

## **Review of 1st half of course**

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- **A thumbnail outline of major elements**
- **Intended as a study guide**
- **Emphasis on key points to be mastered**

## **Major Elements Covered**

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- **Modeling of production possibilities**
- **Valuation Issues**
  - \* over time
  - \* evaluation criteria
- **Optimization of production and cost**
  - \* marginal analysis
  - \* constrained optimization
  - \* linear programming
- **Sensitivity Analysis**
- **Use of Excel, LP programs (not on mid-term)**

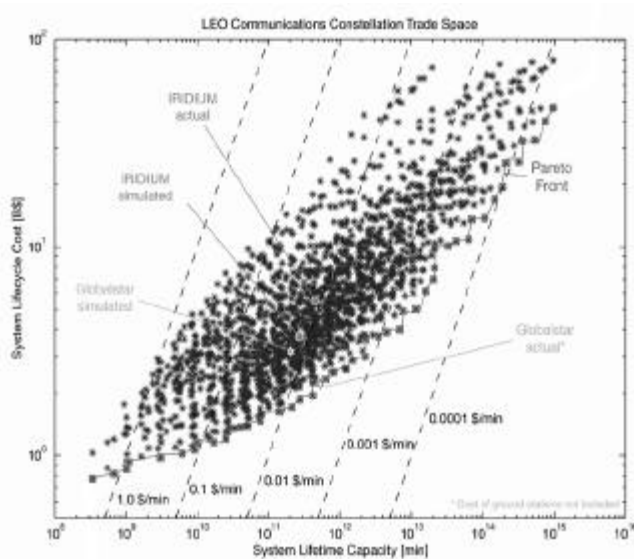
## Modeling of Production Possibilities

- **Basic Concept: Production Function**
  - \* locus of technical efficiency
  - \* defined in terms of technology only
- **Characteristics**
  - \* marginal products, marginal rates of substitution
  - \* isoquants -- loci of equal production
  - \* returns to scale ( $\neq$  economies of scale!)
  - \* convexity of feasible region? Know when!
- **Generally defined by systems models that define possibilities (eg: satellite systems)**

Engineering Systems Analysis for Design  
Massachusetts Institute of Technology

Richard de Neufville, Joel Clark, and Frank R. Field  
Marginal Analysis  
Slide 3 of 15

## Trade Space



## Valuation Issues -- over time

- **Resources have value over time**
  - \* Discount rate (DR) ,  $r$  %/period
  - \* Formulas;  $e^{rt}$  for continuous compounding
- **Choice of discount rate defined by best alternatives, at the margin**
- **DR ~ 10% or more -- long term benefits beyond 20 years have little consequence**
- **Money may change value via inflation**
- **Make sure you are comparing like with like**

## Valuation issues-- criteria

- **Many types -- none best for all cases**
  - \* Net Present value -- no measure of scale
  - \* Benefit/ Cost -- sensitive to recurring costs
  - \* Cost / Effectiveness -- no notion of value
  - \* Internal Rate of Return -- ambiguity, does not reflect actual time value of money
  - \* Pay-Back Period -- omits later returns
- **Choose according to situation (if allowed)**
- **In practice, people may use several criteria**

## Optimization -- Marginal Analysis

- **Economic efficiency merges technical opportunities (Prod. Fcn) and Values (Costs)**
- **For continuous functions, convex feasible region in domain of isoquants**
  - \* **Optimum when MP/MC ratios all equal**
  - \* **Expansion path is locus of resources that define optimal designs**
  - \* **Cost function: Cost = f(Optimum Production)**
  - \* **Economies of Scale ( $\neq$  increasing returns to scale)**
- **Good Concepts, often not applicable in detail**

## Optimization -- Dealing with Constraints

- **Equality Constraints:**
  - \* **Lagrangean Equation**
  - \* **Lagrangean multipliers = shadow prices**
- **Inequality Constraints**
  - \* **Kuhn-Tucker conditions**
  - \* **Shadow prices or slack = zero (complementary slackness)**

## Optimization -- LP

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- **Standard Form**
- **Conditions**
  - \* linearity of Objective function
  - \* additivity; non-negativity
- **Formulation issues**
  - \* Activities
  - \* Non-linearities (approximations valid only for convex feasible regions)
  - \* "fixed charge" problems
- **Solution cost = f(number of constraints)**

## Typical Formulations: "Transportation" Problem

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**Objective = Minimize cost of moving a single commodity from sources "i" to uses "j"**  
$$= \sum C_{ij} X_{ij}$$

**Subject to:**

**Amount shipped  $\leq$  Amount available  $\sum_j X_{ij} \leq S_i$**

**Amount delivered  $\geq$  Uses  $\sum_i X_{ij} \geq S_j$**

**Note: Matrix of constraint coefficients are all 0's and 1's ==> Particularly efficient solutions**

## Typical Formulations: "Blending" or "Diet" Problems

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**Objective = Minimize cost of materials**  

$$= \sum C_i X_i$$

**Subject to:**

**Limits on availability**                       $X_i \leq$  Amounts given  
**Maxima or minima on impurities, trace elements, nutritional requirements, etc...**       $\sum_i a_{ij} X_i \leq = \geq b_j$

**Example:**

–Minimize cost of steel alloy when only so much scrap is available, subject to limitations on carbon content, trace elements, etc.

## Typical Formulations: "Activity" Problems

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**Objective = Minimize cost of production**  

$$\sum C_i X$$

$X_i$  represent "activities", that is, specific ways or fixed ratios of using resources

$$\sum_i a_{ij} \leq = \geq$$

**Example**

–Minimize cost of producing a given amount of a product by using different activities, subject to limitations on the use of resources, etc.

## Sensitivity Analysis

- **Key part of analysis -- because parameters never known exactly, need to explore solution**
- **Two main concepts:**
  - \* **Shadow prices = Change in Objective Function per unit change in constraint**
  - \* **Opportunity cost = Change in OF per unit use of non-optimal decision variable**
- **Shadow prices typically given with ranges**
  - \* **Notice that SP decrease as constraints relaxed or feasible space increased (floors dropped, roofs raised)**

## Computer Programs

- **Excel -- a basic too**
  - \* **Economic valuations**
  - \* **Cost models (as in Exercise 1)**
  - \* **Sensitivity analysis (via Data Tables)**
  - \* **Basic optimization**
- **LP Suites**
  - \* **What's Best, etc**
  - \* **Sensitivity information**

**Good Luck!**

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**The teachers really hope you will do  
excellently!**

**(and make us look good!)**