# Monte Carlo Simulation

Richard de Neufville
Professor of Engineering Systems
and of
Civil and Environmental Engineering
MIT



# Organization

- Presentation Based on text, Appendix D
- Assumes background of ESD 70
  - Refer to this if you did not participate
- Focuses on Sensitivity Analysis
  - As way to identify issues for Simulation
- Key Concept: Rules for exercising flexibility
- Example: Antamina Mine (other Slide show)

## What is Simulation?

- Replicates outcomes of uncertain process (often called "Monte Carlo" simulation)
  - As in "Garage case"
- It provides a way to describe what may occur over range of possible scenarios
- Can use variety of irregular distributions
  - This contrasts with alternate methods (decision tree, lattice analysis) covered later in course



## Use of Simulation is New

- Recent software makes simulation feasible
  - Simple example: Excel Add-in (see ESD 70)
  - Expensive, slick examples: Crystal Ball®; @Risk® …
- 1000's of repetitions in seconds
- Often, model of consequences simple, for example, spreadsheet modeling profits
  - Example: Garage Case
- More Complicated: See Antamina case

## Requirements for Simulation

- Distributions for Key parameters
  - May be observed, estimated, assumed, or guessed

- Examples:
  - Observed: Rainfall, river flows over years
  - Estimated: Technical Cost Models (of mine ops)
  - Assumed: Market data (historical price of metal)
  - Guessed: Judgment (ore quantity, quality)



## Recommended Process (Appendix D)

Step

1. Produce a standard valuation model
2. Perform a standard sensitivity analysis (one variable at a time)
3. Perform a probabilistic sensitivity analysis
4. Introduce distributional shapes for uncertain numbers
5. Introduce dependence between uncertain numbers
6. Introduce dynamically changing numbers
7. Model flexibility via rules for exercising flexibility

Steps 4 and 5 optional

Steps will be illustrated with reference to garage example

# Step 1: Valuation Model

| SYSTEM PARAMETERS | | | |
|---|---|---|---|
| Capacity per level | 200 | cars | |
| Number of levels | 6 | levels | [DESIGN PARAMETERS] |

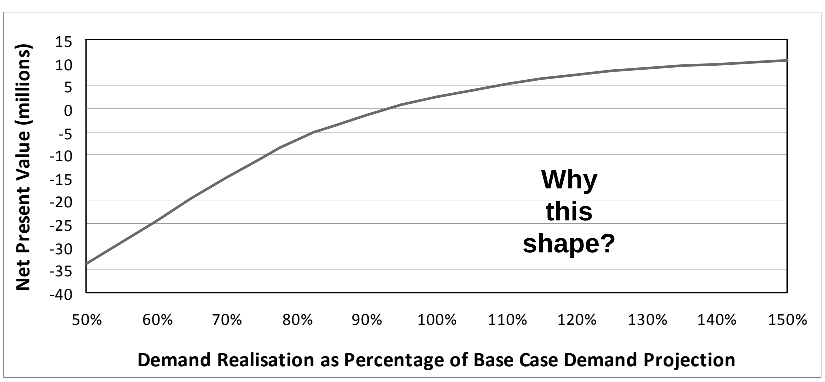
| PERFORMANCE CALCULATION | | | | | |
|---|---|---|---|---|---|
| **Year** | **0** | **1** | **2** | **…** | **15** |
| Demand | | 750 | 893 | … | 1,634 |
| Capacity | | 1,200 | 1,200 | … | 1,200 |
| Revenue | | 7.5 | 8.9 | … | 12.0 |
| Operating costs | 0.0 | 3.6 | 3.6 | … | 3.6 |
| Land leasing and fixed costs | 3.3 | 3.3 | 3.3 | … | 3.3 |
| Cashflow | -3.3 | 0.6 | 2.0 | … | 5.1 |
| Discounted cashflow | -3.3 | 0.5 | 1.7 | … | 1.2 |
| Present value of cashflow | 26.7 | | | | |
| Capacity cost for up to two levels | 6.8 | | | | |
| Capacity costs for levels above 2 | 17.4 | | | | |
| Net present value | 2.5 | | | | |

**Figure D.1**



# Step 2: Standard Sensitivity Analysis

- **One dimensional**




- **Figure D.2**

# Step 2: Tornado Diagram

| INPUT VARIABLE | Base case | %-deviation in Tornado diagram | |
|---|---|---|---|
| Demand deviation from projection | 100% | 50 | % |
| Average annual revenue | $10,000 | 20 | % |
| Average operating costs | $3,000 | 15 | % |
| Land lease and other fixed costs | $3,330,000 | 5 | % |
| Capacity cost | $13,600 | 10 | % |
| Growth of capacity cost above level 2 | 10% | 20 | % |
| Average capacity utilization | 80% | 10 | % |

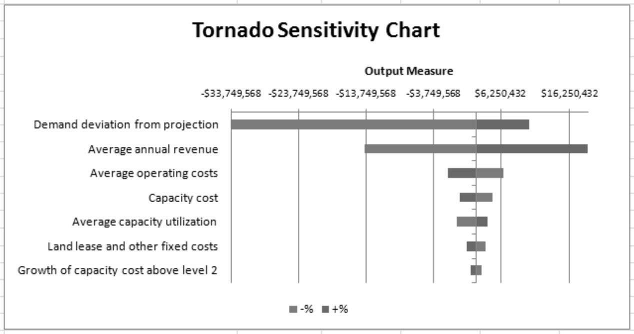



**Figure D.3**

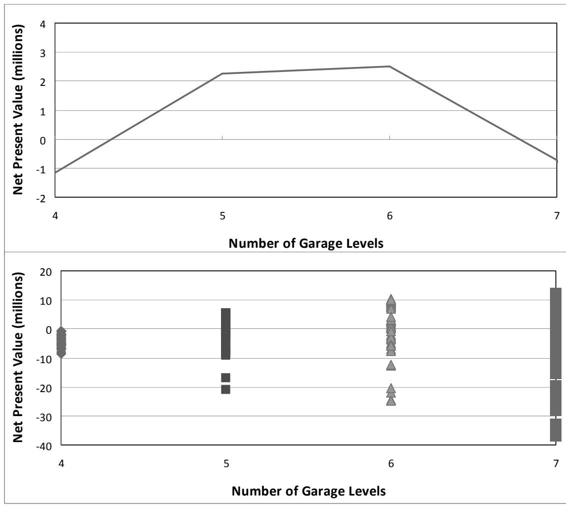


# Step 3: Probabilistic Sensitivity Analysis

| Trial | Demand deviation | Average annual revenue | Average operating costs | Land lease and other fixed costs | Capacity cost | Growth of cap cost above 2 levels | Average capacity utilization | NPV |
|---|---|---|---|---|---|---|---|---|
| 1 | 134% | $11,016 | $3,140 | $3,495,435 | $12,399 | 9.88% | 79.69% | $17,625,663 |
| 2 | 141% | $11,852 | $3,260 | $3,304,375 | $13,795 | 9.84% | 79.06% | $23,221,598 |
| 3 | 86% | $10,193 | $3,312 | $3,430,938 | $13,068 | 10.43% | 79.50% | -$4,718,694 |
| 4 | 83% | $9,055 | $3,283 | $3,354,325 | $14,036 | 9.90% | 76.09% | -$15,456,972 |
| 5 | 83% | $10,114 | $3,047 | $3,316,798 | $12,685 | 11.52% | 87.40% | -$3,934,426 |
| 6 | 62% | $8,111 | $3,283 | $3,206,261 | $13,100 | 11.02% | 84.27% | -$36,456,858 |
| 7 | 76% | $10,515 | $2,647 | $3,386,862 | $14,260 | 11.81% | 83.77% | -$6,799,900 |
| 8 | 68% | $8,343 | $3,440 | $3,269,874 | $14,830 | 9.71% | 78.82% | -$32,839,236 |
| 9 | 114% | $10,272 | $3,291 | $3,410,315 | $14,852 | 10.11% | 85.44% | $5,120,553 |
| 10 | 68% | $10,151 | $3,413 | $3,437,877 | $14,110 | 10.78% | 85.28% | -$24,320,813 |
| 11 | 68% | $9,664 | $3,153 | $3,163,520 | $13,841 | 11.51% | 82.62% | -$21,314,749 |
| 12 | 129% | $11,189 | $2,636 | $3,430,040 | $14,713 | 10.59% | 75.51% | $16,118,317 |
| 13 | 132% | $11,832 | $2,695 | $3,179,077 | $14,469 | 11.09% | 84.10% | $30,373,102 |
| 14 | 114% | $10,250 | $2,586 | $3,189,684 | $14,311 | 9.34% | 75.61% | $9,916,978 |
| 15 | 105% | $9,754 | $2,641 | $3,218,399 | $14,147 | 9.14% | 72.32% | $2,261,573 |
| 16 | 63% | $9,595 | $3,089 | $3,240,407 | $13,222 | 9.92% | 77.66% | -$22,983,479 |
| 17 | 147% | $10,881 | $2,702 | $3,374,710 | $13,140 | 10.25% | 79.15% | $20,546,268 |
| 18 | 91% | $8,783 | $3,275 | $3,222,284 | $12,731 | 9.97% | 86.09% | -$10,350,864 |
| 19 | 67% | $9,320 | $3,182 | $3,477,948 | $14,446 | 9.57% | 81.29% | -$26,886,841 |
| 20 | 67% | $8,961 | $3,302 | $3,206,613 | $14,770 | 9.87% | 75.37% | -$26,433,140 |

**Figure D.4 – variables moving together, not one at a time**

# Optimization in Context of Uncertainty




**Figures D.8 and D.9**

**Top is Expected NPV, Bottom Graph shows range of outcomes.**

**Notice the difference in scale between the two graphs!**

**In this case, the uncertainties may dominate the choice.**



# Step 4: SA for Shape of Distribution

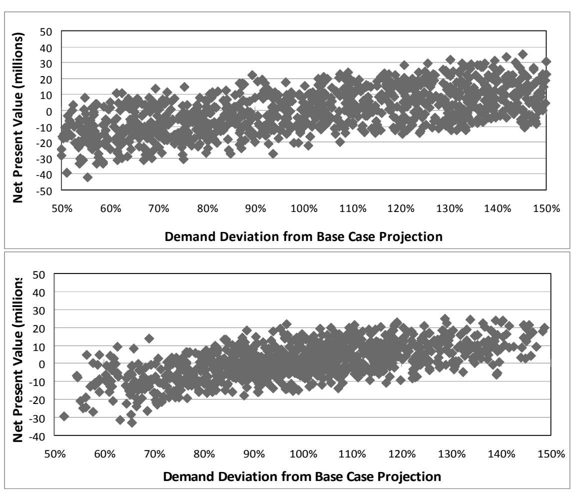



**Figure D.13**

**Top: uniform distribution**

**Bottom: triangular distribution**

## Shape of output distributions

- Traditional (NPV) analysis is:
  "numbers in, numbers out"
- Uncertainty analysis is:
  "shapes in, shapes out"
- Useful representation of "shape out" is the cumulative distribution, the "target curve"
- (also known as the VARG –- value at risk and gain curve)



## Target Curve

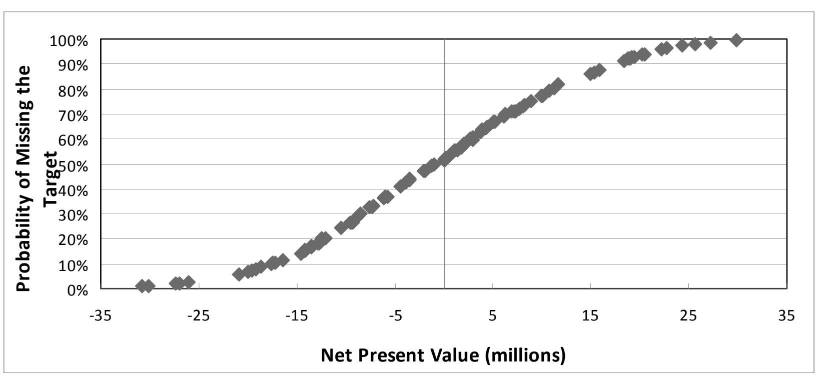



Figure D.14

## Step 6: Introduce dynamic scenarios

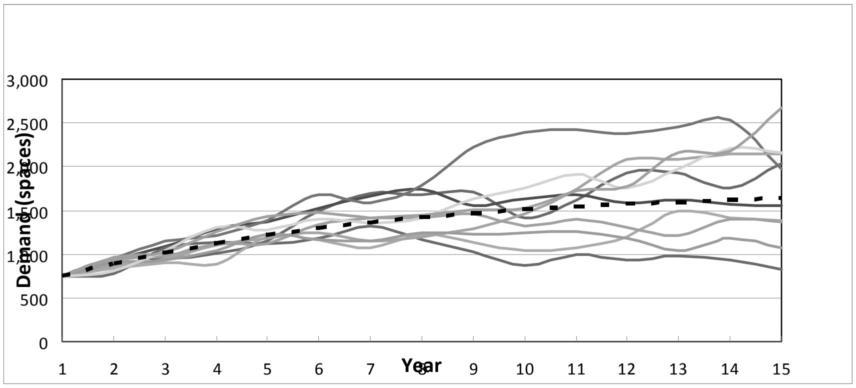



**Figure D.18**



## Step 7: Rule for exercising flexibility

- When should we exercise flexibility?
- In simulation, this time cannot be calculated
- Why?
- Because number of possible future paths, states are too large to be searched
- Procedure: set up a priori conditions for when to exercise flexibility
- These known as "rules to exercise flexibility"

## Example of Rule for exercising flexibility

**Consider Parking Garage**

- **"Expand if, over 2 years, observed demand > capacity "**
- **Why would this make sense?**

  **=> Because, want some assurance that growth is 'permanent'**
- **How could this be improved?**

  **=> Change rule toward end of life? No addition in last 5 years?**



## Creation of Target Curve

- **At end of Simulation, we have many trials**
- **What is probability of each?**
- **They are equal**
- **How do we get pdf?**
- **"Binning" the outcomes by ranges (= bins)**
  - **Percent of samples in a bin = P(outcome in bin)**
- **Note: We look at all simulations!**
  - **But binning process easily automated (see ESD 70)**

## Typical results

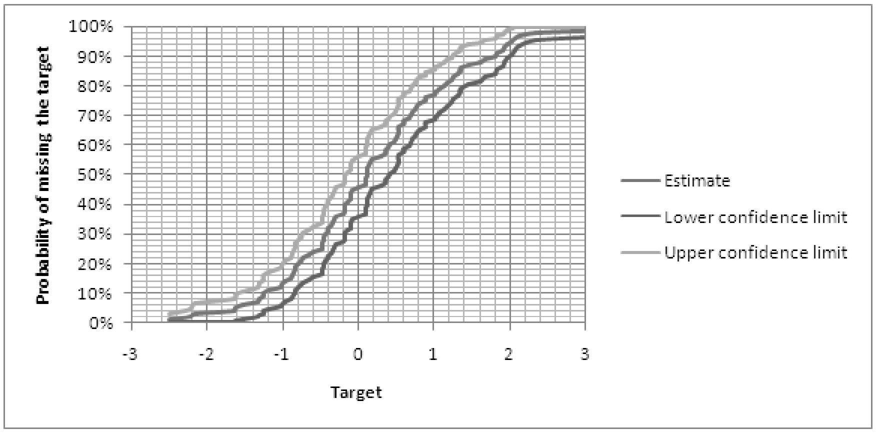



**Figure D.19**



## Value of Flexibility by Simulation

- **Step 1: Get distribution of consequences for plan or design without flexibility**
  **=> NPV, EV(NPV); also Target Curve**
- **Step 2: Repeat above, but considering availability of flexibility, and its exercise at desired times**
  **=> new NPV pdf, EV(NPV); Target Curve**
- **Step 3: Value of Flexibiity is difference ; comparing Target Curves shows source of value**

## Take-Aways

- Simulation useful to represent pdfs of outcomes (target curves) define value of flexibility
  - Computationally efficient
- Can deal with all kinds of uncertainties
- Relatively easy to explain to decision-makers
  - No complicated math or
  - No confusing decision trees (see later in course)
  - No high-powered theory (see later in course)

CAN BE A VERY GOOD APPROACH