# Valuation of Financial Options (1)

Richard de Neufville
Professor of Engineering Systems and of
Civil and Environmental Engineering
MIT



# Outline

- Developing an intuitive sense of value
  - Asymmetric payoffs (limited losses)
  - Value different from immediate payoff
  - Value increases with: volatility of asset, time to expiration
  - Current stock price and option strike price also affect value
- Motivating Example of Value
  - Replicating outcomes
  
  => Value Independent of Objective probabilities!!!
- Arbitrage
  - Arbitrage Enforced Pricing as key concept

# Definitions of Key Features

- S = price (of stock, commodity, etc) at any time
- S* = price at time you exercise option
- K = strike price at which item can be bought (call) or sold (put)
- t = time remaining until option expires
- ß = standard deviation of returns (volatility)
- r = risk-free rate of interest



# Financial Options: Payoff

- Payoff = amount received from exercise of option

- Call Option Payoff
- If exercised, option owner buys stock for a set price
  - Gets stock worth S* dollars
  - Pays strike price of K dollars
  - Net position = S*– K
- If unexercised, net payoff is zero

- Net Payoff of Call Option:
  - Maximum of either 0 or S*– K = net payoff for call
  - Expressed as: Max [0, S*– K]
  - This is ASYMMETRIC

# Financial Options: Value

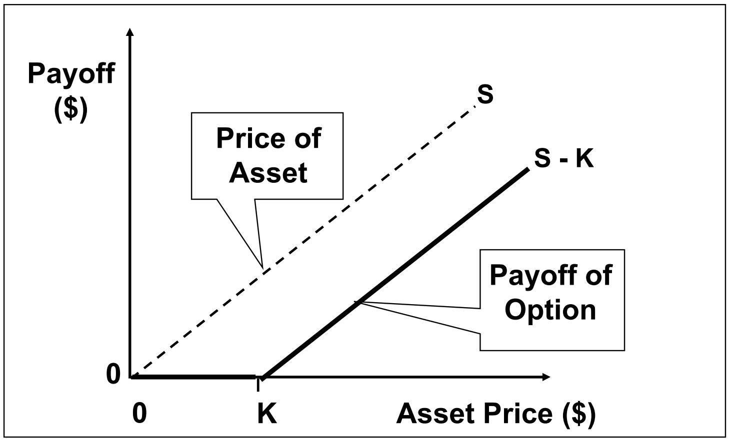
- Value often exceeds Payoff
  - Because variability of stock price can increase payoff of option
  - There is thus an expectation of greater value than immediate payoff
- Calculation of Value
  - requires sophisticated analysis
  - Determination of method for calculating value of options won Nobel Prize
  - Subject of Next Lectures



# Sources of Value in Options

- Need to build up to valuation
  - Identify interesting features
  - Examine influences of value
  - Combine findings into valuation framework
- Start by looking at payoffs from options
  - Payoff structure influences on value
  - Generally, payoff and value of options are different
- Calls and then Puts

# Payoff Diagram for Call Option






# Valuation of Options

- How much should you pay to acquire an option?
- Payoff diagrams show for a given strike price
  - Call payoff increases with asset price
  - Put payoff decreases with asset price
- Immediate payoff generally does not reflect full value of option
  - Owner exercises only when advantageous
  - Must compare immediate exercise value with waiting

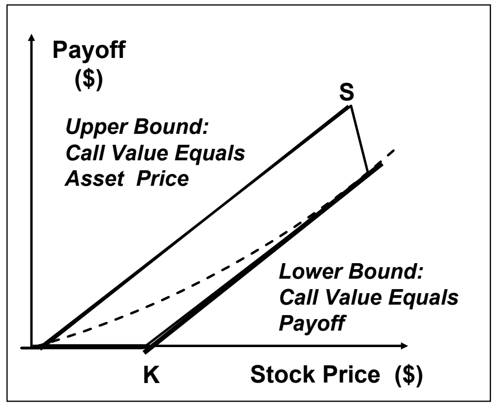
# Boundaries on Price

Some logical boundaries on the price of an American call

**Price ≥ 0**
Otherwise buy option immediately

**Price ≤ S**
Stock yields S*
Option yields S*- K
Option worth less than stock

**Price ≥ S - K**
Or buy and exercise immediately



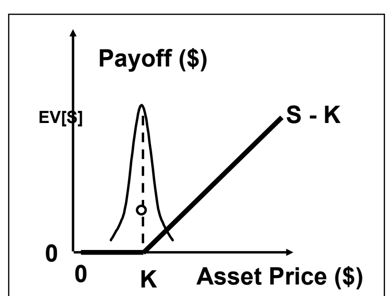



# Why Immediate Payoff and Value Differ

- Consider an option "at the money", that is, current price of asset equals strike price: (S = K)
  Immediate exercise payoff is zero

  However, <u>if you wait</u>:
  You might have higher payoff
  Worst is zero payoff
  (same as immediate exercise)

  Value of waiting not reflected in immediate exercise

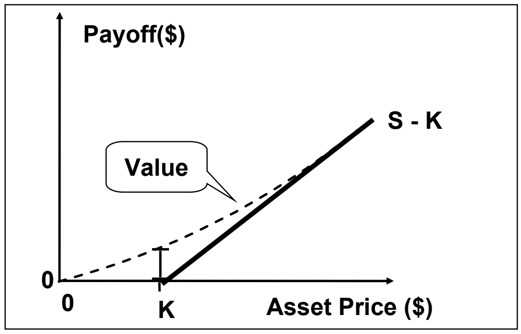



○ = value of option

# Examining Value for All Stock Prices

- Value exceeds immediate exercise payoff

- Asymptotically approaches payoff for increased S
- If asset has no expectations: value = 0 = value of option

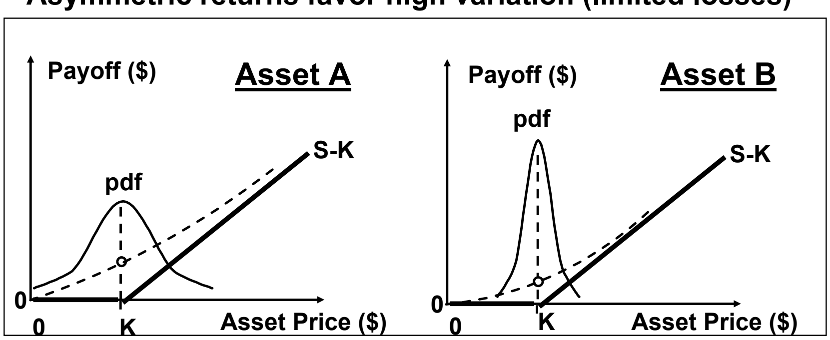





# Option Value Increases with Volatility

- Two at the money options (S=K) on different assets
  Both options have average around payoff = 0
  Asset A with greater volatility has more opportunity for large net payoffs: so it will have higher expected value

  Asymmetric returns favor high variation (limited losses)



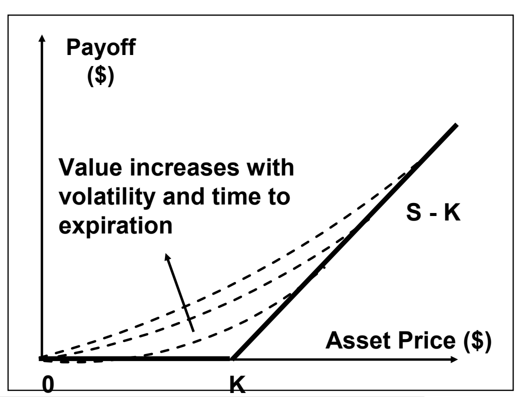
# Impact of Time

- Increasing time to expiration increases value of American option (those you can exercise any time)
  - Ability to wait allows option owner to benefit from asymmetric returns
  - Longer- term contains shorter-term options, plus more time, must be better
- Compare a 3 and 6 month American call
  - Can exercise 6 month call at same time as 3 month
  - Can wait longer with 6 month
  - Which is more valuable? Must be longer one...
- Time impact less clear for European options
  - Forced to wait to exercise
  - Could miss out on profitable opportunities



# Generalized American Call Option Value

- For a set strike price, value increases with
  - Stock price increases
  - Volatility
  - Time
- Increased strike price
  - Reduces likelihood of payoffs
  - Reduces call option value

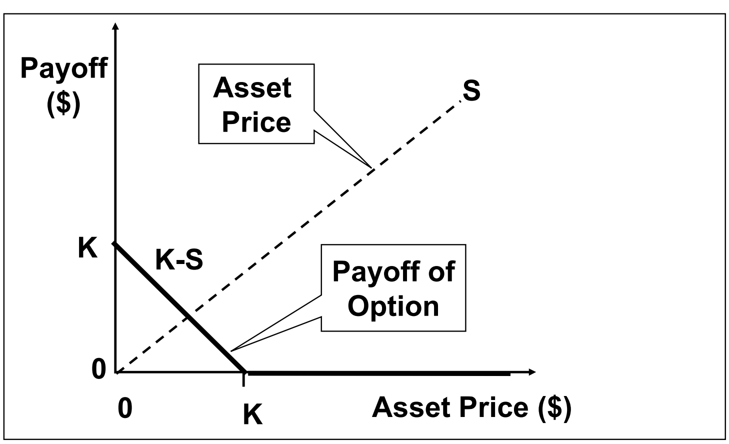


# Put Option Payoff

- Recall: Put Option gives person right to sell an asset for a predetermined "strike" price, K
  – Will do so only when value of asset, S, is less than K
- If exercised, put option owner sells asset
  – Sells asset worth S* dollars
  – Receive strike price of K dollars
  – Net position = K – S*
- If unexercised, net payoff is zero
- Net payoff for put = Max [0, K–S*]

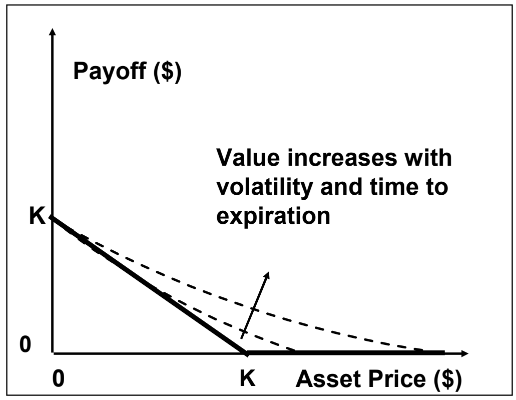


# Payoff Diagram for Put Option



# Generalized American Put Option Value

- For a set strike price, put option value increases with
  - Stock price declines
  - Volatility
  - Time
- Increased strike price
  - Increases likelihood of payoffs
  - Increases put option value






# Option Valuation: Motivating Example

- The valuation of this very simple option has fundamentally important lessons
- The key idea is that it is possible to replicate options payoffs using a portfolio of other assets
  - Since option and portfolio has same end payoffs, then
  - The value of option = value of portfolio
- Surprisingly, value of option does NOT depend on probability of payoffs!
  - Contrary to intuition associated with probabilistic nature
  - This surprising insight is basis for options analysis
- => Arbitrage enforced valuation (to be defined)

# A Simple 1-Period Option

- Asset has a Current price, S = $100
- Price at end of period either $80 or $125
- One-period call option to buy asset at Strike price, K = $110
- What is the value of this option?
- More precisely, what is the price, C, that we should pay for this option?



# Portfolio of Equal Outcomes

- The valuation is based on the idea that we can construct another asset that will have same payoffs
- This is a “replicating portfolio”
- By construction, the set of assets in portfolio will counter-balance each other to give same payoffs as the option
- Therefore: value of option = value of portfolio
- What does this portfolio look like?

## Components of Replicating Portfolio

- Think about what a call option provides: It enables owner to get increased value of asset
  - If exercised, call option results in stock ownership
- However, options provides these benefits without much money! Payment for asset delayed until option is exercised
  - Ability to delay payment is equivalent to a loan
- Therefore: A Call option is like buying stock with borrowed money
- This analogy is basis for "Replicating Portfolio"



## Call Option Cost and Payoffs

- Fair Cost of Option, C, is its value. This is what we want to determine
- If end of period asset price, S > K, strike price: payoff of the option = S - K
  If end of period asset price S < K, strike price: payoff of the option = 0

| | Start | End | End |
|---|---|---|---|
| Asset Price | 100 | 80 | 125 |
| Buy Call Strike = 110 | - C | 0 | (125 - 110) = 15 |

## Replicating Portfolio Cost and Payoffs

- Replicating Portfolio consists of:
  Asset bought at beginning of period
  Financed in part by borrowed money
- Amounts of Stock bought and money borrowed arranged so that payoffs equal those of option
- If S > K, want stock and loan payment to net to positive return
- If S < K, want stock and loan payment to net to zero
  Note: This is first crucial point of arrangement!



## Table of Portfolio Cost and Payoffs

| | Start | End | End |
|---|---|---|---|
| Asset price | 100 | 80 | 125 |
| Buy Stock | -100 | 80 | 125 |
| Borrow Money | 80/(1+r) | - 80 | - 80 |
| Net | -100 + 80/(1+r) | 0 | 45 |

**Important Issue: What discount rate, r, should be used?**

## Comparing Costs and Payoffs of Option and Replicating Portfolio

- If S < K, both payoffs = 0 and are automatically equal.
- If S > K, portfolio payoff is a multiple of call payoff (in this case, ratio is 3:1)
- Thus, payoff of this multiple of calls (3) = portfolio payoff

| Period | Start | End | End |
|---|---|---|---|
| Asset Price | 100 | 80 | 125 |
| Buy Call | - C | 0 | (125-110) = 15 |
| Buy Asset And Borrow | -100 + 80/(1 + r) | 0 | 45 |



## Implications of: Option = Portfolio

- The obvious result is that if payoffs of having Option or Portfolio are equal, their values are equal
- Most important, however, is fact that person selling option can counter- balance this with a portfolio of equal value, so seller of option cannot lose!
- Note: Since above transaction has no risk, appropriate DISCOUNT RATE = RISK FREE RATE!
- Such a no- risk situation is known as ARBITRAGE

## Arbitrage- Enforced Pricing

- The possibility of setting up a risk-free portfolio to balance option absolutely defines prices of option
- This is known as "Arbitrage - Enforced Pricing"
- If you pay C* for option, where C* > C (price defined by portfolio), than someone could sell you options and be sure to make money -- until you lower price to C.
- Conversely, if you sell option for $C_*$ < C, then someone could buy them and make money until price = C
- Thus: C is price that must prevail, as calculated using risk- free discount rate!! THIS IS CRUCIAL INSIGHT!!!



## Value of Option

- Value of Option = Value of Portfolio
- This is easy to define, using risk-free discount rate, Rf
  - Calculation below assumes Rf = 10% (for easy calculation)
- C = (1/3)[ -100 + 80/ (1 + Rf)] = $ 9.09

| Period | Start | End | End |
|---|---|---|---|
| Asset Price | 100 | 80 | 125 |
| Buy 3 Calls | - 3C | 0 | 45 |
| Buy Asset And Borrow | -100 + 80/(1 + r) | 0 | 45 |

## Value independent of actual probability!

- Note Carefully!!
- Nowhere in the calculation of the option value is there any statement about actual probability that high or low payoffs (125 or 80) would occur.
- With arbitrage-enforced pricing, actual probabilities do not matter!
- This is a remarkable result. It is counter-intuitive. Since options deal with risks, it is very surprising.
- What does matter is the RANGE of possible payoffs.



## Summary

- Value of options INCREASES WITH RISK
- Value of Options is determined by Arbitrage -- This is the fundamental observation
- Arbitrage creates portfolio to match option payoffs -- being risk- free, evaluated at risk-free rate