in computer programs

- Note similarity to NPV
  - Estimate cash-flows (end-of-period option value)
  - Discount to present (using risk-free rate)

C = [p*Cu + (1-p)*Cd]/(1+r)

- Why does this work?
  - Started out on premise that NPV does not work for options
  - Model adjusts cash-flows such that risk-free rate is proper discount rate
  - Probability p is the mechanism for adjusting the cash-flows

Comparing Traditional NPV to Options Valuation

- Traditional NPV procedure
  - Estimate cash flows
  - Discount at risk-adjusted rate from CAPM

- Traditional NPV does not work for options

- Option valuation handles risk-adjustment differently
  - Estimate cash-flows and adjust for risk
  - Discount at risk-free rate

- Options procedure also known as risk neutral valuation
  - Critical concept of derivatives field

Summary

- Options cannot be valued using NPV
  - Risk constantly changes
  - Proper risk-adjusted discount rate cannot be determined
- Options valuation procedures use risk-neutral valuation
  - Adjust cash flows and apply risk-free rate
  - Versus adjust discount rate and apply to cash-flows
- Black-Scholes is compact, but limited
  - Values European calls
  - Put-call parity works for valuing puts
- Binomial model more general
  - A recursive technique
  - More complicated, but can be automated

Appendix: Observed Option Price Influences

- Combined list of influences

  Underlying price (S)
  Strike price (K)
  Time to expiration (t)
  Risk-free rate of interest (r)
  Range (volatility) of stock price changes
  Dividends (D)
  American vs European options
  (ability to exercise early)

Appendix: Impact of Individual Factors on Option Value

Factor/Option Type	American Call	American Put	European Call	European Put
Underlying Price	+	-	+	-
Striking Price	-	+	-	+
Time to Expiration	+	+	?	?
Volatility of Underlying	+	+	+	+
Risk-free rate of interest	+	-	+	-
Dividends	-	+	-	+

Appendix: Rationale Behind Influence Factors Table

- **Stock price**
  - The greater the stock price (S) relative to striking price (K), the more likely a call (put) will be in (out of) the money.
- **Striking price**
  - The greater the striking price (K) relative to stock price (S), the less likely a call (put) will be in (out of) the money.
- **Time to expiration**
  - For American options, an option with a longer term to expiration is the same as an option with a shorter term, plus additional time.
  - European options cannot be exercised until the expiration date, so the extra time could cause harm relative to the shorter term option.

Appendix: Rationale Behind Influence Factors Table (2)

- **Volatility of underlying stock**
  - Since options have a zero downside and a positive upside, increased volatility increases the likelihood of finishing in the money.
- **Risk-free rate**
  - The striking price is paid or received in the future, and its present value is reduced by increased interest rates.
  - For calls, the striking price is paid in the future.
  - For puts, the striking price is received in the future
- **Dividends:**
  - Stock prices adjust downward for dividend payments. This reduces (increases) the likelihood a call (put) will finish in the money.