Real Options Analysis: Where are the Emperor's Clothes?

Adam Borison, Stanford University

05/17/03 DRAFT

Presented at Real Options Conference, Washington, DC July 2003
http://www.realoptions.org/abstracts/abstracts03.html

Abstract

In the more than 25 years since the term "real options" was coined by Stewart Myers, several approaches have been proposed for calculating the real option value of a potential corporate investment. Unfortunately, the assumptions underlying these various approaches and the conditions that are appropriate for their application are often not spelled out. Where they are spelled out or can be inferred, they differ widely from approach to approach and are even contradictory. Furthermore, the difficulties in implementing the different approaches are rarely discussed, and the pros and cons of alternative approaches are not explained.

This paper attempts to help remedy this situation by describing, contrasting, and…yes…criticizing the major proposed analytic approaches for applying real options, typically termed "real options analysis" or "real option valuation." The emphasis is on three fundamental issues surrounding each proposed approach:

1. Applicability: what does the calculated real option value represent, and where is it appropriate to use this calculation?
2. Assumptions: what are the notable assumptions underlying the approach, and what is the evidence regarding the validity of these assumptions?
3. Mechanics: what steps are involved in applying the approach, and what are the associated difficulties?

The paper concludes with observations about the relative strengths and weaknesses of the proposed approaches, and specific recommendations on which ones to use in what circumstances.

1. Introduction

The term "real options" was coined by Stewart Myers in 1977. It referred to the application of option pricing theory to the valuation of non-financial or "real" investments with learning and flexibility, such as multi-stage R&D, modular manufacturing plant expansion and the like. (Myers, 1977) The topic attracted moderate, primarily academic, interest in the 1980's and 1990's, and a number of articles were published on theory and applications.

Beginning in the mid-1990's, interest in the concepts of value and the techniques of valuation increased substantially. Real options began to attract considerable attention from industry as a potentially important tool for valuation and strategy. Beginning in the oil and gas industry and extending to a range of other industries, management consultants and internal analysts began to apply real options intermittently, and in some cases regularly, to major corporate investment issues. An annual real options symposium for both academics and practitioners was first organized in 1996, and continues to this day. Several practitioner books on the topic, many simply titled *Real Options*, have appeared, and more are in the works. Most mainstream academic finance texts now mention real options prominently. Conferences on the topic, with both academic and industry participants, are held regularly. The increasing number of academic articles on real options is now matched by an increasing number of stories in such mainstream publications as *Business Week* and *USA Today*. All in all, real options has made a transition from a topic of modest academic interest to considerable, active academic and industry attention.

Despite this transition, the state of the topic on an analytical level from a potential practitioner's point of view is problematic. There appears to be a great deal of agreement about the appeal of the underlying concepts. However, a variety of contradictory approaches have been suggested for implementing real options in practice. The assumptions underlying these different approaches and the conditions that are appropriate for their application are typically not spelled out. Where they are spelled out or can be inferred, they differ widely from approach to approach. The difficulties in implementing the approaches are rarely discussed, and the pros and cons of alternative approaches are not explained.

This situation leaves potential practitioners in troubling circumstances. In principle, the concept seems valuable and has inherent appeal. However, in practice, there is a good chance that one could apply an inappropriate approach and/or apply an approach inappropriately. The result is not simply a lack of academic correctness, but incorrect investment decisions and lost value.

The purpose of this article is to help remedy this situation by describing, contrasting, and…yes…critiquing the major proposed analytic approaches to applying real options, typically termed "real options analysis" or "real option valuation." The emphasis is not on mathematical details, but on three fundamental issues surrounding each proposed approach:

1. Applicability: what does the calculated "real option value" represent, and where is it appropriate to use this calculation?
2. Assumptions: what are the notable assumptions underlying the approach, and what is the evidence regarding the validity of these assumptions?
3. Mechanics: what steps are involved in applying the approach, and what are the associated difficulties?

It is important to point out that the task of answering these fundamental questions is made more difficult by the fact that they are rarely addressed explicitly by the proponents of each approach, so answers must often be inferred. Such inferences can be a matter of potential dispute.

A single example is used to elucidate these points and contrast the approaches. The example is very general. We consider a firm evaluating an uncertain investment. The uncertainty is perceived to come primarily from two sources. One uncertainty is the size of the business that the investment will create. Another uncertainty is the profitability of that business. Both uncertainties are dynamic, and some learning is expected to occur over time. The firm can commit to the investment immediately, reject it, or invest in an opportunity to learn (an option) before making the commit/reject decision. For simplicity, we assume that the costs associated with creating the business, specifically the cost of the investment and the option, are not a major source of uncertainty.

For the purposes of illustrating the mechanics of the approaches, we rely on an oil and gas version of this example. However, the results are not limited to oil and gas, and it would be just as easy to create a pharmaceutical, high tech or other industry version. In the oil and gas version, the investment being considered by an oil and gas firm is acquisition of an undeveloped natural gas field in the Western US. The "size of the business" uncertainty is represented by the amount of natural gas in the field. Proved reserves are currently estimated by the firm at 100 BCF. The "profitability of the business" uncertainty is represented by the future price of natural gas. The firm can acquire and develop the field for $175M, decide never to acquire and develop the field, or acquire an option to acquire and develop it in two years for $20M. For simplicity, development is assumed to take essentially zero time so revenues begin immediately after the acquisition and development decision. The right will expire if not exercised at the end of 2 years. In the 2-year period, the value of the field may change as uncertainty evolves regarding the amount and price of natural gas. Information about the amount may be obtained through geologic testing and test drilling, while information about the price may be obtained through market observation.

After describing the various approaches and illustrating them with this example, the article concludes with observations about the relative strengths and weaknesses of the proposed approaches.

2. The Classic (No Arbitrage, Market Data) Approach

The best place to start in describing real options approaches is with what can be termed the "classic" approach. In this context, the term" classic" refers to the most direct application of "classic" option pricing from finance theory to non-financial or real investments. The most extensive exposition of this approach is the work by Amram and Kulatilaka (1999), although the book is largely conceptual with relatively few details. Many others have recommended this approach at times, including Brennan and Schwartz (1985), Trigeorgis and Mason (1987), Trigeorgis (1999) and Copeland, Koller and Murrin (1994).

2.1 Applicability

Real options approaches have at their core the calculation of a value. In the case of the classic approach, this value clearly represents "financial market value" (Amram and Kulatilaka, 1999, p. 30) or "internal valuations of strategic business opportunities that are aligned with valuations in the financial markets." The calculated value represents an estimate of the incremental shareholder wealth created by an investment or, said another way, what the incremental investment would trade for in the capital markets.

When evaluating an investment from the point of view of a firm, this concept needs to be couched in terms of strategy or decisions. Although not stated explicitly, the calculated value can be viewed as a decision threshold. When acting in the interest of (potentially) diversified shareholders in the firm, the investment should be bought if available for less than the estimated value or sold if marketable for more than the estimated value. This action will increase the wealth of firm's shareholders. Taking this perspective, the classic approach is applicable to decisions about firm investments that are made from the point of view of diversified shareholders in the firm.

Amram and Kulatilaka discuss various kinds of investments, including those where flexibility and/or uncertainty has relatively little impact. They note that real options analysis is not needed in these cases. In contrast, it is needed in staged investments with considerable uncertainty and the possibility of learning. (Amram and Kulatilaka, 1999, p. 24)

2.2 Assumptions

The classic approach makes the standard replicating-portfolio assumption of financial option pricing. Specifically, this approach assumes that a portfolio of traded investments can be constructed to replicate the returns of the option in question, and therefore that the option can valued based on standard no-arbitrage arguments. As Amram and Kulatilaka state clearly "Options are valued in a transparent, no-arbitrage manner." (Amram and Kulatilaka, 1999, p. 38). Although it is not emphasized, the classic approach also generally assumes that the traded replicating portfolio behaves in standard ways. In particular, the approach assumes that asset price movements can be described by geometric Brownian motion so that standard financial tools, such as Black-Scholes, can be applied..

Amram and Kulatilaka wrote the first real options book that relied heavily on the replicating portfolio assumption. However, this assumption figures prominently in earlier articles or chapters by several other authors. For example, Brennan and Schwartz, 1985 is often mentioned as one of the first articles on real options applications. In applying real options to a capital project, these authors state explicitly (Brennan and Schwartz, 1985, p. 137) "…the cash flows from the project can then be replicated by a self-financing portfolio of riskless bills and futures contracts." Later, they repeat this point (Brennan and Schwartz, 1985, p. 154): "The explicit analysis rests on the assumption that such [replicating self-financing] portfolios may be formed by trading in futures contracts in the output commodity…"

Trigeorgis and Mason, 1987 is another often-cited article on the application of real options. In this article, an example is provided of the application of real options to valuation of a manufacturing plant (Trigeorgis and Mason, 1987, p. 15). Again, the existence and characteristics of a replicating portfolio are called out explicitly. "Following traditional practice, let S be the listed stock price of an identical plant." This view is echoed in a more recent article by Trigeorgis (1999) in a simple oil E&P example (Trigeorgis - 1999, p. 10). "Let S be the price of oil, or generally of a *twin security* that is traded in the financial markets and has the same risk characteristics as (i.e., is perfectly correlated with) the real project under consideration (such as the stock price of a similar unlevered oil company)."

The valuation book by Copeland, Koller and Murrin is widely regarded as the standard in the industry. In the 2nd edition of this book, the authors state explicitly (Copeland, Koller and Murrin, 1994, p. 453) that the real options approach is built around the idea of a replicating portfolio: "…the option-pricing approach….combines the desirable features of both the NPV and DTA [decision tree analysis] approaches. From the NPV approach, it borrows the idea that we must find a comparable (perfectly correlated) security to correctly evaluate risk, and from the DTA approach it uses decision nodes (not rigid event nodes) to model flexibility." Interestingly, these authors appear to have changed their views since that time. Their new view is discussed in Section 4 below.

Unfortunately, none of these authors presents any reasoning based on principles or any empirical evidence regarding the validity of the "replicating portfolio" argument; that is, the contention that a traded replicating portfolio of financial assets exists for a typical corporate investment in real assets. There has been considerable attention paid to co-variances between individual classic stocks and the overall market; that is, betas. However, there has been relatively little attention paid to co-variances among individual classic stocks, and even less between individual classic real and financial assets. There is little reason to believe on principle that an individual classic corporate investment, such as a specific manufacturing plant or R&D project, would be highly correlated with a particular stock. In fact, given that firms typically oversee numerous investments (in a way, that is the whole idea of a firm), the opposite is likely to be true. In the absence of arguments based on principle or evidence, this is a difficult assumption to accept.

Interestingly, the well-known corporate finance authorities Brealey and Myers explicitly say that replicating portfolio/no arbitrage arguments cannot be used to justify the application of real options. "…do the valuation techniques developed for…financial options always work for *real* options? When we introduced option pricing models…, we suggested that the trick is to construct a package of the underlying asset and a loan that would give exactly the same payoffs as the option….But many assets are not freely traded. This means that we can no longer rely on arbitrage arguments to justify the use of option models."

Furthermore, the proponents of the classic approach see fit to apply it in cases where the replicating portfolio argument appears to be decidedly weak. For example, Amram and Kulatiliaka apply the approach to an investment in a specific mill (Amram and Kulatilaka, 1999, p. 42). In this example, reference is made to a "tracking" or replicating portfolio of textile mill stocks. While there may be some relationship between the behavior of the specific mill and this portfolio, the portfolio in no way "replicates" the risk of the specific mill. The risk of the specific mill is likely to be higher than the proposed tracking portfolio. It may even be negatively correlated with that portfolio if the more successful the competition is, the lower the value of the specific mill. Another application involves an investment in a specific piece of vacant land where the proposed tracking portfolio is a basket of REIT's. Once again, there is very little reason to believe the value of a specific piece of vacant land is highly correlated with the value of a basket of REITS. In neither case is an argument in favor of the replicating portfolio concept made based on principle or evidence.

Amram and Kulatilaka acknowledge difficulty with the tracking portfolio concept, and recognize the existence of "tracking error" due in part to what is termed "private" or untracked risk. No details are provided about the importance of this tracking error, although there is one graph without a scale that that shows "market-priced risk" representing say 70% of the total change in value over a short time interval. The remaining 30% is divided into "leakages" (e.g., dividends), basis risk and private risk. The implication is that private risk represents perhaps 10% of the total risk of a typical corporate investment, and the overall discussion indicates that the authors view tracking error as relatively unimportant. The authors conclude rather vaguely; "The bottom line is that both financial and real option valuation can be less precise in practice than in theory because certain asset and market features can prevent the Law of One Price from holding. The consequences of this imprecision depend on the specifics of the firm and its industry."

Elsewhere, Amram and Kulatilaka argue for careful specification of private and market-priced risk, and note that the effect of private risk can be captured. "The effect of private risk in an option valuation model can be quantified, but it is not tracked by traded securities." (Amram and Kulatilaka, 1999, p. 54) However, no guidance is provided on how to take private risk into account and deal with tracking error. In the examples provided by the authors, all risk, including private risk, is treated in the same manner using standard option pricing (Amram and Kulatilaka, 1999, p. 152).

2.3 Mechanics

Given the strong, standard "replicating portfolio/no arbitrage" assumptions of this approach, the application is straightforward (Kulatilaka and Amram, 1999, p. 99).

1. Identify the replicating or tracking portfolio, and calculate its price and volatility
2. Size the investment relative to the replicating portfolio
3. Apply standard financial option pricing tools, particularly Black-Scholes

Kulatilaka and Amram (1999, p. 143) provide an example valuing a start-up, essentially an option on an established firm. The replicating portfolio includes an established firm in this industry with a current price and substantial historical volatility data. The investment is sized relative to this established firm based on relative sales. Then, Black-Scholes is applied. In this example, no uncertainty in the ability to create such a business or the size of the business is incorporated, and no indication of the role of private risk is provided.

In our oil and gas example, the first step is to find a replicating portfolio (or single replicating asset) that tracks the value of the Western U.S. natural gas investment in question. There are many oil and gas firms that concentrate on natural gas in the Western U.S., but none whose fortunes are tied specifically to the field in question. Some of the candidates include BR (Western US), Cabot (Western US), Chesapeake (Western natural gas), Devon Energy & Mitchell Energy (Western US), and Newfield (Western US, mostly natural gas) and XTO (Western US, mostly natural gas). After some consideration, we conclude that Newfield (NYSE: NFX) is the closest analog to the investment in question.

Newfield is currently (early May 2003) priced at 36.15 with a market cap of $1.88 billion (according to www.cbsmarketwatch.com). Short-term options (out 7 months) are available for Newfield which can be used to estimate volatility, but no long-term options are available. Historical annual volatility is in the 20-25 range, whereas implied annual volatility is in the 25-30 range (according to www.ivolatility.com). Consequently, 25 appears to be a good volatility figure.

The second step is to "size" the investment relative to Newfield. Newfield is currently estimated to have proved reserves of 1200 BCF (according to www.hoovers.com). The investment under consideration is estimated to have proved reserves of 100 BCF. Consequently, our investment can be thought of as 1/12 or 8.33% of Newfield and the investment option can be thought of as a three-year option on 8.33% of the shares of Newfield with an exercise price of $150 million.

Using Black-Scholes, the parameters are as follows:

- Current value: 8.33% of $1.88 billion or $157 million
- Time to maturity: 2 years
- Risk free interest rate: 3%
- Volatility: 25%
- Exercise price: $175 million

The resulting Black-Scholes option value is $19 million.

The interpretation of this result is straightforward. Based on the value of analogous traded assets, the investment under consideration is worth $157 million. Acquiring it for $175 million will cause a $18 million loss in value for shareholders. The option to acquire/develop the investment in two years for $175 million is worth $19 million. It's a closer call but acquiring the option for $20 million will also cause a loss in shareholder value, in this case $1 million. The appropriate value-maximizing strategy is to acquire neither the investment or the option.

The rationale behind using the $157 million and $19 million figures as guides for decision-making comes directly from the replicating portfolio/no arbitrage assumptions. In this example, the approach presumes that an exactly identical underlying investment and option …with the same financial outcomes in all states of the world…is available to the firm's shareholders at prices of $157 million and $19 million respectively. If these investments can be obtained at these prices, there is no reason to pay higher prices, in particular $175 million and $20 million, for them.

As this example illustrates, the classic approach…like classic financial option pricing…is very powerful and remarkably straightforward. Assuming the assumptions are appropriate, the main difficulties in application are 1) identifying and characterizing the replicating portfolio and 2) sizing the investment relative to that portfolio.

3. The Subjective (No Arbitrage, Subjective Data) Approach

The classic approach makes explicit use of no-arbitrage arguments, and a clearly-identified replicating portfolio to provide the inputs into standard financial option-pricing calculations. Other authors have proposed a closely-related approach that is also based on no-arbitrage arguments and the use of standard option-pricing tools from finance, but does not include the explicit identification of a replicating portfolio. Instead, this approach is based entirely subjective estimates of inputs. The best examples of this approach are the *Real Options* book by Howell et. al. and the three *Harvard Business Review* articles by Luehrman.

3.1 Applicability

In his initial article on valuation, Luehrman (1997) begins by pointing out that valuation is key to resource allocation and resource allocation is key to firm performance. While firm performance is not explicitly defined, the strong implication is that it refers to market performance; that is, return to shareholders. The real options approach, then, provides a yardstick whereby resources can be allocated to maximize shareholder value. This is a view held by most, if not all, real options practitioners.

In this article, Luehrman discusses a variety of investments and valuation approaches. He proposes that one class of investments, those he calls opportunities, should be approached using real options. These are staged investments where the initial investments typically do not provide cash flow but instead provides the right to make further investments.

3.2 Assumptions

The assumptions underlying the subjective approach are essentially the same as with the classic approach, although they are generally not mentioned as prominently. Specifically, this approach assumes the existence of a replicating portfolio and therefore the applicability of no arbitrage arguments. In addition, it assumes that geometric Brownian motion describes the dynamics of the value of this replicating portfolio.

Howell et. al. pay little attention to assumptions, and their appropriateness, until late in their book. In a section titled "Using real options when we shouldn't," they note that the two key assumptions are that "the risks of an option can be hedged away" (replicating portfolio/no arbitrage) and "the relevant uncertainties are random walks…" (geometric Brownian motion). (Howell et al., 2001, pp. 193-4). However, in the many examples in this book, no attempt is made to identify the replicating portfolio or defend the random walk assumption. Nor is there any discussion about the impact of divergence from these assumptions.

Luehrman treats assumptions similarly in the first of his 1998 articles. In referring to option value calculations, he states "…the Black-Scholes option-pricing model that generated the numbers in the table makes some simplifying assumptions of its own. They include assumptions about the form of the probability distribution that characterizes project returns. They also include assumptions about the tradability of the underlying project assets; that is, about whether those assets are regularly bought and sold. And they include assumptions about the ability of investors to continually adjust their investment portfolios. When the Black-Scholes assumptions fail to hold, this framework still yields qualitative insights but the numbers become less reliable." (Luehrman, 1998a, p. 14) As with Howell, examples are provided but no attempt is made to justify the applicability of these assumptions in the examples. And like Amram and Kulatilaka, no information is provided to quantify the degree of error introduced when the assumptions are less than perfect. .

Unlike the classic approach described in Section 2, the approach proposed by Howell and Luehrman relies on subjective assessments, as opposed to data from traded markets, for value calculations. For example, to determine the current value of the underlying asset, they recommend a discounted cash flow calculation rather than an attempt to "size" the investment relative to an equivalent traded investment. No attempt is made to justify the use of these subjective assessments as appropriate proxies for traded market values.

The combination of a reliance on the replicating portfolio assumption and the use of subjective data seems odd and inconsistent. Suppose for example that one is considering an investment that has an equivalent in the capital markets; that is, a replicating portfolio. At the extreme, this could be a share of traded stock. In this case, it seems inappropriate to rely on subjective assessments for the value of this investment. On the other hand, suppose that the investment has no replicating portfolio. In this case, the assumptions underlying this approach are violated and we are limited to "qualitative results."

3.3 Mechanics

Although the key assumptions behind the subjective approach are effectively identical to the classic approach, the mechanics are quite different. They are:

1. Subjectively estimate the price and volatility of the underlying asset
2. Apply standard financial option pricing tools, particularly Black-Scholes

In applying this approach to our oil and gas example, we make no attempt to identify a replicating portfolio. Instead, we subjectively estimate the current value and volatility of the underlying asset. Because the estimates are subjective, it is difficult to argue that a particular set of estimates is right or wrong. For this example, we adopt the view that the subjective data comes from general industry experience. Currently (May 2003), proved natural reserves in production are valued at around $2.25 per thousand CF or $2.25 million per billion CF. Consequently, the value of the investment is $225 million (excluding the cost of acquiring and developing it). The volatility of major oil and gas firms is currently around 25% (according to ivolatility.com) but is projected to increase, so we will adopt a figure of 30%.

Using these subjective estimates, the inputs to Black-Scholes are:

- Current value: $225 million
- Time to maturity: 2 years
- Risk free interest rate: 3%
- Volatility: 30%
- Exercise price: $175 million

The resulting Black-Scholes value is $71 million.

As with the classic approach, the interpretation of this result is straightforward. Based on subjective inputs, the underlying investment under consideration is worth $225 million. Acquiring it for $175 million will create $50 million in value for shareholders. On the other hand, based on the subjective inputs, the option on the investment is worth $71 million. Acquiring the real option for $20 million will create $51 million in value for shareholders. Both the investment and the option are highly valuable, but the appropriate value-maximizing strategy is to acquire the option and increase shareholder wealth by an estimated $51 million.

As with the classic approach, the rationale behind using the $225 million and $71 million figures as guides stems from the replicating portfolio/no arbitrage assumptions. However, unlike the classic approach, these estimates were not based on comparison with traded assets but instead on subjective estimates. And no guidance is provided as to how the risks of the corporate investment could be hedged.

While both the classic and subjective approaches use the same basic computational tool, the results are decidedly different. The classic approach valued the underlying investment at $157 million and the option at $19 million, while the subjective approach valued the underlying investment at $225 million and the option at $71 million. Given the simplicity of the calculation, it is quite clear that the differences due simply to fundamentally different inputs. This difference highlights an interesting conceptual difficulty with the subjective approach.

If there is in fact a replicating portfolio for the corporate investment in question, then the answer obtained using the subjective approach may diverge widely from the answer that would be obtained based on using that replicating portfolio. The subjective approach is based on subjective inputs, and does not emphasize the use of traded assets to develop inputs or even to validate subjective inputs. On the other hand, if there is no such replicating portfolio, the meaning of an option value calculated on the basis of no arbitrage using such a portfolio is unclear.

The primary difficulty associated with the classic approach is identifying the replicating portfolio and then sizing the investment with respect to that portfolio. The subjective approach eliminates this difficulty by simply skipping the replicating portfolio step. The difficulty that is introduced, assuming the underlying assumptions are valid, is developing valid, subjective estimates of the input parameters.

4. The MAD (Equilibrium-Based, Subjective Data) Approach

The classic approach is firmly connected to standard option pricing. It is based on the existence of a traded replicating portfolio, and built on data from that replicating portfolio The subjective approach takes a half-step away from standard option pricing. It is based on the existence of a traded replicating portfolio, but built on subjectively assessed data…although the use of this data is not explicitly justified.

The MAD approach extends this progression. It takes a full step away from standard option pricing, and it justifies this step explicitly. Specifically, the MAD approach does not rely on the existence of a traded replicating portfolio as with option pricing. Instead, proponents of this approach argue that the same, weaker assumptions that are used to justify the application of net present value (or discounted cash flow) to "fixed" corporate investments can be used to justify the application of option value (or real options) to flexible corporate investments. Furthermore, they argue that the same source of input data for the valuation calculations…namely subjective assessment…is appropriate.

Copeland and Antikarov's *Real Options* book (2001) provides the most complete description of this approach, and even provides a name for the key assumption: Marketed Asset Disclaimer or MAD. A similar view is taken by Trigeorgis (1999) and even by the luminaries Brealey and Myers in their classic (2000) *Principles of Corporate Finance* text.

4.1 Applicability

Copeland and Antikarov, like Amram and Kulatilaka, view shareholder value as the metric underlying real options. "We take value creation as a company's objective." (Copeland and Antikarov, 2001, p. 3) They also point out that all investors in the firm will agree on a decision rule to keep investing until marginal return no longer exceeds marginal cost of capital. (Copeland and Antikarov, 2001, p. 56) Consequently, we can interpret the output of this approach as an estimate of value created for the firm's (potentially) diversified investors and a guide for decision-making.

Copeland and Antikarov argue that NPV, the dominant approach for valuing investments, "systematically undervalues every investment opportunity" because of its failure to incorporate management flexibility. They argue that "real options will replace NPV as the central paradigm for investment decisions" within ten years. (Copeland and Antikarov, 2001, p. v and vi.) Clearly, these authors believe their approach to real options is applicable to every, or nearly every, major corporate investment decision where value maximization is the goal.

Trigeoris (1998, p. 24) takes a similar view. The goal of the firm is to maximize its market value, and consequently the wealth of its shareholders. NPV provides inadequate guidance in this effort because of its inability to value flexibility and learning appropriately. Real options analysis expands the concept of NPV to include these factors, and provides a more accurate estimate of value for virtually all corporate investments.

4.2 Assumptions

As noted earlier, the 2nd edition of the valuation text by Copeland, Koller and Murrin makes a strong case for the classic approach and the importance of finding a traded replicating portfolio. However, in the 3rd edition, the thinking of these authors has shifted. They argue instead for the MAD approach, and dismiss the need for a traded replicating portfolio. "The option pricing approach gives the correct value because it captures the value of flexibility correctly by using an arbitrage-free replicating portfolio approach. But where does one find the twin security? We can use the project itself (without flexibility) as the twin security, and use its NPV (without flexibility) as an estimate of the price it would have if it were a security traded in the open market. After all, what has better correlation with the project than the project itself? And we know that the DCF value of equities is highly correlated with their market value when optionality is not an issue. We shall use the net present value of the project's expected cash flows (without flexibility) as an estimate of the market value of the twin security. We shall call this the marketed asset disclaimer." (Copeland, Koller and Murin, 2000, p. 406) This is truly remarkable departure from reliance on capital market data.

Copeland and Antikarov reiterate this point in their real options book. "We are willing to make the assumption that the present value of the cash flows of the project without flexibility (i.e., the traditional NPV) is the best unbiased estimate of the market value of the project were it a traded asset." (Copeland and Antikarov, p. 94) Presumably, the option value based on this NPV is then the best unbiased estimate of the market value of the option were it also a traded asset.

Copeland and Antikarov justify their assumption based on comparison with NPV. They note that the use of NPV to value a corporate investment assumes traded assets of comparable risk (same beta), and they argue that "MAD makes assumptions no stronger than those used to estimate the project NPV in the first place." (Copeland and Antikarov, 2001, p. 67)

Trigeorgis and Mason, 1987, use this "MAD" argument as well. They argue that the existence of a 'twin security' is implicitly assumed in traditional NPV analysis for purposes of estimating the required rate of return on a project. That is, they argue that the assumptions underlying the application of option pricing to real assets are no stronger than the assumptions underlying the application of NPV to real assets.

Trigeorgis repeats and expands this argument in his 1998 book in a section (Trigeorgis, 1988, p. 127) titled *Justification of the Options Analogy*. "Can the standard techniques of valuing options on the basis of no-arbitrage equilibrium, using portfolios of traded securities to replicate the payoff to options, be justifiably applied to capital projects where projects may not be traded? As Mason and Merton (1985) point out, the answer is affirmative if we adopt the same assumptions used by standard DCF approaches – including NPV – which attempt to determine what an asset or project would be worth if it were to be traded. Recall that in DCF analysis we identify for each project a twin security with the same risk characteristics [note that the term "twin security" in the context of DCF typically refers to the same beta, whereas the term in the context of option pricing typically refers to the same return in all states of the world] which is traded in the financial markets, and use its equilibrium required expected rate of return – typically by estimating the project's co-variability with the market from the prices of a twin security and applying the CAPM – as the appropriate discount rate. The 'correctness' of using NPV (value maximization) rests, of course, on the assumption of market completeness."

Brealey and Myers view is similar. "When we value a real option by the risk-neutral method, we are calculating the option's value if it could be traded. This exactly parallels standard capital budgeting….a DCF calculation of project NPV is an estimate of the project's market value if the project could be set up as a mini-firm with shares traded on the stock market. The certainty equivalent (i.e., risk-neutral) value of a real option is likewise an estimate of the option's market value if it were traded." (Brealey and Myers, 2000, p. 636-637)

Following this argument, the calculated NPV of the "fixed" corporate investment is an estimate of the value that the asset would have if it were traded. The only market data used in this calculation is the risk-adjusted cost of capital or discount rate. Similarly, the calculated real option value of the "flexible" corporate investment is an estimate of the value that the flexible asset would have if it were traded.

Following these guidelines ensures that shareholder value is maximized using management's subjective inputs on an expected value basis with respect to comparable investments; that is, investments of comparable risk. However, there is an important issue here with respect to "no arbitration" conditions. The use of option pricing ensures that the "Law of One Price" is maintained internally between the asset and the option itself. However, the use of subjective data for all inputs other than the cost of capital means that arbitrage opportunities may be available between the corporate investment and traded investments if any related traded investments are available.

Copeland and Antikarov are very explicit that the data used in this approach is entirely subjective, with the exception of the discount rate. Trigeorgis is less clear, but his examples make extensive use of subjectively-assessed base values and volatilities, so one can only surmise that his approach is akin to Copeland and Antikarov. Like Amram and Kulatilaka, Trigeorgis uses real estate in one example (Trigeorgis, 1998, p. 347) Unlike Amram and Kulatilaka, he makes no effort to identify a twin security. Instead, he simply applies option pricing to the subjectively-assessed base value and volatility of the investment.

In addition to MAD, Copeland and Antikarov make a second important assumption. Namely that asset prices follow geometric Brownian motion or GBM (Copeland and Antikarov, 2001, p. 219). This provides the rationale for using binomial lattice…a random walk…for calculating value. Copeland and Antikarov build their argument on Samuelson's proof that "property anticipated prices fluctuate randomly" and therefore that change in asset value follows a random walk even if underlying processes affecting cash flows do not follow random walks (i.e., are mean-reverting, cyclical or the like). "Even the most complex set of uncertainties that my affect the cash flows of a real options project can be reduced to a single uncertainty—the variability of the value of the project through time. Samuelson's proof that properly anticipated prices fluctuate randomly implies that no matter how strange or irregular the stochastic pattern of future cash flows may be, the value (wealth relative) of the project will follow a normal random walk through time with constant volatility."

This assumption parallels the MAD assumption. The underlying asset itself is not traded. The initial NPV is an estimate of the value that the asset would have if it were traded. The GBM behavior is an estimate of the future behavior that the asset would have if it were traded. And the real option value is an estimate of the value the option would have if the underlying asset were traded and behaved this way.

Copeland and Antikarov argue (2001, p. 415) that the MAD and GBM approach allows for much greater applicability of real options. "Most asset option-pricing applications are limited to those situations where the option value depends on the market price of a world commodity, such as oil, coal, copper, nickel, gold or zinc. By using the marketed asset disclaimer and a lattice approach, however, it is possible to solve a much wider set of problems than ever before, in a way that is fairly easy to understand from a manager's point of view." He is certainly correct on this point, but the more important question is whether these wider applications are correct.

4.3 Mechanics

The steps in the MAD approach, following Copeland and Antikarov (2001, p. 248) are as follows:

1. Build a spreadsheet cash-flow model of the cash flow of the underlying asset using subjectively-estimated inputs, and calculate its NPV using a CAPM-based beta
2. Subjectively estimate the uncertainty associated with inputs to this model, and conduct a Monte Carlo simulation of the model
3. Use the resulting distribution to build a risk-neutral binominal lattice, and estimate the value using this lattice

In our example, the two uncertainties are the amount and price of natural gas. Let us assume that the firm's geologists have studied the proposed acquisition extensively. They believe the amount of gas is distributed lognormally with a mean of 145 BCF and a standard deviation of 30 BCF. The 5% value is 100 BCF (proved reserves) and the 95% value is 200 BCF. We assume that this uncertainty is effectively resolved after one year of intense geologic study and test drilling. The second uncertainty is the price of natural gas. The Henry Hub price is currently \$5.25 per thousand CF. Their economists believe that natural gas price changes are normally-distributed with mean of 2% and a standard deviation of 4%. Variable operating costs are estimated to be \$2.00 per thousand CF.

Following the procedure above, our first step is to build a spreadsheet model of the value of the underlying asset (excluding any costs for acquiring the underlying investment or the option) based on our subjective inputs. Production is assumed to go on for 10 years, with 10% of the reserves being produced each year. These amounts are discounted based a CAPM-based WACC using the average WACC for oil and gas firms (13%). Given the base assumptions, the value of the underlying asset if it were acquired and developed today is \$322 million. Assuming production starts in 2005 (if the option acquired and exercised), the value is \$268 million.

We then use Monte Carlo simulation to vary the inputs to this model, and derive information on the expected value and volatility on annual return. The net result is a mean value of 14% annual return with a standard deviation of 25% on annual return. This information is used to build a binomial tree of the value of the underlying asset (assuming production starts in 2005) over two years as shown below. The probability of up is 0.492 and the probability of down is 0.508.

		\$ 450.43
	\$ 347.30	
\$ 267.79		\$ 267.79
	\$ 206.48	
		\$ 159.21

Solving this binomial tree using a risk-neutral approach, we generate the following lattice of option values by scenario. Black indicates wait, red indicates abandon, and greenindicates exercise. The current value of the option is \$113 million.

		$ 275.43
	$ 182.72	
$ 113.17		$ 92.79
	$ 45.69	
		$ -

Based on this analysis, acquiring and developing the investment itself for $175 million will generate $147 million ($322 million less $175 million) in incremental shareholder value. Acquiring the option for potential exercise in two years for $20 million will generate $93 million ($113 million less $20 million) in incremental shareholder value. Consequently, the value maximizing decision is to make the investment.

The interpretation of the value calculations is somewhat different than in the classic and subjective approaches since we no longer make reference to a traded replicating portfolio. The $322 million figure is our estimate of the current equilibrium value of the underlying asset if traded, based on subjective inputs. The $113 million figure is our estimate of the current value of an option on this underlying asset, again based on the same subjective inputs. If the option can be acquired for $20 million, then we estimate that $93 million in shareholder value would be created in equilibrium. However, the $113 million estimate is based entirely on subjective data, such as the natural gas price forecast. Furthermore, the $93 million gain cannot be realized in a risk-free transaction involving an equivalent traded asset. Instead, the value is a forecast of what will be achieved in equilibrium over time.

The major difficulty with this approach is the considerable disconnect from the marketplace. The calculations are entirely internally consistent, but no effort is made to tie assessments to market valuations. The approach would be the same if the underlying asset were a coin flip (where clearly there is no twin security), IBM stock (where clearly there is a twin security), or an undeveloped natural gas field (where elements of the investment may have twins in the market). In this example, we see clearly that no effort was made to determine if a twin security existed or to base subjective assessments on market valuations. For example, our subjective assessment was that natural gas prices will rise at 2% per year, whereas the futures market may currently be trading natural gas futures at an entirely different value. The bottom line is that, with this approach, the underlying asset and the option are priced consistently but they may both be mispriced relative to the market. Beyond this conceptual difficulty, the major difficulty with this approach…as with the subjective approach, is developing the subjective inputs.

5. The Revised Classic (Two Investment Types) Approach

5.1 Applicability

The classic and subjective approaches are based on fairly limiting no arbitrage assumptions, but leave unsaid what to do when those assumptions do not apply. In fact, given the breadth of examples, the implication is that this is a very rare occurrence and that real options can be applied appropriately to just about any investment. The MAD approach, on the other hand, states very explicitly that the assumptions are not restrictive, and that real options can be applied to all corporate investments, irrespective of the existence of a replicating portfolio.

In contrast, the revised classic approach states explicitly that the assumptions underlying real options are restrictive. They suggest that the classic finance-based real options approach can be applied where these assumptions apply, and that management science-based approaches such as dynamic programming and decision analysis be applied where they do not. Real options should be used where investments are dominated by market-priced or public risks, and dynamic programming/decision analysis should be used where investments are dominated by corporate-specific or private risks.

In the former case, the meaning of the calculated real option value is precisely the same as in the classic case. The calculated value represents incremental shareholder value, and it can be used as an objective, no-arbitrage-based threshold for maximizing shareholder value.

In the latter case, the meaning of the calculated value is somewhat more complicated. Investments in this case are dominated by private risks, and subjective assessments are used to evaluate those risks. If the value is calculated using managerial time and risk preferences, the calculated value simply represents the investment's certain equivalent based on management's utility function. On the other hand, assuming time and risk preference are handled appropriately, the calculated value represents an estimate of incremental shareholder value using management estimates of the inputs. In practice, the choice of discount rate may introduce inaccuracy into this calculation.

This view is elaborated most extensively by Dixit and Pindyck (1994). Importantly, it is also the view taken most recently by Amram and Kulatilaka (2000), earlier strong proponents of the classic approach. Consequently, we use the term "revised classic" approach and view this as a replacement for the classic approach.

5.2 Assumptions

Dixit and Pindyck (1994) refer to option pricing with the more general term "contingent claims analysis." They state explicitly that "The use of contingent claims analysis requires one important assumption: stochastic changes in V [the value of the project in question] must be *spanned* by existing assets in the economy. Specifically, capital markets must be sufficiently "complete" so that, at least in principle, one could find an asset or construct a dynamic portfolio of assets…the price of which is perfectly correlated with V. This is equivalent to saying that markets are sufficiently complete that the firm's decisions do not affect the opportunity set available to investors. The assumption of spanning should hold for most commodities, which are typically traded on both spot and futures markets, and for manufactured goods to the extent that prices are correlated with the values of shares or portfolios. However, there may be cases in which this assumption will not hold; an example might be a project to develop a new product that is unrelated to any existing ones, or an R&D venture, the results of which may be hard to predict." (Dixit and Pindyck, 1994, p. 147) Based on this view, they propose (p. 121) the use of dynamic programming where spanning is not a reasonable assumption and contingent claims analysis where it is.

Amram and Kulatilaka (2000) make a similar argument in the context of "tracking." Real options are applied where an investment's behavior can be reasonably tracked by traded assets. Real options are relevant to only to "the subset of strategic options in which the exercise decision is largely triggered by market-priced risk." (Amram and Kulatilaka, 2000, p. 17) Decision analysis is applied where an investment cannot be tracked by market risks.

In the latter case involving decision analysis, the authors provide no guidance about a key issue. In particular, they do not indicate what discount rate is appropriate to reflect shareholder risk preference. Common practice in decision analysis is to use the weighted-average-cost-of-capital (WACC). This is designed to reflect shareholder access to investments of comparable (but not equivalent) risk; that is, essentially investments with the same beta. However, it is unclear in this context whether WACC should reflect the overall risk of the investment or just the private risk, since this is viewed as the dominating factor. Although it is not spelled-out explicitly, we have adopted the latter view.

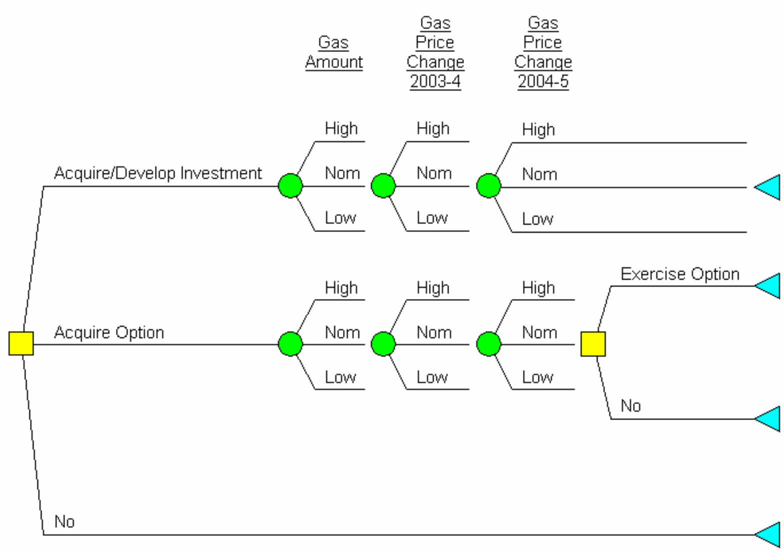
5.3 Mechanics

The revised classic approach involves the following steps:

1. Determine if the investment in question is dominated by public or private risks
2. If public risks, apply the classic approach.
3. If private risks, apply decision analysis
    a. Build a decision tree representing the investment alternatives
    b. Assign probabilities and values to the risks based on subjective judgment
    c. Apply a spreadsheet cash-flow mode at each tree endpoint, and calculate NPV using the appropriate WACC rate
    d. "Roll back" the tree to determine the optimal strategy and its associated value

Amram and Kulatilaka (2000) use pharmaceutical R&D as an example of an investment dominated primarily by private risk, namely science and technology. They use oil and gas development as an example of an investment dominated primarily by market risk, namely oil and gas prices. Although our example is also oil and gas, it has one important element in common with the pharmaceutical example – scientific or technological uncertainty. While gas price is an issue, the dominant uncertainty is really the amount of gas recovered as determined by geology, technology and the like. Based on this view, we determine that the investment is dominated by private risks and should be evaluated using decision analysis.

Following the decision analysis approach, we set up a decision tree that shows the investment decisions and the evolution of the two major uncertainties over time. The tree indicates that there are three fundamental alternatives. First, the investment can acquired now and the field developed. The realized value will depend on the gas price and gas amount uncertainty. Second, the option can be acquired. Information about gas price and amount will be obtained over a two-year period, and then a decision must be made whether to exercise the option or to acquire and develop the field. Third, the investment and option can be rejected.



The data for the two uncertainties is based on subjective assessment as in the MAD approach above. The amount of gas is assessed to be log-normally distributed with a mean of 145 BCF and a standard deviation of 30 BCF. In the tree, this uncertainty is represented by a discrete distribution of high, nominal and low states with the appropriate moments. This uncertainty is resolved in 2004. The change in the price of gas is assessed to be normally-distributed with a mean of 2% and a standard deviation of 4%. Again, this uncertainty is represented by a discrete distribution of high, nominal and low states with the appropriate moments.

As noted above, the proponents of the revised classic approach suggest the use of decision analysis for investments dominated by private risk. In this example, the dominant risk is the amount of gas in this particular field. By the definition of private risk, this risk is clearly not closely correlated with any individual investment or combination of investments in the capital market. Furthermore, it would be reasonable to conclude that this risk shows no correlation with the market as a whole; that is, it has zero beta. Consequently, for the purposes of this example, we have concluded that the risk-free rate represents the appropriate WACC.

Solving the tree, we determine that the value of the investment is $299 million net of the $175 million acquisition/development cost. The value of the option is $280 million net of the acquisition cost. The value-maximizing decision is to make the investment.

The primary, conceptual difficulty with the revised classic approach is separating all investments, in various shades of grey, into black and white, namely "all market risk" and "all private risk." Once this separation is achieved, then the difficulties faced are those of the classic approach in one case and those of standard decision analysis in the other. In the latter case, the major problem is developing calibrated subjective inputs from experts. There is the added complication of interpreting decision analysis results in shareholder value terms..

6. The Integrated (Two Risk Types) Approach

The approaches outlined above originated with practitioners in finance looking to expand to real, rather than financial, investments. The integrated approach, on the other hand, resulted primarily from practitioners in management science looking to incorporate capital market considerations, and shareholder value in particular, into their evaluation of corporate strategy.

Like several of the approaches mentioned above, the integrated approach acknowledges that there are two types of risk associated with most corporate investments: public or market and private or corporate. However, rather viewing private risk as a source of error as in the classic approach or forcing investments entirely into one category or the other, the integrated approach acknowledges that most realistic problems have both kinds of risk and is designed to address that situation. This approach was first described in depth in Smith and Nau (1995) and in Smith and McCardle (1998), although the authors specifically refer to their approach as the integration of option pricing and decision analysis not as a real options approach per se. Other authors such as Constantides (1978) and Luenberger (1998) have made similar, although less fully-developed, arguments.

6.1 Applicability

Given its origin in management science rather than finance, the integrated approach is based on a somewhat different philosophy than the other approaches. This approach recognizes that firms have a variety of stakeholders, owners and managers in particular. It presumes the owners and managers of the firm considering an investment have a single set of beliefs and preferences. It then establishes a goal of making investment (and financing) decisions to maximizing the utility of these owners and managers.

How does this goal compare to the "maximize wealth of diversified shareholders" objective noted above. "When markets are complete, investment decisions can be made solely on the basis of market information and all owners, regardless of their beliefs and preferences, will agree on appropriate project values and management strategies." (Smith and Nau, 1995, p. 812). Under these conditions, maximizing the utility of the owners and managers is the same as maximizing the wealth of the owners.

Things are somewhat more complicated when markets are incomplete because the beliefs (probability assessments) and preferences (risk attitudes) of the individual classic participants may be important. There are many ways that these beliefs and preferences can be incorporated. In practice, the integrated approach adopts a view that is completely consistent with the goal taken by the other real options approaches. In particular, the integrated approach takes as its goal the maximization of the wealth of diversified shareholders (as with the other approaches), where these shareholders agree that the probability assessment of private risks will be made by firm managers (not owners) as the agents of the owners. The risk tolerance of firm managers is ignored, and the risk neutrality of diversified owners with respect to private risks is adopted.

Essentially, firm management takes as its goal the maximization of the wealth of firm owners. Where subjective assessments are required, management supplies those assessments. Given this view, the integrated approach can be applied to evaluate any corporate investment with this goal in mind.

6.2 Assumptions

Jim Smith and Bob Nau pioneered the concept of public (or market) and private uncertainties, and were the first to fully develop an integrated approach for addressing both such uncertainties in an axiomatic fashion. "The basic idea of the integrated valuation procedure is to use option pricing methods to value risks that can be hedged by trading existing securities and decision analysis procedures to value risks that cannot be hedged by trading." (Smith and McCardle, 1998, p. 199). Their approach is to "mark to market" the portion of value of any asset driven by public risks, and to rely on judgment for the portion of value of any asset driven by private risks.

Smith and Nau explicitly lay out the assumptions underlying this approach. The fundamental assumption is that the market is "partially complete." By this, the proponents mean that it is complete with respect to the set of risks called public risks. Specifically, it is possible to hedge any public risk in any state of the world. The integrated approach makes relatively few assumptions regarding the form of these risks (e.g., geometric Brownian motion), although there are some limiting assumptions involving the relationship between public and private risks.

To implement this approach, Smith and Nau develop what may be termed a "risk-adjusted decision tree." In this tree, public and private risks are identified explicitly.

Opportunities for hedging public risks are incorporated, and opportunities for arbitrage are removed.

Luenberger develops a similar approach using binomial lattices. (Luenberger, 1998, p. 458). Much earlier, Constantinides (1978) outlined a related approach. He argued that corporate "projects" can be valued by first adjusting for market risk by adjusting the drift (essentially a risk-neutral approach) and then discounting the cash flows at a risk-free rate. This is quite similar in philosophy to the integrated approach, although Constantinides makes a variety of restrictive assumptions, such as limiting asset price movements to a Wiener process.

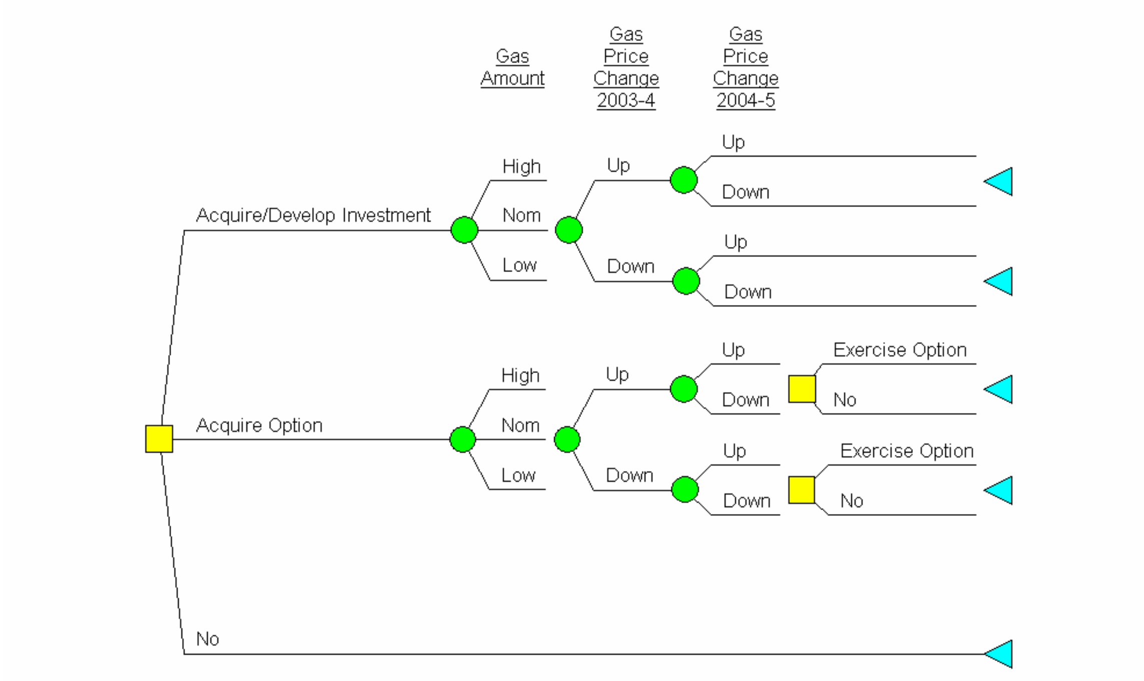
The integrated approach appears to be the only one that takes the view that corporate investments typically involve a mix of public and private risks, and that an accurate valuation depends on addressing both. It presents an approach that covers the range of corporate investments as a continuum…reducing to traditional decision analysis in one extreme (all private risks) and finance theory or option pricing in the other (all public risks). Perhaps because this approach stands so squarely in the middle of both finance theory and decision theory, it seems to be remarkably unfamiliar to either the finance or management science community.

6.3 Mechanics

The integrated approach involves the following steps:

1. As with the revised classic approach for private risks, build a decision tree representing the investment alternatives
2. Identify each risk as either public or private
3. For public risks, identify the replicating portfolio and assign "risk-neutral" probabilities
4. For private risks, assign subjective probabilities
5. Apply a spreadsheet cash-flow mode at each tree endpoint, and calculate NPV using the risk-free rate
6. "Roll back" the tree to determine the optimal strategy and its associated value

In our example, we build a decision tree similar to the one identified above in the revised classic approach. However, in the case of the integrated approach, we explicitly identify "amount of gas" as a private risk and "price of gas" as a public risk. As a private risk, the treatment of "amount of gas" remains the same as above. For the price of gas, however, we do not use subjective assessments. Instead, as a public risk, we identify the replicating portfolio and impose no-arbitrage conditions. This is reflected in the tree below where the private risk is represented as a three-branch discrete distribution and the public risk is represented as a binomial lattice with up-down movements each year.



The replicating portfolio for gas price is clearly gas price. The current price of natural gas is $5.25. There is an extensive market for gas price futures and options through 2009. Gas price options imply an annual volatility of around 19%, and previous studies have indicated a convenience yield for oil of 7%. Using this information, we develop the following binomial model of gas price changes for the 2003-4 and 2004-5 periods. The probability of up is 0.53 and the probability of down is 0.47.

		6.8229
	5.985	
5.25		4.5486
	3.99	
		3.0324

We then incorporate this binomial model of gas prices in the tree together with the three-state model of gas amount and roll the tree back at the risk free rate. The result is that the investment itself is worth $225 million (less the $175 million acquisition/development cost) and the option is worth $125 million (less the $20 million acquisition cost). Both are good investments, but the value maximizing decision is to acquire the option. The gas price risk can be effectively hedged in the gas market, leaving shareholders exposed only to the private amount of gas risk.

7. Summary of the Different Approaches

7.1 Applicability

The real options approaches described above have considerable differences. However, to a large extent, they all share a similar goal. Specifically, they are primarily intended to help firm management select investments that maximize the wealth of the firm's shareholders. They each provide a number or "real option value" that provides the buy/sell price for the investment being evaluated. If the investment is bought for less than this price, shareholder wealth is increased. If it is sold for more than this price, shareholder wealth is increased.

In some cases, the meaning of this threshold value is unambiguous. For the approaches built around the replicating portfolio concept, namely the classic and subjective approaches and the revised classic approach applied to market-driven investments, it represents the price of an exactly-equivalent asset currently available to the firm's investors. Consequently, the recommendation constitutes a risk-free increase in wealth for shareholders. In the case of the classic approach and the revised classic approach applied to market-driven investments, this risk-free increase is identified with specific traded assets. In the case of the subjective approach, such traded assets are purported to exist but they are not identified so the risk-free increase is in a sense a management estimate.

For the MAD approach and for the revised classic approach applied to privately-driven investments, the value represents management's subjective estimate of the equilibrium value of the investment and its associated option if and when traded (with some potential inaccuracy in the case of the revised classic approach due to treatment of the discount rate). For the integrated approach, it represents the combination of the market value for market-driven components of the investment and management's subjective estimate of the equilibrium market value for privately-driven components of the investment.

There are also differences in the types of investments each approach is appropriate for. The classic and subjective approaches are based on the replicating portfolio argument; consequently, they are applicable to those investments where a suitable replicating portfolio exists. (In the extreme, one may believe that all investments have such a replicating portfolio, but the point still holds.) The other approaches are more general; in principle, they are applicable to any investment.

7.2 Assumptions

Although the approaches share a common goal, they differ markedly in the assumptions involved in applying each approach. The key assumptions are in two key areas: the nature of capital markets and the source of data. These differences are summarized below.

The classic approach assumes that capital markets are complete with respect to all corporate investments; that is, that all corporate investments have equivalents in the capital markets and therefore can be effectively hedged through a traded tracking portfolio. The data for valuing the investment is market data from that traded portfolio, and the value calculated is the “no arbitrage” value of the investment. In my view, this is a consistent, but inaccurate, world-view. Consider our example. Certainly, there are major differences between the risks affecting the value of a particular undeveloped natural gas property and the risks affecting the value of Newfield, or of any individual company or combination of companies. Presumably, one major reason that corporate investments are made is that they are not duplicated in the capital markets, and it is important for a valuation process to reflect this reality.

The subjective approach also assumes that capital markets are complete. However, the data for establishing a “no arbitrage” value for the investment is drawn from subjective judgment. This eliminates the task of finding the tracking portfolio. In my opinion, this is not only an inaccurate world-view (like the classic approach) but an inconsistent one as well. Consider the following thought experiment. Suppose our example had actually involved acquiring a portion of a publicly-traded natural gas company such as Newfield. Further, suppose the name of the company had not been revealed but could be uncovered. Would it make sense to estimate the Black-Scholes parameters from general information about the investment (industry, size, etc.) or simply to find out the name of the company and use the available market data? I suggest the latter. However, this approach would have you do the former. Presumably, one should have a good sense of the investment or investments in the replicating portfolio before basing a valuation on that portfolio.

The MAD approach assumes that capital markets are absent with respect to corporate investments; that is, that corporate investments do not have equivalents in the capital markets and cannot be effectively hedged. It then calculates an estimate of the value that the investment would have if traded, and the data is drawn from subjective judgment. In my opinion, this is a consistent, but inaccurate, world-view. Consider another thought experiment. Suppose our example had not involved drilling for an unknown amount of gas but drilling for an unknown amount of IBM stock certificates. Would it make sense to estimate the value of the stock certificates by building a spreadsheet model of IBM’s business? Or would it make sense to obtain that value directly from the capital markets? I suggest the latter. However, this approach would have you do the former. Many corporate investments are driven in part by the same factors that drive the investments in capital markets, and for which there are established market valuations. If corporate investments are being evaluated from the point of view of diversified shareholders, it would presumably be important to incorporate these valuations.

The revised classic approach assumes that capital markets are partially complete with respect to investments; that is, some corporate investments have market equivalents and can be effectively hedged and others do not have market equivalents and cannot be effectively hedged. For investments in the former category, it applies the classic approach using market data. For investments in the latter category, it applies decision analysis using subjective judgment. In my opinion, this is a reasonably consistent, but crude, world-view. Once again, consider our example. The value of the investment in question was affected by both a private risk, the amount of gas in this particular field, and a market risk, the price of gas. It seems artificial to describe this investment as being dominated by either one or the other; it is simply not that black and white. And the valuation obtained from artificially placing this investment wholly in either camp would differ widely. Presumably, it would be preferable to reflect the fact that investments come in shades of gray with respect to the relative significance of market and private risks.

The integrated approach assumes that capital markets are partially complete with respect to individual uncertainties or risks; that is, some risks have market equivalents and can be effectively hedged and other risks do not have market equivalents and cannot be hedged. Each corporate investment may be affected by both types of risks. For market risks, "no arbitrage" option pricing is applied and market data is used. For private risks, decision analysis is applied and subjective judgment is used. In my opinion, this is a consistent and reasonably accurate world-view.

The table below summarizes each approach in terms of its view of capital markets and the data sources used.

Approach	Nature of Capital Markets	Data Source
Classic	Complete with respect to all corporate investments	Capital market
Subjective	Complete with respect to all corporate investments	Subjective judgment
MAD	Absent with respect to all corporate investmetns	Subjective judgment
Revised Classic	Complete with respect to market-dominated corporate investments; absent with private-dominated corporate investments	Capital market for market-dominated investments; subjective judgment for private-dominated investments
Integrated	Complete with respect to market risks of corporate investments; absent with respect to private risks of corporate investments	Capital market for market risks; subjective judgment for private risks

7.3 Mechanics

The different assumptions lead to different mechanics. The mechanics of the classic and subjective approaches are perhaps the simplest, given use of the powerful Black-Scholes algorithm. The mechanics of the classic approach are only slightly more involved than the subjective approach, since it involves the added task of actually finding the replicating portfolio. The MAD and revised classic approaches both require a moderate amount of effort in spreadsheet modeling and option pricing. The integrated approach requires perhaps the most effort, since spreadsheet modeling is required and each individual risk must be evaluated and modeled separately.

The results obtained from the different approaches following these mechanics are summarized below.

Approach	Net Value of Investment	Net Value of Option	Value-Maximizing Recommendation
Classic	(18)	(1)	Neither
Subjective	50	51	Option
MAD	147	93	Investment
Revised Classic	299	280	Investment
Integrated	50	125	Option

This table tells a very important story. Specifically, the differences discussed above as far as applicability, assumptions and mechanics are not minor. Instead, they lead to dramatic differences in valuation; that is, orders of magnitude and sign differences. And they lead to differences in strategy; that is, each of the three alternatives (investment, option, neither) is the recommended alternative for at least one approach.

8. Conclusion

The summary above illustrates what could be called the unfortunate state of confusion in the state of real options analysis. There are several different approaches. They differ in fundamental ways. And they provide fundamentally different results. In the face of such differences, it would be difficult to blame a potential practitioner who simply responded with "a pox on all your houses" and concluded that little if anything of value could be obtained from this type of analysis. However, this response would be unfortunate as well. Instead, we must critically evaluate the overall usefulness of each approach.

My opinion of each approach based on quality of results and ease of use, and my overall recommendation is summarized below. The scale is simple. Red is bad, yellow is fair, and green is good.

Approach	Quality of Results	Ease of Use	Overall
Classic	Red	Green	Red
Subjective	Red	Green	Red
MAD	Red	Yellow	Red
Revised Classic	Yellow	Yellow	Yellow
Integrated	Green	Red	Yellow

The classic, subjective, and MAD approaches all have significant problems with inaccurate and inconsistent assumptions that make them effectively unacceptable for practical use in valuation or strategy applications involving corporate investments. Their relative ease of use does not counterbalance this effect.

The revised classic approach has an overly-simplistic black-or-white world view, but may be useful when approximate results are acceptable and resources are limited. If one accepts this world-view and is willing to assign a particular corporate investment either to the "entirely market-dominated" or "entirely private-dominated" categories…and if there is insufficient time or need for more accurate results, then this approach is appropriate.

The integrated approach is based on the most accurate and consistent theoretical and empirical foundation, but requires more effort as a result. For applications where quality and credibility are important, this approach is appropriate.

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