

Utility Assessment

- **Basic Axioms**
- **Example**
- **Interview Process**
- **Procedures**
 - Traditional
 - New
- **Discussion**

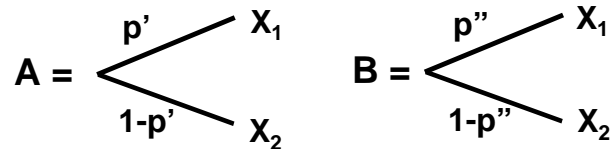
Utility Function - $U(X)$

- **Definition:**
 - $U(X)$ is a Special $V(X)$,
 - Defined in an Uncertain Environment
- **It has a Special Advantage**
 - Units of $U(X)$ DO measure relative preference
 - CAN be used in meaningful calculations

Basic Axioms of U(X) (1)

- **Probability**

- Probabilities exist - can be quantified
- More is better

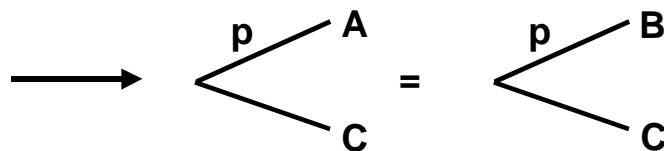


If $X_1 > X_2$; **A > B if $p' > p''$
is preferred to**

Basic Axioms of U(X) (2)

- **Preferences**

- Linear in Probability
(substitution/independence) - Equals can be substituted if a subject is indifferent between A and B



Not a good assumption for small p (high consequences) !

Cardinal Scales (1)

- **Units of interval are equal, therefore averages and arithmetic operations are meaningful**
- **Two types exist**
 - **Ratio**
Zero value implies an absence of phenomenon
e.g., Distance, Time
note: $F'(x) = a F(x)$
defines an equivalent measure (e.g., meters and feet)

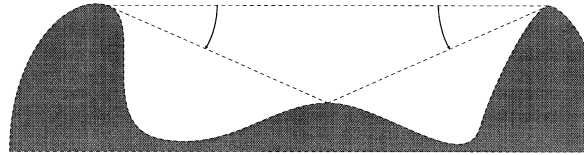
Cardinal Scales (2)

- **Ordered Metric**
Zero is relative, arbitrary for example: Temperature
- **define two points:**
 - 0 degrees C - freezing point of pure water
 - 100 degrees C - boiling point of pure water at standard temperature and pressure
 - 0 degrees F - freezing point of salt water
 - 100 degrees F - What?

Note: $f'(x) = a f(x) + b$ (e.g. $F = (9/5) C + 32$)
equivalent measures under a positive linear transformation

Consequences of Utility Axioms

- Utility exists on an ordered metric scale
- To measure, sufficient to
 - Scale 2 points arbitrarily
 - obtain relative position of others by probability weighting -- Similar to triangulation in surveying
 - For Example: Equivalent = $(X^*, p; X_*)$



How do we Measure Utility?

- Since it is empirical -- Measure
- Since it is personal -- Measure Individuals
- Solution: Some form of Interview
 - oral
 - computer based

Interview Issues (1)

- **Put person at ease**
 - this individual is expert on his values
 - his opinions are valued
 - there are no wrong answers
 - **THIS IS NOT A TEST!!**

- **Scenario relevant to**
 - person
 - issues to be evaluated

Interview Issues (2)

- **Technique for obtaining equivalents:
BRACKETING**

- **Basic element for measurement:
LOTTERIES**

Nomenclature

- **Lottery**

A risky situation with outcomes 0_j
at probability p_j

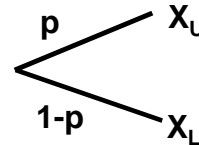
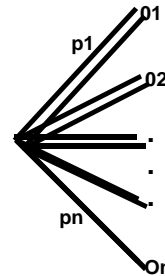
Written as $(0_1, p_1; 0_2, p_2; \dots)$

- **Binary Lottery**

A lottery with only two branches,
entirely defined by X_U, p_U, X_L

$$p(X_L) = 1 - P_U$$

Written as $(X_U, P_U; X_L)$



Nomenclature (cont'd)

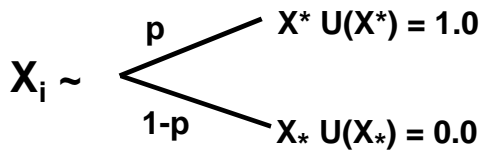
- **Elementary Lottery**

Lottery where one outcome equals zero,
that is, the status quo

written as (X,p)

Utility Measurement Traditional Method

- **Certainty Equivalent - Balance X_i and a lottery**
 - Define X^* - best possible alternative on the range
 - Define X_* - worst possible alternative on the range
 - Assign convenient values - $U(X^*) = 1.0$; $U(X_*) = 0.0$
 - Conduct data collection/interview to find X_i and p
 - Note: $U(X_i) = p$
 - Generally $p = 0.5$
 - 50:50 lotteries



- Repeat, substituting new X_i into lottery, as often as desired e.g. $X_2 = (X_1, 0.5; X)$

Utility Measurement New Method (1)

- **Avoid Certainty Equivalents to Avoid “Certainty Effect”**
- **Consider a “Lottery Equivalent”**
 - Rather than Comparing a Lottery with a Certainty
 - Reference to a Lottery is Not a Certainty



- **Vary “ P_e ” until Indifferent between Two Lotteries. This is the “Lottery Equivalent”**

Utility Measurement New Method (2)

- **Analysis** $(X^*, P_e; X_*) \sim (X_i, P; X_*)$

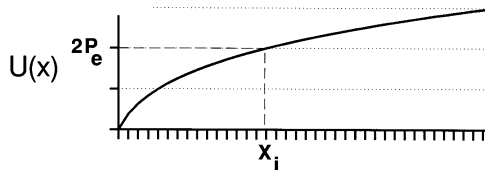
$$P_e U(X^*) + (1-P_e)U(X_*) = P U(X_i) + (1-P) U(X_*)$$

$$P_e [U(X^*) - U(X_*)] = P [U(X_i) - U(X_*)]$$

$$P_e = P \frac{U(X_i) - U(X_*)}{U(X^*) - U(X_*)}$$

$$U(X_i) = P_e/P; \text{ or } U(X_i) = 2 P_e \text{ when } P = 0.5$$

- **Graph**



- **Big Advantage - Avoids Large Errors (+/- 25% of "Certainty Equivalent" Method)**

Example of Measurement

- **Scenario**
Your rich, eccentric relative offers you X for sure or a 50:50 chance to get _____

- **Bracketing**
if X = _____
would you take it?
would someone else?

Your indifference point is

Other person's is

- **Interpretation:** 1

U(x)
0

Lotteries: Central to Utility Measurement

- **Uncertainty**
 - Basis for Assessment of Utility
 - Motivates Decision Analysis
- **Lottery - Formal Presentation of Uncertain Situation**
- **Utility Assessment -**
Compares Preference of Alternative of Known Value with Alternative of Known Value
- **How Does One Extract Utility Information from Interview Data?**
- **How Does One Construct Lottery Basis for Interview?**

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“Buying and Selling Lotteries”

- **Observable Feature of Daily Existence**
- **Obvious One Include:**
 - Buying Lottery tickets
 - Gambling; Other Games of Chance
 - Purchase of Insurance
- **Subtler Ones Are:**
 - Crossing a Street against the Lights
 - Exceeding the Speed Limit
 - Illegal Street Parking
 - Smoking; Overeating; Drug-Taking
- **Question: How to Analyze This Behavior?**

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Two Basic Lottery Transactions (1)

- **Buying of Lotteries**
 - In Absence of Transaction, Subject “Holds” an Object of Value
 - In Exchange for the Lottery, Subject Gives Up Valued Object
 - Buying “Price” Defines Net Value of Purchased Lottery

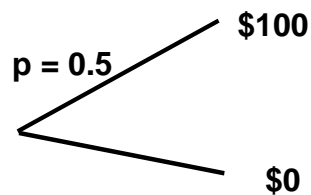
Two Basic Lottery Transactions (2)

- **Selling of Lotteries**
 - In Absence of Transaction, Subject “Holds” a Lottery
 - In Exchange for the Lottery, Subject Receives a Valued Object
 - Selling “Price” Defines Value of Sold Lottery

- **Analytically Distinct Transactions;
Must be Treated Differently**

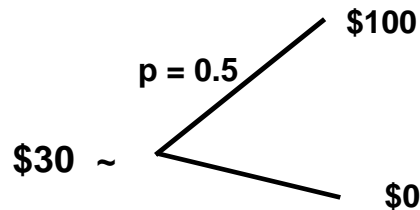
Selling Lotteries (1)

- Generally Easier to Understand
- Initially, Subject Holds a Lottery
Example, You Own a 50:50 Chance to Win \$100



Selling Lotteries (2)

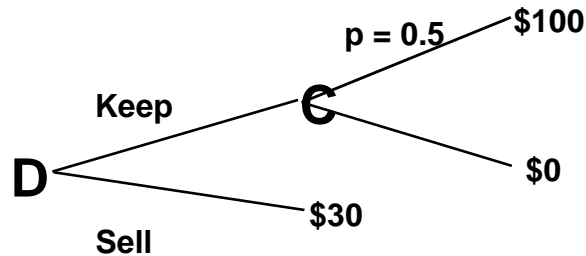
- Subject Agrees to Exchange (Sell) this Lottery for No Less Than SP = Selling Price Example: \$30



This is Called an "Indifference Statement"

Selling Lotteries - Alternative View (1)

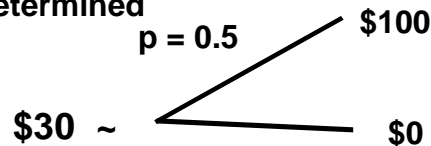
- Another way to look at lottery transactions is to express them as decision analysis situations. Selling a lottery can be represented as follows:



- When the two alternative strategies are equally valued, then we can construct an indifference statement using the two sets of outcomes.

Selling Lotteries Alternative View (2)

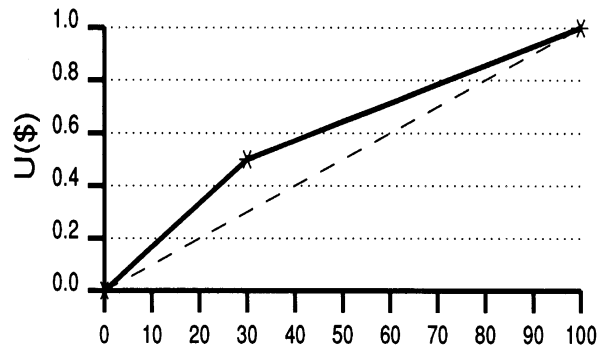
- Based on this Indifference Statement, Utility Values can be determined



- Set $U(\$0) = 0.0$ and $U(\$100) = 1.0$.
- Translate the Indifference Statement into a Utility Statement: $U(\$30) = 0.50 U(\$0) + 0.50 U(\$100)$
- Solve for $U(\$30)$

$$U(\$30) = 0.50 (0) + 0.50 U(\$100) = 0.50$$

Selling Lotteries -- Graph



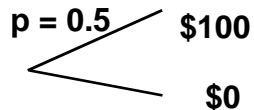
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Buying Lotteries (1)

- The “Other” Side of the Transaction
- Subtle, but Critical Analytical Difference
- Source of Difference:
Buying Price Changes Net Effect of Lottery
- Example: Look at the Buyer in the Last Example

This Lottery was Purchased for \$30

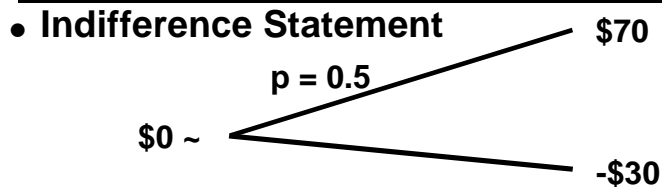


What is the Appropriate Indifference Statement?

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Buying Lotteries (2)

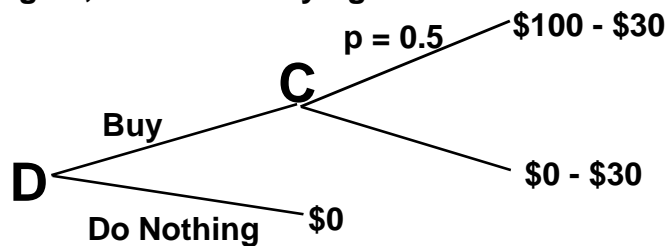


Must Explicitly Consider “Do Nothing” vs Net Outcomes

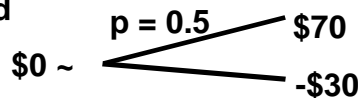
- Note:
Net Outcomes, Not Original Outcomes,
Determine Indifference Statement
 - Set $U(-\$30) = 0$; $U(\$70) = 1$
 - $U(\$0) = 0.5 U(-\$30) + 0.5 U(\$70)$
 - $U(\$0) = 0.5$

Buying Lotteries (3)

- Again, recast the buying situation as a decision tree



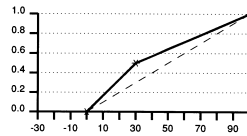
- If the buyer is just indifferent between the two decision outcomes, then the following indifference statement must hold



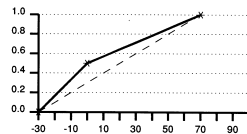
Buying Lotteries (4)

- Resulting Utility Function is Different

Seller



Buyer

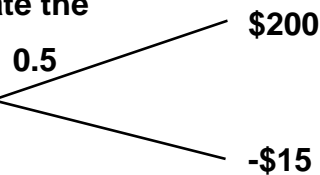


- This Should Not be Surprising. If the Utility Functions were Not Different, the Transaction would Not Have Taken Place!

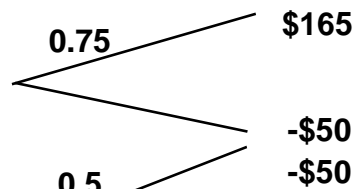
Exercises: Buying and Selling Lotteries

- Given a Transaction, Generate the Indifference Statement

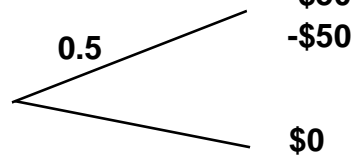
Buy this Lottery for \$35



Sell this Lottery for \$50



Pay Someone \$30 to Take This Lottery



Indifference Statements

Let

$$U(\$165) = 1$$

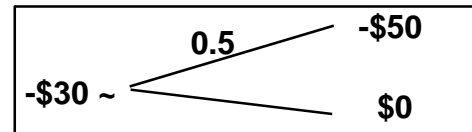
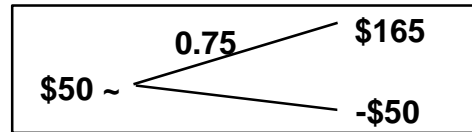
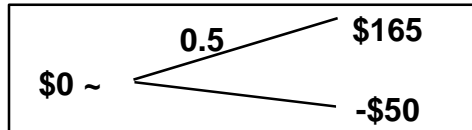
$$U(-\$50) = 0$$

Then

$$U(\$0) = 0.50$$

$$U(\$50) = 0.75$$

$$U(-\$30) = 0.25$$



Utility Result

